Standing Waves in Strings

APPARATUS

1. Buzzer (vibrating at a given frequency) mounted on a board with a pulley
2. Electronic balance
3. 2 Strings, one light and one heavy
4. Set of known masses (slotted type)
   (4 × 100 g, 4 × 50 g, 2 × 20 g, 2 × 10 g, 1 × 5 g, 2 × 2 g, 1 × 1 g)
5. A pan to support the known masses
6. Meter stick
7. 30 cm ruler

INTRODUCTION

We shall state, very briefly, some of the properties of traveling and standing waves in strings under tension which form the basis for this laboratory exercise. The student should refer to the textbook for a more complete discussion.

Traveling Waves

Consider a string along which there is a transverse traveling wave moving from left to right. In figures (1a), (1b), and (1c), we see how the string looks at slightly different times. Despite the fact that the ”wave” is moving from left to right, each particular point on the string is moving up and down, and all the points on the string undergo this transverse oscillatory motion with the same amplitude. The wavelength λ (lambda) and the amplitude A of the wave are shown in Fig. (1a).

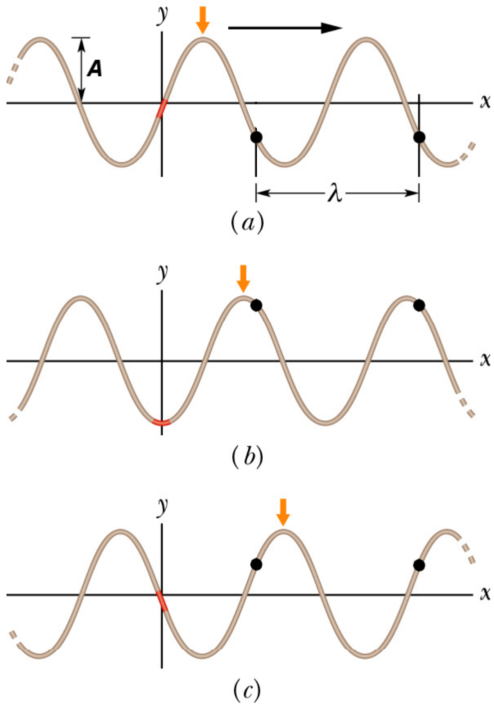

Figure 1: A traveling wave in a string at successive times. In each picture, the black dots represent the same two points on the string while the arrow points to the propagating crest with constant phase.

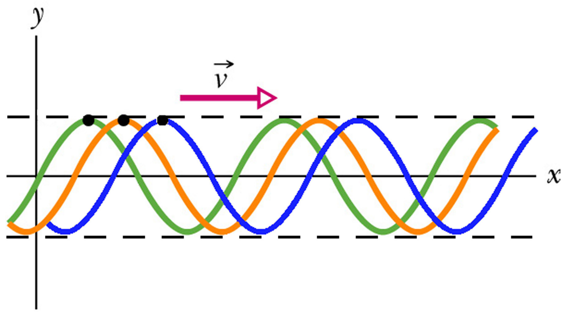

Figure 2: Traveling wave in a string at several successive times; horizontal dashed lines show the envelope of the motion.

Because all the points on the string are moving, the entire string would look like a blur to the eye. All that could be distinguished would be the envelope (or extremes) of the motion. This is illustrated in Fig. 2 where the solid lines represent the string at different times and the dotted lines are the envelope of the motion which indicates the outline of what the eye would see.

Standing Waves

When there are two traveling waves in the string going in opposite directions, the resultant motion of the string can be quite different than the one just described for a single traveling wave. Each wave will try to make any given point on the string undergo an oscillation of the type described above, and the actual motion of the point will be the sum of two such oscillations. Now consider the special but important case in which the two traveling waves have the same amplitude and wavelength (but are still traveling in opposite directions). We show two such waves separately in Figure 3(a) and (b).

