Math 164 (Spring 2008)

Solutions for Homework #7

<u>Section 5.3</u>: 11, 13, 15, 17, 19, 23, 27

#11 $\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{32}=0.03125$

#13 $P($"Both are left-handed"$)=(0.13)(0.13)=0.0169$

$P($"At least one is right-handed"$)$

$=1-P($"Neither is right-handed"$)$

$=1-P($"Both are left-handed"$)=1-(0.13)(0.13)=1-0.0169=0.9831$

#15 Note that the ELISA is given to five people who <u>do not</u> have the HIV antibody. For any of such an individual, the probability that the ELISA comes back negative is 0.995.

(a) $P($"All are negative"$)$

$=P(negative)\cdot P(negative)\cdot P(negative)\cdot P(negative)\cdot P(negative)$

$=(0.995)(0.995)(0.995)(0.995)(0.995)\approx 0.9752$

(b) $P($"at least one positive"$)=1-P($"all negative"$)$

$=1-(0.995)(0.995)(0.995)(0.995)(0.995)\approx 1-0.9752=0.0248$

#17

(a) $(0.99718)(0.99718)\approx 0.99437$

(b) Similarly, we have $(0.99718)^5\approx 0.98598$

(c) $P($"At least one will not live to be 41 years old."$)$

$=1-P($"All live to be 41 years old"$)$

$=1-(0.99718)^5\approx 1-0.98598=0.01402$

Since this is lower than the usual cutoff of 0.05, we conclude that it is unusual that at least one of five randomly selected 40-year-old males will not live to be 41 years old.

#19

(a) $(0.99)(0.99)=0.9801$

(b) $(0.99)^6\approx 0.9415$

(c) $P($"At least one has RH-."$)$

$=1-P($"None have RH-"$)$

$=1-P($"All six are RH+"$)$

$=1-(0.99)^6\approx 1-0.9415=0.0585$

#23 $P($"At least one detects an incoming ballistic missile"$)$

$=1-P($"None detects an incoming ballistic missile"$)$

$=1-P($"All four fail to detect an incoming ballistic missile"$)$

$=1-(0.1)^4=1-0.0001=0.9999$

This means that the probability that an incoming ballistic missile manages to elude the defense system is 0.0001. Though this is lower than the usual cutoff of 0.05, one may nevertheless feel that 0.0001 is not low enough in such life-or-death situation.

#27

(a) $(0.29)(0.29)(0.29)=0.024389\approx 0.0244$

(b) $P($"At least one has not driven while under the influence of alcohol"$)$

$=P($"At least one is behaving himself"$)$

$=1-P($"All are driving while under the influence of alcohol"$)$

$=1-(0.29)^3=1-0.024389=0.97561\approx 0.9756$

(c) $P($ "None has driven while under the influence of alcohol" $)$
$= P($ "All three are behaving themselves" $)$
$= (0.71)(0.71)(0.71) \approx 0.3579$

(d) $P($ "At least one has driven while under the influence of alcohol" $)$
$= 1 - P($ "None has driven while under the influence of alcohol" $)$
$= 1 - P($ "All are behaving themselves" $)$
$= 1 - (0.71)(0.71)(0.71) \approx 1 - 0.3579 = 0.6421$

<u>Section 5.4</u>: 3, 4, 5, 6, 7, 8, 9, 11, 13, ~~15~~, 17, 21, 37 ← Disregard #15.

#3 $P(F \mid E) = \dfrac{P(E \text{ and } F)}{P(E)} = \dfrac{0.6}{0.8} = 0.75$

#4 $P(F \mid E) = \dfrac{P(E \text{ and } F)}{P(E)} = \dfrac{0.21}{0.4} = 0.525$

#5 $P(F \mid E) = \dfrac{N(E \text{ and } F)}{N(E)} = \dfrac{420}{740} \approx 0.568$

#6 $P(F \mid E) = \dfrac{N(E \text{ and } F)}{N(E)} = \dfrac{380}{925} \approx 0.411$

#7 $P(E \text{ and } F) = P(E)\,P(F \mid E) = (0.8)(0.4) = 0.32$

#8 $P(E \text{ and } F) = P(E)\,P(F \mid E) = (0.4)(0.6) = 0.24$

#9 $P($ "Earn more than \$75,000 per year" $) = 0.184$

$P($ "Earn more than \$75,000 per year" $\mid$ "Earned a bachelor's degree" $) = 0.350$

There two are not equal, which means that the event "Earn more than $75,000 per year" and the event "Earned a bachelor's degree" are not independent of each other.

#11 $P(club) = \dfrac{N(club)}{N(S)} = \dfrac{13}{52} = \dfrac{1}{4}$

$P(club \mid black) = \dfrac{N(club \text{ and } black)}{N(black)} = \dfrac{13}{26} = \dfrac{1}{2}$ (Note: All clubs are black.)

#13 $P(rainy \mid cloudy) = \dfrac{P(rainy \text{ and } cloudy)}{P(cloudy)} = \dfrac{0.21}{0.37} \approx 0.568$

#15 (The problem was erroneous. You were told to disregard this problem.)

