Math 164 (Spring 2008)

Solutions for Homework #4

1. Stanford-Binet Intelligence Quotient (IQ) scores are known to form a bell-shaped distribution with a mean of 100 and a standard deviation of 15.

(a) What does the Empirical Rule has to say about the percentage of people with an IQ score between 70 and 130?

Sol: Note that 70 is two standard deviations to the left of the mean 100, and 130 is two standard deviations to the right of the mean 100. The Empirical Rule says that approximately 95% of people have an IQ between 70 and 130.

(b) What does Chebyshev's Inequality has to say about the percentage of people with an IQ score between 70 and 130? Does this contradict with your answer to (a)? Why or why not. (Be very precise and be careful in your wording in answering (b). Key words: "at least".)

Sol: Compute $\left(1-\frac{1}{k^2}\right)100\% = \left(1-\frac{1}{2^2}\right)100\% = \frac{3}{4}\cdot 100\% = 75\%$.

So Chebyshev's Inequality states that ***at least*** 75% of people have an IQ between 70 and 130. (Note that this is quite a conservative statement.)

2. Section 3.3, Problem 3.

Class			Class Midpoint x_i	Freq. f_i	$x_i f_i$	$\bar{x}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2 f_i$
10	-	19	15	8	120	32.83333	-17.83333	2544.2222222
20	-	29	25	16	400	32.83333	-7.83333	981.7777778
30	-	39	35	21	735	32.83333	2.16667	98.5833333
40	-	49	45	11	495	32.83333	12.16667	1628.3055556
50	-	59	55	4	220	32.83333	22.16667	1965.4444444
				60	1970			7218.333333
				$\bar{x} =$	32.83333		$s^2 =$	122.3446328
							$s =$	11.0609508

Answer: $\bar{x} \approx \$32.83$ $s \approx \$11.06$

3. Section 3.3, Problem 4.

Class			Class Midpoint x_i	Freq. f_i	$x_i f_i$	μ	$x_i - \mu$	$(x_i - \mu)^2 f_i$
1	-	5	3.5	11	38.5	14.57143	-11.07143	1348.3418367
6	-	10	8.5	0	0.0	14.57143	-6.07143	0.0000000
11	-	15	13.5	5	67.5	14.57143	-1.07143	5.7397959
16	-	20	18.5	6	111.0	14.57143	3.92857	92.6020408
21		25	23.5	1	23.5	14.57143	8.92857	79.7193878
26		30	28.5	2	57.0	14.57143	13.92857	388.0102041
31		35	33.5	1	33.5	14.57143	18.92857	358.2908163
36	-	40	38.5	2	77.0	14.57143	23.92857	1145.1530612
				28	408.0			3417.8571429
				$\mu =$	14.57143		$\sigma^2 =$	122.0663265
							$\sigma =$	11.0483631

Answer: $\mu \approx 14.6$ points $\sigma \approx 11.0$ points

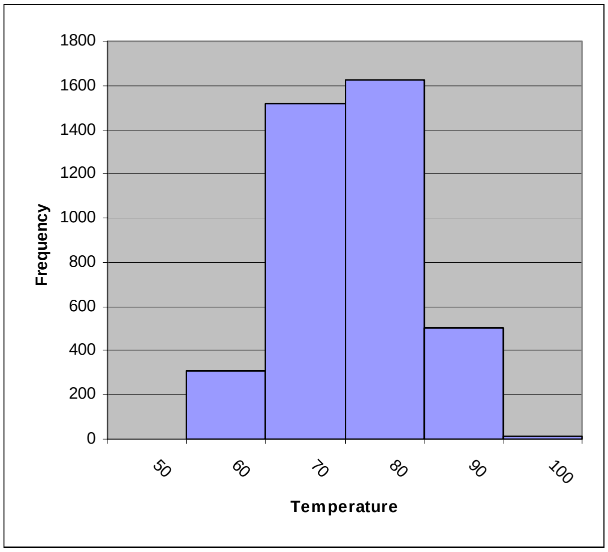
4. Section 3.3, Problem 9.