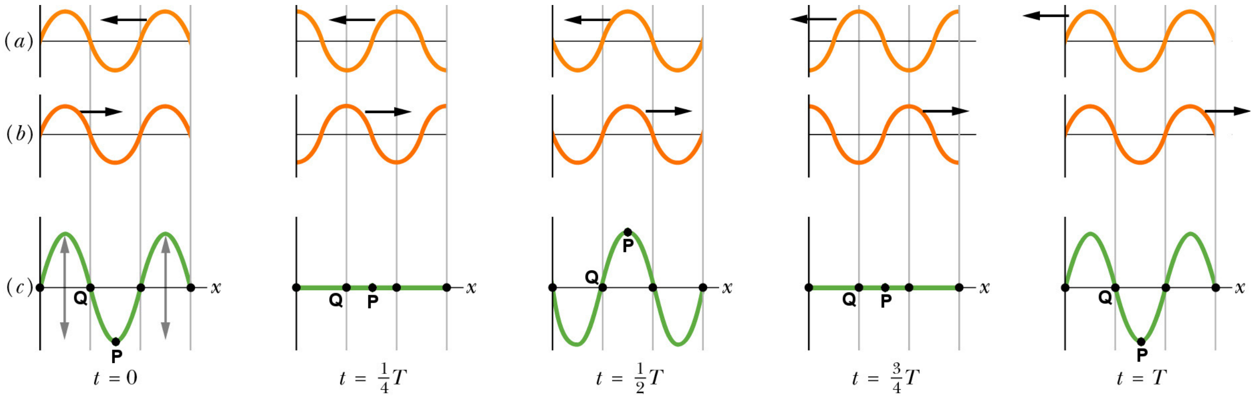

Figure 3: Two traveling waves (a) and (b) going in opposite directions generate a standing wave (c).

There is a moment (t=0 in Fig. 3) where the two counterpropagating waves are in a position to constructively interfere with each other so that the displacement of each location of the string is twice the displacement of the case of only one propagating wave. One quarter of a period later (t=$\frac{1}{4}$T in Fig. 3), the waves have moved with respect to each other for half a wavelength, causing them to destructively interfere, so that there is no displacement of any part of the string at all, momentarily. This procedure repeats itself as the waves are constantly traveling into opposite directions. Consequently, there are points (point Q in Fig. (3c)) where the string does not move at all, because at every time, the contributions from each of the waves cancel each other exactly, and there are points (point P in Fig. (3c)) which oscillate at twice the amplitude of the original waves.

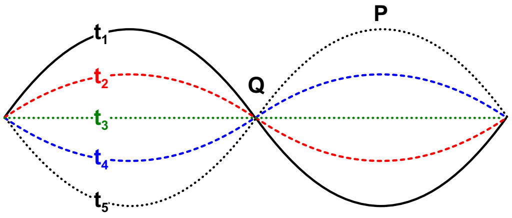

Figure 4: Appearance of a standing wave at times t = t_1, t_2, etc. Corresponding to Fig. 3: $t_3 = \frac{1}{4}T$ and $t_5 = \frac{1}{2}T$

The above situation may be summarized by saying that the two traveling waves arrive at point P **in phase** and at point Q **out of phase**, and thereby produce a large oscillation at P and no motion at Q. There are many points like P and Q on the string, and other points where the waves arrive partially out of phase and produce a motion with amplitude less than that at P. At successive times t_1, t_2, etc. the string will look like the solid curves labeled t_1, t_2, etc. in Fig. 4. Points like Q which never move, because the two waves are out of phase, are called **nodes**, and points like P which have large amplitude of transverse motion because the two waves are in phase are called **antinodes**. The distance between two adjacent nodes is equal to one half wavelength of the traveling waves.

