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CHAPTER 6 Time-Varying Fields and Maxwell's Equations

Faraday's Law

$v = -\frac{d\Phi}{dt}$,

$\oint_C \vec{E} \bullet d\ell = -\frac{d}{dt}\int_S \vec{B} \bullet d\vec{s}$ (Basic Formula)

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (point form)

Transformer emf

$\oint_c \vec{E} \cdot d\ell = -\int_s \frac{\partial B}{\partial t} . d\vec{s}$

(Integral form)

Motional or flux-cutting emf

$v = \oint_C (\vec{u} \times \vec{B}) \bullet d\vec{\ell}$

General form of Faraday's law

$\oint_C \vec{E}' \bullet d\vec{\ell} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} + \oint_C (\vec{u} \times \vec{B}) \cdot d\vec{\ell}$

(Both motional and flux-cutting)

Displacement Current Density

$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$

Differential form of Maxwell's equations

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\oint_S \vec{B} \cdot d\vec{s} = 0$

Boundary Condition for tangential component of E

$E_{1t} = E_{2t}$ (V/m)

Boundary Condition for tangential Component of H

$\hat{a}_{n2} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$ (A/m)

Boundary Condition for normal component of D

$D_{1n} - D_{2n} = \rho_s$ (C/m^2)

Boundary Conditions Between Two Lossless Media

$E_{1t} = E_{2t}$

$H_{1t} = H_{2t}$

$D_{1n} = D_{2n}$

$B_{1n} = B_{2n}$

Boundary Conditions Between a Dielectric (Medium 1) and A Perfect Conductor(Medium 2)

in Medium 1	in Medium 2
$E_{1t} = 0$	$E_{2t} = 0$
$\hat{a}_n \times \vec{H}_1 = \vec{J}_s$	$H_{2t} = 0$
D1n =	$D_{2n} = 0$
$B_{1n} = 0$	$B_{2n} = 0$

Potential Functions

$\vec{B} = \nabla \times \vec{A}$

$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$ (V/m)

Time Harmonic Functions and Phasors

$I(t) = I_0 \cos(\omega t + \varphi)$ (Sinusoidal)

$I_s = I_0 e^{j\varphi}$ (Phasor)

$I(t) = \text{Re}\left[I_s e^{j\omega t}\right]$ (Time-domain)

Time harmonic Maxwell's equations in Phasor Form

$\nabla \times \vec{E} = -j\omega\mu\vec{H}$

$\nabla \times \vec{H} = J + j\omega \in \vec{E}$

$\nabla \cdot \vec{E} = \frac{\rho_v}{\in}$

$\nabla \cdot \vec{H} = 0$

Homogenous wave equation for E

$\nabla^2 \vec{E} - \frac{1}{u^2{}_p} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$

Homogenous wave equation for H

$\nabla^2 \vec{H} - \frac{1}{u^2{}_p} \frac{\partial^2 \vec{H}}{\partial t^2} = 0$

CHAPTER 7 : PLANE ELECTROMAGNETIC WAVES

Homogeneous vector

$u_p = \frac{1}{\sqrt{\mu \in}}$ (Phase velocity)

k = $\frac{2\pi}{\lambda}$ (phase constant in terms of wavelength)

$f\lambda = u_p$

Intrinsic impedance of medium

$\eta = \frac{\sqrt{\mu}}{\sqrt{\in}}$ (Intrinsic impedance)

$H(z,t) = \hat{a}_y \frac{E_0^+}{\eta} \cos(\omega t - kz)$

(Magnetic field intensity)

Doppler Effect

$f' = f(1 + \frac{u}{c} \cos\theta)$

Doppler Shift

$\Delta f = f' - f = \frac{fu}{c} \cos\theta$

TEM, Transverse Electromagnetic waves

$\vec{E}(R) = \vec{E}_0 e^{-j\vec{k}.\vec{R}}$

Finding H from E of a uniform plane wave and vice versa

$\vec{H} = \frac{1}{\eta} \hat{a}_k \times \vec{E}$

$\vec{E} = \eta \vec{H} \times \hat{a}_k$

Polarization of Plane Waves

$\vec{E}(z,t) = \hat{a}_x E_{10} \cos(\omega t - kz) + \hat{a}_y E_{20} \cos(\omega t - kz + \delta)$

Condition for Linear Polarization

$\delta = 0$, or $\delta = \pi$

Condition for Circular Polarization

$E_{10} = E_{20}$ and $\delta = \pm 90^0$

(+90 -> Left hand , –90 -> Right hand)

Elliptic Polarization

Cases excluding linear and circular polarizations

Plane waves in Lossy Media

$\nabla^2 \vec{E} - \gamma^2 \vec{E} = 0$ (Wave equation)

$\vec{E} = \hat{x} E_0 e^{-\gamma z}$ (Simplified solution)