(a)

Class			Class Midpoint x_i	Freq. f_i	$x_i f_i$	μ	$x_i - \mu$	$(x_i - \mu)^2 f_i$
50	-	59	55	1	55.0	80.93498	-25.93498	672.6231792
60	-	69	65	308	20020.0	80.93498	-15.93498	78208.4633977
70	-	79	75	1519	113925.0	80.93498	-5.93498	53505.2342569
80	-	89	85	1626	138210.0	80.93498	4.06502	26868.6563704
90		99	95	503	47785.0	80.93498	14.06502	99505.8704452
100		109	105	11	1155.0	80.93498	24.06502	6370.3771490
				3968	321150.0			265131.2247984
				$\mu =$	80.93498		$\sigma^2 =$	66.8173450
							$\sigma =$	8.1741877

Answer: $\mu \approx 80.9°$ F, $\sigma \approx 8.2°$ F

(b)



(c) $\mu - 2\sigma$ and $\mu + 2\sigma$. Calculate: $80.9 - 2 \cdot 8.2 = 64.5$, $80.9 + 2 \cdot 8.2 = 97.3$. Thus, according to the Empirical Rule, roughly 95% of days in the month of August will have a high temperature between $64.5°$ F and $97.3°$ F

5. Section 3.3, Problem 17.

Sol: (You don't need any statistics, though this is related to what is called "weighted mean" in this section.) $\dfrac{(4)(3.50)+(3)(2.75)+(2)(2.25)}{4+3+2} \approx 2.97$

Answer: \$2.97 per pound

6. Section 3.4, Problem 9. (Use z-scores.)

Sol: 75-inch man: $z = \dfrac{75-69.6}{2.7} = 2.0$.

70-inch woman: $z = \dfrac{70-64.1}{2.6} \approx 2.27$

Answer: The woman.

7. Section 3.4, Problem 10. (Use z-scores.)

Sol: 68-inch man: $z = \frac{68-69.6}{2.7} \approx -0.593$.

62-inch woman: $z = \frac{62-64.1}{2.6} \approx -0.808$

Note that $-0.593 > -0.808$.

Answer: The man.

8. Section 3.4, Problem 13. (Don't forget to interpret each percentile!)

Sol:

(a) $i = \left(\frac{40}{100}\right)(51+1) = 20.8$, so $P_{40} = \frac{20^{th} + 21^{st}}{2} = \frac{325.5 + 333.2}{2} = 329.35$

So approximately 40% of the states have violent crime rates below 329.35.

(b) $i = \left(\frac{95}{100}\right)(51+1) = 49.4$, so $P_{95} = \frac{49^{th} + 50^{th}}{2} = \frac{730.2 + 793.5}{2} = 761.85$

So approximately 95% of the states have violent crime rates below 761.85.

(c) $i = \left(\frac{10}{100}\right)(51+1) = 5.2$, so $P_{10} = \frac{5^{th} + 6^{th}}{2} = \frac{173.4 + 221.0}{2} = 197.2$

So approximately 10% of the states have violent crime rates below 197.2.

(d) There are 48 states that are lower than Florida. So the percentile of Florida is $\frac{48}{51} \approx 0.9412$. **Answer:** 94^{th} percentile.

(e) $\frac{40}{51} \approx 0.7843$. **Answer:** 78^{th} percentile.

9. Section 3.4, Problem 14. (Don't forget to interpret each percentile!)

Sol:

(a) $i = \left(\frac{30}{100}\right)(51+1) = 15.6$, so $P_{30} = \frac{15^{th} + 16^{st}}{2} = \frac{275.8 + 285.6}{2} = 280.7$

So approximately 30% of the states have violent crime rates below 280.7.

(b) $i = \left(\frac{85}{100}\right)(51+1) = 44.2$, so $P_{85} = \frac{44^{th} + 45^{th}}{2} = \frac{646.3 + 658.0}{2} = 652.15$

So approximately 85% of the states have violent crime rates below 652.15.

(c) $i = \left(\frac{5}{100}\right)(51+1) = 2.6$, so $P_{10} = \frac{2^{nd} + 3^{rd}}{2} = \frac{108.9 + 110.2}{2} = 109.55$

So approximately 5% of the states have violent crime rates below 109.55.

(d) There are 45 states that are lower than New Mexico. So the percentile of Mew Mexico is $\frac{45}{51} \approx 0.8824$. **Answer:** 88^{th} percentile.

(e) $\frac{15}{51} \approx 0.2941$. **Answer:** 29^{th} percentile.

