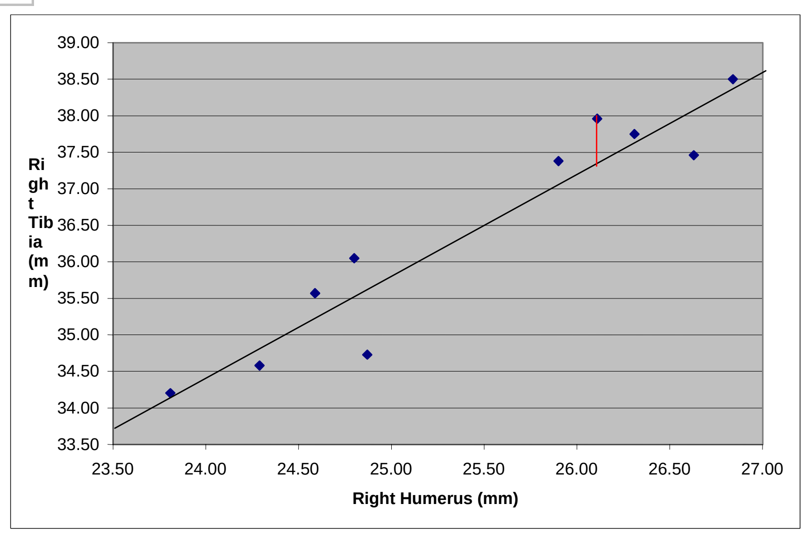
Math 164 (Spring 2008)

Solutions for Homework #5

1.

Right Humerus (mm) x	Right Tibia (mm) y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	s_x	s_y	$\frac{x-\bar{x}}{s_x}$	$\frac{y-\bar{y}}{s_y}$	$\left(\frac{x-\bar{x}}{s_x}\right)\left(\frac{y-\bar{y}}{s_y}\right)$
24.80	36.05	-0.5400000	-0.2909091	0.2916000000	0.0846280992	1.0412685	1.5216205	-0.5185982503	-0.1911837293	**0.0991475475**
24.59	35.57	-0.7500000	-0.7709091	0.5625000000	0.5943008264	1.0412685	1.5216205	-0.7202753476	-0.5066368827	**0.3649180568**
24.59	35.57	-0.7500000	-0.7709091	0.5625000000	0.5943008264	1.0412685	1.5216205	-0.7202753476	-0.5066368827	**0.3649180568**
24.29	34.58	-1.0500000	-1.7609091	1.1025000000	3.1008008264	1.0412685	1.5216205	-1.0083854867	-1.1572590116	**1.1669631916**
23.81	34.20	-1.5300000	-2.1409091	2.3409000000	4.5834917355	1.0412685	1.5216205	-1.4693617092	-1.4069927580	**2.0673812837**
24.87	34.73	-0.4700000	-1.6109091	0.2209000000	2.5950280992	1.0412685	1.5216205	-0.4513725512	-1.0586799011	**0.4778590479**
25.90	37.38	0.5600000	1.0390909	0.3136000000	1.0797099174	1.0412685	1.5216205	0.5378055929	0.6828843832	**0.3672590406**
26.11	37.96	0.7700000	1.6190909	0.5929000000	2.6214553719	1.0412685	1.5216205	0.7394826902	1.0640569435	**0.7868516912**
26.63	37.46	1.2900000	1.1190909	1.6641000000	1.2523644628	1.0412685	1.5216205	1.2388735979	0.7354599087	**0.9111418633**
26.31	37.75	0.9700000	1.4090909	0.9409000000	1.9855371901	1.0412685	1.5216205	0.9315561163	0.9260461889	**0.8626639912**
26.84	38.50	1.5000000	2.1590909	2.2500000000	4.6616735537	1.0412685	1.5216205	1.4405506953	1.4189417411	**2.0440575116**
278.74	399.75	0.0000000	0.0000000	10.8424000000	23.1532909091					**9.5131612822**
25.3400000	36.3409091			1.0842400000	2.3153290909					**0.9513161282**
				1.0412684572	1.5216205476					

Right Humerus (mm) x	Right Tibia (mm) y	x^2	y^2	xy
24.80	36.05	615.0400	1299.6025	894.0400
24.59	35.57	604.6681	1265.2249	874.6663
24.59	35.57	604.6681	1265.2249	874.6663
24.29	34.58	590.0041	1195.7764	839.9482
23.81	34.20	566.9161	1169.6400	814.3020
24.87	34.73	618.5169	1206.1729	863.7351
25.90	37.38	670.8100	1397.2644	968.1420
26.11	37.96	681.7321	1440.9616	991.1356
26.63	37.46	709.1569	1403.2516	997.5598
26.31	37.75	692.2161	1425.0625	993.2025
26.84	38.50	720.3856	1482.2500	1033.3400
278.74	399.75	7074.1140	14550.4317	10144.7378



(A) See the graph.

(B) $\bar{x} = 25.340\,\text{mm}$ and

$\bar{y} = 36.3409091\,\text{mm} \approx 36.341\,\text{mm}$.