The motion pictured in Fig. 4 is an example of a standing wave, and it looks quite different to the observer than the traveling wave of Fig. 2. It is easy to see the nodes in standing waves, and thereby make a direct determination of the wavelength. When a string is held fixed at two particular points, then any standing waves which exist in the string will have nodes at those two fixed points. Thus, if the fixed points are Q and Q' in Figure 5, then the standing waves shown in (5a), (5b), (5c) are all possible because they each have nodes at Q and Q'. The waves in figures (5a), (5b), and (5c) are examples of standing wave patterns (also called "modes of oscillation") for a string fixed at Q and Q'.

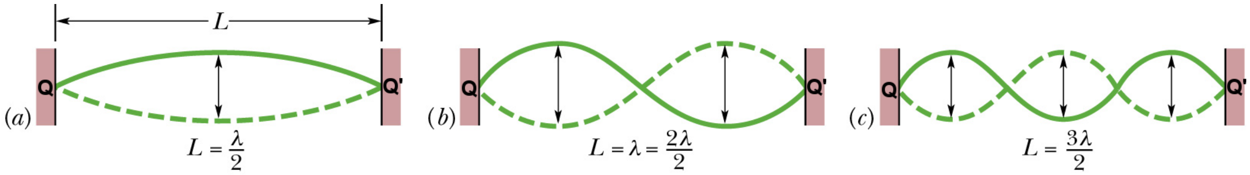

Figure 5: Three possibilities of standing waves (modes of oscillation) on a string of fixed length L.

For waves in strings the wavelength λ is related to the frequency f and velocity v by:

$$v = f\lambda \tag{1}$$

The velocity of a transverse wave in a string is given by

$$v = \sqrt{\frac{F}{\mu}} \tag{2}$$

where F is the tension in the string and μ is the mass per unit length of the string. Equating equations (1) and (2), one gets:

$$f = \frac{1}{\lambda}\sqrt{\frac{F}{\mu}} \tag{3}$$

In the setup used in this experiment, the tension is generated by a mass that pulls on the string over a pulley, the tension in the string is therefore $F = mg$ with g=9.806 m/s^2. By varying this tension, the sound velocity (and thus, the wavelength corresponding to a fixed frequency) will be so altered that several standing wave patterns can be found. That is, over the string's entire length there will be two, three, or more nodes.

Determination of the Frequency of a Source Using a Standing Wave.

Standing waves are to be set up in a stretched string by the vibrations of a buzzer driven by an alternating current. The frequency of vibration is 120 Hertz.

You are supposed to vary the tension in the string, by varying the mass m (masses of the pan and the slotted masses) suspended over the pulley, until resonance is reached. Then, record the value of the suspended mass, and the distance L, between nodes of the standing wave. In each case the wavelength is equal to twice the distance between neighboring nodes, i. e. $\lambda = 2L$. Make sure that the nodes are as sharp and distinct as possible. The sound of the vibrator will indicate to some extent when resonance is reached and the amplitude will reach a maximum value. Also, at resonance, adding or removing 5 grams should reduce the amplitude of the standing wave.

PROCEDURE

1. Using one of the strings determine the tension in the string and the wavelength for as many different standing wave patterns (at least four) as possible. It is suggested to start with the light string.
2. From these data plot two curves on the same sheet:
   (a) Plot tension on the X axis and wavelength on the y axis.
   (b) Plot tension on the X axis and (wavelength)^2 on the y axis.
3. Determine the mass per unit length of the string. Then calculate the average frequency. Do this by first determining the slope of the second of the above graphs and then interpret its significance with the aid of equation (3).
4. Repeat 1-3 using the other string.
5. In each case, assume f=120 Hz to be the correct value of the frequency and determine the percentage error in the calculated frequency:

$$\frac{f_{measured} - f}{f} \times 100\%$$

Questions (to be answered in your report):

1. Did you observe longitudinal or transverse waves in this experiment?
2. In any two cases above, calculate the velocity of the wave in the string.
3. What is the shape of each curve plotted?
4. Does each curve agree with equation (3)?
5. When there are three or more loops, why is it better to use one of the inner loops to measure L, rather than one of the loops formed at either end of the string?