10. Section 3.4, Problem 15.

Sol:

(a) You may use $s^2 = \dfrac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$ or $s^2 = \dfrac{\sum (x_i - \bar{x})^2}{n-1}$

Inches of Rain x_i	x_i^2
0.97	0.9409
1.14	1.2996
1.85	3.4225
2.34	5.4756
2.47	6.1009
2.78	7.7284
3.41	11.6281
3.48	12.1104
3.94	15.5236
3.97	15.7609
4.00	16.0000
4.02	16.1604
4.11	16.8921
4.77	22.7529
5.22	27.2484
5.50	30.2500
5.79	33.5241
6.14	37.6996
6.28	39.4384
7.69	59.1361
79.87	379.0929

$$s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1} = \frac{379.0929 - \frac{(79.87)^2}{20}}{20-1} = 3.1648450$$

$$s = \sqrt{s^2} = \sqrt{3.1648450} \approx 1.7790011$$

	Inches of Rain x_i	$\bar{x}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
	0.97	3.9935	-3.02	9.1415523
	1.14	3.9935	-2.85	8.1424623
	1.85	3.9935	-2.14	4.5945923
	2.34	3.9935	-1.65	2.7340623
	2.47	3.9935	-1.52	2.3210523
	2.78	3.9935	-1.21	1.4725823
	3.41	3.9935	-0.58	0.3404722
	3.48	3.9935	-0.51	0.2636823
	3.94	3.9935	-0.05	0.0028622
	3.97	3.9935	-0.02	0.0005522
	4.00	3.9935	0.01	0.0000423
	4.02	3.9935	0.03	0.0007022
	4.11	3.9935	0.12	0.0135723
	4.77	3.9935	0.78	0.6029523
	5.22	3.9935	1.23	1.5043023
	5.50	3.9935	1.51	2.2695423
	5.79	3.9935	1.80	3.2274123
	6.14	3.9935	2.15	4.6074623
	6.28	3.9935	2.29	5.2280823
	7.69	3.9935	3.70	13.6641123
	79.87			60.13
$\bar{x} =$	3.9935		$s^2 =$	3.1648450
			$s =$	1.7790011

Then $z = \dfrac{x - \bar{x}}{s} = \dfrac{0.97 - 3.9935}{1.7790011} \approx -1.70$ This means that the rainfall of 0.97 inches is 1.70 times the standard deviations below the mean.

(b) $Q_1 = \dfrac{2.47 + 2.78}{2}\text{in} = 2.625\,\text{in}$ $Q_2 = \dfrac{3.97 + 4.00}{2}\text{in} = 3.985\,\text{in}$

$Q_3 = \dfrac{5.22 + 5.50}{2}\text{in} = 5.360\,\text{in}$

(c) $\text{IQR} = Q_3 - Q_1 = 5.360\,\text{in} - 2.625\,\text{in} = 2.735\,\text{in}$

(d) lower fence: $2.625\,\text{in} - 1.5(2.7353\,\text{in}) = -1.4775\,\text{in} \approx -1.478\,\text{in}$

upper fence: $5.360\,\text{in} + 1.5(2.7353\,\text{in}) = 9.46295\,\text{in} \approx 9.463\,\text{in}$

There are no outliers.

11. Section 3.4, Problem 19.

Sol: The sorted data are shown in the accompanying table.

$$Q_1 = \frac{429+437}{2}\text{min} = 433.0\,\text{min} \qquad Q_3 = \frac{489+490}{2}\text{min} = 489.5\,\text{min}$$

$$\text{IQR} = Q_3 - Q_1 = 489.5\,\text{min} - 433.0\,\text{min} = 56.5\,\text{min}$$

upper fence: $489.5\,\text{min} + 1.5(56.5\,\text{min}) = 574.25\,\text{min} \approx 574\,\text{min}$

Answer: The cutoff is 574 minutes.

