Math 164 (Spring 2008)

Solutions to Homework #3

<u>Section 3.1</u>: (this section again!!)

46, 47, 49, 50, 52

#46 The mean of all 20 scores was 82, thus the sum of all 20 scores was $82 \times 20 = 1640$. Likewise, since the mean of the 19 readable scores is 84, the sum of these 19 readable scores is $84 \times 19 = 1596$. Therefore, the unreadable score must be $1640 - 1596 = 44$.

#47 $34 \times 6 = 204$

#49

(a) $(30+30+45+50+50+50+55+55+60+75)/10 = 500/10 = 50$.
So the mean is 50 (i.e. 50 thousand dollars)
Since there are 10 data points, which is even, the median is the average of the 5th and the 6th data points, namely $(50+50)/2 = 50$.
The mode is 50.
Answer: Mean = \$50,000, Median = \$50,000, Mode = \$50,000.

(b) The new data set becomes
32.5, 32.5, 47.5, 52.5, 52.5, 52.5, 57.5, 57.5,. 62.5, 77.5.
Mean = 52.5, Median = 52.5, Mode = 52.5.
Answer: Mean = \$52,500, Median = \$52,500, Mode = \$52,500.
(So all three measures increased by \$2,500.)

(c) Multiply each of the original data points by a factor of 1.05, we get the new data set as
31.50, 31.50, 47.25, 52.50, 52.50, 52.50, 57.75, 57.75, 63.00, 78.75
The sum is 525.00, so the mean is $525.00/10 = 52.5$.
The median is $(52.5+52.5)/2 = 52.5$. The mode is 52.5.
Answer: Mean = \$52,500, Median = \$52,500, Mode = \$52,500.
(All three measures increased by 5%.)

(d) The new data set is
30, 30, 45, 50, 50, 50, 55, 55, 60, 100
The sum is 525, with the mean being $525/10 = 52.5$. The median is still 50, and the mode is still 50.
Answer: Mean = \$52,500, Median = \$50,000, Mode = \$50,000.
(The median and the mode didn't change, but the mean has creased by \$2,500.)

#50

(a) Sum is 376. So the mean is $376/5 = 75.2$.
(b) There are 5 data points (odd!), so the median is the middle one (the 3rd), i.e. 71.
(c) The data point 95 is quite a bit to the far right, so the data appear to skew to the right. The median is a better measure of the central tendency in skewed case.
(d) 79.2. (You do the computation.)
(e) The mean increases by 4.

#52 $\dfrac{0.94+0.76}{2} = \dfrac{1.70}{2} = 0.85$. The midrange is not resistant. For example, by increasing only the largest value in the data set, with all remaining values intact, the midrange obviously will increase.

Section 3.2: 11, 13, 15, 17, 22, 46

#11 $s^2 = 36$, $s = 6$ (Note: Sample, so make sure you use $n-1$!!!)

#13 $\sigma^2 = 16$, $\sigma = 4$ (Note: Population! So make sure you use N.)

#15 $s^2 = 196$, $s = 14$ (Note: Sample, so make sure you use $n-1$!!!)

#17 (Note that this is a sample! Use $n-1$ in computing variance.)

Range: $\$462 - \$236 = \$226$.

Sample variance: $9962.916667 \approx 9962.9$ (dollar squared)

Sample standard deviation: $\sqrt{9962.916667} \approx 99.81441112 \approx 99.8$, so \$99.8.

#22 First, observe that I, III, IV all have a mean of 53 and a median of 53. By looking at the four histograms, it appears that (a) (b) (c) all roughly have the center near 53, while the center of (d) is closer to 60. Now focus on the values of standard deviations of I, III, IV in order to sort out how they match (a) (b) (c). You notice that (c) ranges from 52.78 to 53.33, either one is within at most 0.33 of the supposed mean of 53; In particular, the standard deviation of (c) cannot be as big as 11 or 1.3, so it must be that (c) is IV. Now, between (a) and (b), clearly (a) is far more spread out, ranging from 27 to 82, so it is reasonable to expect (a) to be III. Thus: :(a) III (b) I (c) IV (d) II

#46

(a) $R = 75 - 30 = 45$, $\sigma^2 = 160$, $\sigma = 12.6$. More precisely,

$$R = \$45{,}000,\ \sigma^2 = 160{,}000{,}000\ dollar^2,\ \sigma \approx \$12{,}600.$$

(b) $R = 77.5 - 32.5 = 45$, $\sigma^2 = 160$, $\sigma = 12.6$. More precisely,

$$R = \$45{,}000,\ \sigma^2 = 160{,}000{,}000\ dollar^2,\ \sigma \approx \$12{,}600$$

(Note that all three remain the same as in (a).)

(c) $R = 78.75 - 31.50 = 47.25$, $\sigma^2 = 176.4$, $\sigma = 13.3$. More precisely

$$R = \$47{,}250,\ \sigma^2 = 176{,}400{,}000\ dollar^2,\ \sigma \approx \$13{,}300$$

(Note that from (a) to (c), R and σ increase by a factor of 1.05 (i.e. increase by 5%), while σ^2 increases by a factor of 1.05^2, i.e. a factor of 1.1025.

(d) $R = 100 - 30 = 70$, $\sigma^2 = 341.3$, $\sigma = 18.5$. More precisely,

$$R = \$70{,}000,\ \sigma^2 = 341{,}250{,}000\ dollar^2,\ \sigma \approx \$18{,}500$$

Here I used the more efficient formula $\sigma^2 = \dfrac{\sum x_i^2 - \frac{(\sum x_i)^2}{N}}{N}$. For example, in (a),

$\sigma^2 = \dfrac{26600 - \frac{(500)^2}{10}}{10} = 160$. Also be careful about the unit and numerical values of σ^2. E.g. $\sigma^2 = 160$ means the variance is 160,000,000 dollar^2.

(a)			(b)			(c)			(d)	
Salary (thousands of dollars) x_i	x_i^2		x_i	x_i^2		x_i	x_i^2		x_i	x_i^2
30	900		32.5	1056.25		31.50	992.2500		30	900
30	900		32.5	1056.25		31.50	992.2500		30	900
45	2025		47.5	2256.25		47.25	2232.5625		45	2025
50	2500		52.5	2756.25		52.50	2756.2500		50	2500
50	2500		52.5	2756.25		52.50	2756.2500		50	2500
50	2500		52.5	2756.25		52.50	2756.2500		50	2500
55	3025		57.5	3306.25		57.75	3335.0625		55	3025
55	3025		57.5	3306.25		57.75	3335.0625		55	3025
60	3600		62.5	3906.25		63.00	3969.0000		60	3600
75	5625		77.5	6006.25		78.75	6201.5625		100	10000
500	26600		525	29162.5		525	29326.5		525	30975
	160			160			176.4			341.25
	12.64911			12.64911			13.2815662			18.4729532