(C)

$$s_x = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} = \sqrt{\frac{10.8424\,\text{mm}^2}{11-1}} = \sqrt{1.08424\,\text{mm}^2} = 1.0412684572\,\text{mm} \approx 1.041\,\text{mm}$$

$$s_y = \sqrt{\frac{\sum (y-\bar{y})^2}{n-1}} = \sqrt{\frac{23.1532909091\,\text{mm}^2}{11-1}} = \sqrt{2.31532909091\,\text{mm}^2} = 1.5216205476\,\text{mm} \approx 1.522\,\text{mm}$$

(D)

$$s_x = \sqrt{\frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}} = \sqrt{\frac{7074.1140\,\text{mm}^2 - \frac{(278.74\,\text{mm})^2}{11}}{11-1}} = \sqrt{\frac{7074.1140\,\text{mm}^2 - \frac{77695.98076\,\text{mm}^2}{11}}{11-1}}$$

$$= \sqrt{\frac{7074.1140\,\text{mm}^2 - 7063.2716\,\text{mm}^2}{11-1}} = \sqrt{1.08423999999\,\text{mm}^2} = 1.041268457\,\text{mm} \approx 1.041\,\text{mm}$$

$$s_y = \sqrt{\frac{\sum y^2 - \frac{\left(\sum y\right)^2}{n}}{n-1}} = \sqrt{\frac{14550.4317\,\text{mm}^2 - \frac{(399.75\,\text{mm})^2}{11}}{11-1}} = \sqrt{\frac{14550.4317\,\text{mm}^2 - \frac{159800.0625\,\text{mm}^2}{11}}{11-1}}$$

$$= \sqrt{\frac{14550.4317\,\text{mm}^2 - 14527.278409\,\text{mm}^2}{11-1}} = \sqrt{2.31532909090947\,\text{mm}^2} = 1.521620548\,\text{mm} \approx 1.522\,\text{mm}$$

(E)

$$r = \frac{\sum\left(\frac{x-\bar{x}}{s_x}\right)\left(\frac{y-\bar{y}}{s_y}\right)}{n-1} = \frac{9.5131612822}{11-1} = 0.95131612822$$

(F)

$$r = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2}\sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}} = \frac{11(10144.7378) - (278.74)(399.75)}{\sqrt{11(7074.1140) - (278.74)^2}\sqrt{11(14550.4317) - (399.75)^2}}$$

$$= \frac{111592.1158 - 111426.315}{\sqrt{77815.254 - 77695.9876}\sqrt{160054.7487 - 159800.0625}} = \frac{165.8008}{\sqrt{119.2664}\sqrt{254.6862}}$$

$$= \frac{165.8008}{(10.9209157)(15.9588909)} = \frac{165.8008}{174.2857028} = 0.951316128$$

(G) The tables gives, for $n = 11$, a critical value of 0.602. With our $r = 0.951316$, we see there is a linear relation.

(H) $b_1 = r \cdot \frac{s_y}{s_x} = 0.951316128\left(\frac{1.5216205476}{1.0412684572}\right) = 1.390171918$

$b_0 = \bar{y} - b_1\bar{x} = 36.3409091\,\text{mm} - (1.390171918)(25.340\,\text{mm}) = 1.113952697$

$\hat{y} = 1.390171918x + 1.113952697\,\text{mm}$

(I) The slope is 1.390171918.

(J) See the line in the piture.

(K) Determine the residual if the length of the right humerus is 26.11 mm and the actual length of the right tibia is 37.96 mm. Also determine if the length of this particular tibia is above or below average for a rat with a right humerus 26.11 mm in length?
$\hat{y} = 1.390171918(26.11\,\text{mm}) + 1.113952697\,\text{mm} = 37.41134\,\text{mm}$. So the actual length of the right tibia, 37.96 mm, has a residual of $37.96\,\text{mm} - 37.41134\,\text{mm} = 0.55\,\text{mm}$, which is positive, and so the length of this tibia is considered above average for a rat with a right humerus 26.11 mm in length.

(L) The residual corresponding to part (K) is shown on the scatter diagram.

(M) With the length of the right humerus determined to be 25.31 mm, the length of the right tibia is estimated as.
$\hat{y} = 1.390171918(25.31\,\text{mm}) + 1.113952697\,\text{mm} = 36.299\,\text{mm} \approx 36.30\,\text{mm}$

(N) $R^2 = r^2 = (0.951316128)^2 = 0.905002376 \approx 90.5\%$

(O) 90.5% of the variation in the lengths of right tibia is explained by the least squares regression line, and the remaining 9.5% of the variation is explained by other factors.

2. Section 4.1,
#11: Nonlinear #12: Linear, negative #13: Linear, positive #14: Nonlinear
#15:(a) III (b) IV (c) II (d) I
#16 (a) IV (b) III (c) I (d) II

3. Section 4.2
#9: Please see the solution in the book.
#10(a) See graph. (b) $\hat{y} = 0.1457x + 1.1370$ (c) See graph.

4. Section 4.3
#3 (a) III (b) II (c) IV (d) I
#4 (a) $R^2 = 10.24\%$
(b) $R^2 = 1.69\%$
(c) $R^2 = 16\%$
(d) $R^2 = 86.49\%$
#5 (See the solution in the book).
#6 57.5% of the variation in 28-day strength is explained by the least-squares regression equation.