1	345
2	346
3	358
4	372
5	429
6	437
7	442
8	442
9	461
10	466
11	466
12	470
13	471
14	480
15	489
16	490
17	505
18	515
19	516
20	549

12. Section 3.4, Problem 22. (Use a class width of 20 when drawing the histogram.)

Sol:

(a) The sorted data are shown in the accompanying table.

$$Q_1 = \frac{21+21}{2} = 21 \qquad Q_3 = \frac{54+54}{2} = 54$$

$$\text{IQR} = Q_3 - Q_1 = 54 - 21 = 33$$

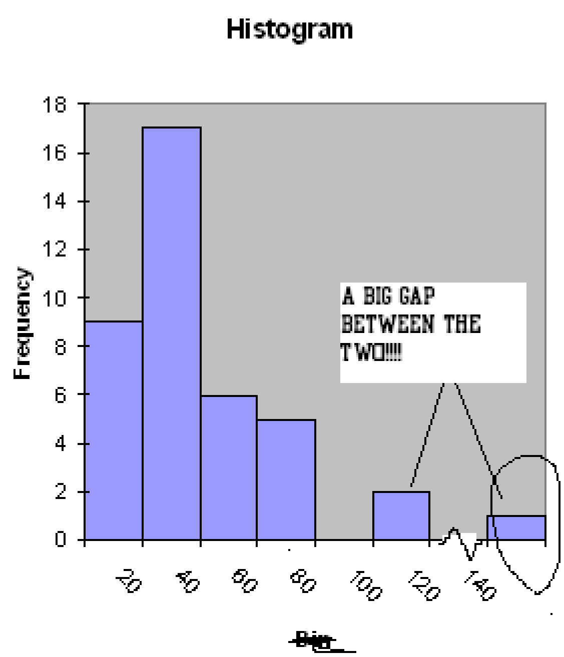
lower fence: $21 - 1.5(33) = -28.5$

upper fence: $54 + 1.5(33) = 103.5$

Answer: Outliers are \$115 and \$1000

(b) See the histogram.

(c) Perhaps there is a student with a deep pocket. Or perhaps the unusually large number 1000 was the result of a typo.



1	7
2	8
3	9
4	10
5	12
6	13
7	14
8	16
9	20
10	21
11	21
12	22
13	22
14	26
15	26
16	27
17	28
18	28
19	32
20	33
21	33
22	35
23	36
24	36
25	38
26	39
27	48
28	51
29	51
30	54
31	54
32	59
33	64
34	65
35	67
36	75
37	80
38	101
39	115
40	1000

13. Section 3.5, Problem 5.

Sol: The sorted data are shown in the accompanying table.

For Q_1, $i = \frac{25}{100}(43+1) = 11$, so $Q_1 = 51$

For Q_2, $i = \frac{50}{100}(43+1) = 22$, so $Q_2 = 55$

For Q_3, $i = \frac{75}{100}(43+1) = 33$, so $Q_3 = 58$

$\text{IQR} = Q_3 - Q_1 = 58 - 51 = 7$

lower fence: $51 - 1.5(7) = 40.5$

upper fence: $58 + 1.5(7) = 68.5$

Min is 42, Max is 69.

The five-number summary is 42, 51, 55, 58, 69

See the solution in the book for the boxplot.

1	42
2	43
3	46
4	46
5	47
6	48
7	49
8	49
9	50
10	50
11	51
12	51
13	51
14	51
15	52
16	52
17	54
18	54
19	54
20	54
21	54
22	55
23	55
24	55
25	55
26	56
27	56
28	56
29	57
30	57
31	57
32	57
33	58
34	60
35	61
36	61
37	61
38	62
39	64
40	64
41	65
42	68
43	69

14. Section 3.5, Problem 9.

Sol: The sorted data are shown in the accompanying table.

For Q_1, $i = \frac{25}{100}(25+1) = 6.5$, so $Q_1 = \dfrac{0.603 + 0.605}{2} = 0.604$

For Q_2, $i = \frac{50}{100}(25+1) = 13$, so $Q_2 = 0.608$

For Q_3, $i = \frac{75}{100}(25+1) = 19.5$, so $Q_3 = \dfrac{0.610 + 0.610}{2} = 0.610$

$\text{IQR} = Q_3 - Q_1 = 0.610 - 0.604 = 0.006$

lower fence: $0.604 - 1.5(0.006) = 0.595$

upper fence: $0.610 + 1.5(0.006) = 0.619$

Min is 0.598, Max is 0.612.

The five-number summary is 0.598, 0.604, 0.608, 0.610, 0.612.

See the solution in the book for the boxplot.

1	0.598
2	0.600
3	0.600
4	0.601
5	0.602
6	0.603
7	0.605
8	0.605
9	0.605
10	0.606
11	0.607
12	0.607
13	0.608
14	0.608
15	0.608
16	0.608
17	0.608
18	0.609
19	0.610
20	0.610
21	0.610
22	0.610
23	0.611
24	0.611
25	0.612