#17

(a) $P(\textit{has no health insurance} \mid \textit{less than } 18 \textit{ years})$

$$= \frac{N(\text{"}\textit{has no health insurance}\text{" AND "}\textit{less than } 18 \textit{ years}\text{"})}{N(\textit{less than } 18 \textit{ years})}$$

$$= \frac{8531}{49473 + 19662 + 8531} = \frac{8531}{77666} \approx 0.110$$

(b) $P(\textit{less than } 18 \textit{ years} \mid \textit{has no health insurance})$

$$= \frac{N(\text{"}\textit{less than } 18 \textit{ years}\text{" AND "}\textit{has no health insurance}\text{"})}{N(\textit{has no health insurance})}$$

$$= \frac{8531}{8531 + 25678 + 9106 + 258} = \frac{8531}{43573} \approx 0.196$$

#21

$P(\text{"Both work"}) = P(\text{"1}^{st}\text{ works" AND "2}^{nd}\text{ works"})$

$= P(\text{"1}^{st}\text{ works"})\, P(\text{"2}^{nd}\text{ works"} \mid \text{"1}^{st}\text{ works"}) = \frac{4}{6}\cdot\frac{3}{5} = \frac{2}{5} = 0.4$

Then

$P(\text{"At least one does not work"}) = 1 - P(\text{"Both work"}) = 1 - 0.4 = 0.6$

#37

	<18	**18--44**	**45--64**	**>64**	
Private	49,473	76,294	52,520	20,685	198,972
Governmen	19,662	11,922	9,227	32,813	73,624
None	8,531	25,678	9,106	258	43,573
	77,666	113,894	70,853	53,756	316,169

$P(\textit{less than } 18 \textit{ years}) = \frac{77668}{316169} \approx 0.246$

whereas in #17(b) we found that

$P(\textit{less than } 18 \textit{ years} \mid \textit{has no health insurance}) \approx 0.196$

This shows that the event "less than 18 years old" and the event "has no insurance" are not independent of each other.

<u>Section 5.5</u>: 5, 6, 9, 11, 13, 15, 17, 19, 21, 23, 25 ← Hand computation! Show work.
31, 33, 35, 39, 47, 53, 55, 59, 63, 69

#5 $5! = 5\times4\times3\times2\times1 = 120$

#6 $7! = 7\times6\times5\times4\times3\times2\times1 = 5040$

#9 $0! = 1$ (This is a matter of definition.)

#11 ${}_6P_2 = 6\times5 = 30$

#13 ${}_4P_4 = 4\times3\times2\times1 = 24$

#15 ${}_5P_0 = 1$

#17 ${}_8P_3 = 8\times7\times6 = 336$

#19 ${}_8C_3 = \frac{8\times7\times6}{1\times2\times3} = 56$

#21 ${}_{10}C_2 = \frac{10\times9}{1\times2} = 45$

#23 ${}_{52}C_1 = \frac{52}{1} = 52$

#25 ${}_{48}C_3 = \frac{48\times47\times46}{1\times2\times3} = 17296$

#31 $6\times4 = 24$

#33 $12! = 479{,}001{,}600$

#35 $8! = 40{,}320$

#39 (a) $10\times10\times10\times10 = 10{,}000$ (b) $\frac{1}{10{,}000} = 0.0001$

#47 $20\times19\times18\times17 = 116{,}280$

#53 This is the same as having six boxes in a row, and you want to choose exactly 2 boxes to mark "B" (with the remaining therefore marked "G"). The answer is therefore

$$_6C_2 = \frac{6\times 5}{1\times 2} = 15$$

#55 There are 1 A, 1 C, 2 I's, 3 S's, 3 T's. Then answer is

$$\frac{(1+1+2+3+3)!}{(1!)(1!)(2!)(3!)(3!)} = \frac{10!}{(1!)(1!)(2!)(3!)(3!)} = 50{,}400$$

#59 The number of ways in which you can grab a handful of 5 balls from 30 balls is $_{30}C_5 = \frac{30\times 29\times 28\times 27\times 26}{1\times 2\times 3\times 4\times 5} = 142{,}506$. Thus the probability of grabbing the right handful in order to win is $\frac{1}{142{,}506}$

#63

$$\begin{aligned}
&P(\textit{reject})\\
&= P(\textit{at least one is defective})\\
&= 1 - P(\textit{none is defective}) = 1 - P(\textit{all four are fine})\\
&= 1 - \left(\frac{116}{120}\right)\left(\frac{115}{119}\right)\left(\frac{114}{118}\right)\left(\frac{113}{117}\right) = 1 - \frac{171{,}845{,}880}{197{,}149{,}680} \approx 1 - 0.871651833 = 0.128348167\\
&\approx 0.1283
\end{aligned}$$

#69 $P(\textit{accept}) = P(\textit{All four are fine}) = \left(\frac{17}{20}\right)\left(\frac{16}{19}\right)\left(\frac{15}{18}\right)\left(\frac{14}{17}\right) = \frac{57{,}120}{116{,}280} \approx 0.4912$