Complex permitivity of lossy media

$$\in_c = \in - j\frac{\sigma}{\omega}$$

Loss tangent of a lossy medium

$\tan\delta_c = \frac{\sigma}{\omega \in}$

Relation between propagation constant and wave number

Propagation constant (γ)

$\gamma = j\omega\sqrt{\mu \in (1 + \frac{\sigma}{j\omega \in})}$

$\gamma = \alpha + j\beta$

α = Attenuation Constant (NP/m)

β = Phase Constant (rad/m)

Solution

$E_x = E_0 e^{-\alpha z} e^{-j\beta z}$

Attenuation Constant and Phase constant of a good conductor

$\alpha = \beta = \sqrt{\pi f \mu \sigma}$

Intrinsic impedance of a good conductor

$\eta_c = \frac{\sqrt{\mu}}{\sqrt{\in_c}} = (1+j)\frac{\alpha}{\sigma} = \sqrt{\frac{\omega\mu}{\sigma}} e^{j45^\circ}$

Depth of Penetration (Skin Depth)

$\delta = \frac{1}{\alpha}$ (General)

$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$ (Good Conductor)

Formula for group velocity in dispersive media

$u_g = \frac{1}{d\beta / d\omega}$

Poynting Theorem

$$\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{s} = -\frac{\partial}{\partial t} \int \left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right) dv - \int \sigma E^2 dv$$

Power flow per Unit area

$\vec{P} = \vec{E} \times \vec{H}$

(Poynting Vector)

Average Power density transmitted by a uniform plane wave in z direction

$\vec{P}_{av}(z) = \hat{a}_z \frac{E_0^2}{2|\eta_c|} e^{-2\alpha z} \cos\theta_\eta$

(Lossy medium)

$\vec{P}_{av}(z) = \hat{a}_z \frac{E_0^2}{2\eta}$ (Lossless medium)

$P_{av} = \frac{1}{2} R_e(\vec{E} \times H^*)$ (Phasor form)

Reflection and transmission coefficients

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ (Reflection Coefficient, Normal incidence)

$\tau = \frac{2\eta_2}{\eta_2 + \eta_1}$ (Transmission Coefficient, Normal Incidence)

$1 + \Gamma = \tau$ (Normal Incidence)

Standing wave ratio

$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$ (Dimensionless)

$|\Gamma| = \frac{S-1}{S+1}$ (Dimensionless)

Electric and Magnetic Standing-Waves in Time Domain

$\vec{E}_1(z,t) = R_e[E_1(z)e^{j\omega t}]$

$= \hat{a}_x 2E_{i0} \sin\beta_1 z \sin\omega t$

$\vec{H}_1(z,t) = \hat{a}_y 2\frac{E_{i0}}{\eta_1} \cos\beta_1 z \cos\omega t$

Oblique Incidence of Plane Waves at Plane Boundaries

Snell's law of reflection

$$\theta_r = \theta_i$$

Snell's law of refraction for

$$\mu_1 = \mu_2 \left| \frac{\sin\theta_t}{\sin\theta_i} = \frac{\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_2}} = \frac{\sqrt{\varepsilon_{r1}}}{\sqrt{\varepsilon_{r2}}} \right.$$

Formula for critical angle

$$\theta_{cc} = \sin^{-1}\left(\frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\right)$$

Transmitted Wave for total reflection

$$\vec{E}_t = \hat{E}_{t0} e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)}$$

$$\vec{E}_t = \hat{E}_{t0} e^{-\alpha_2 z} e^{-j\beta_{2x} x} \quad ,$$

$$\alpha_2 = \beta_2 \sqrt{(\varepsilon_1/\varepsilon_2)\sin^2\theta_i - 1}$$

$$\beta_{2x} = \beta_2 \sqrt{\varepsilon_1 / \varepsilon_2}\, \sin\theta_i$$

The Ionosphere

Effective permitivity of plasma

$$\varepsilon_p = \varepsilon_0 \left(1 - \frac{f_p^{\,2}}{f^2}\right)$$

Propagation constant in a plasma

$$\gamma = j\omega\sqrt{\mu\varepsilon_0}\sqrt{1 - (f_p/f)^2}$$

Perpendicular Polarization

Reflection coefficient for perpendicular polarization

$$\Gamma_\perp = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

Transmission coefficient for perpendicular polarization

$$\tau_\perp = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

Relation between reflection and transmission coefficients for perpendicular polarization

$$1 + \Gamma_\perp = \tau_\perp$$

Parallel Polarization

Reflection coefficient for parallel polarization

$$\Gamma_{II} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

Transmission coefficient for parallel polarization

$$\tau_{II} = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

Reflection and transmission relation between coefficients for parallel polarization

$$1 + \Gamma_{II} = \tau_{II}\left(\frac{\cos\theta_t}{\cos\theta_i}\right)$$

Brewster angle

$$\theta_{BII} = \tan^{-1}\left(\sqrt{\varepsilon_2} / \sqrt{\varepsilon_1}\right)$$

$$(\mu_1 = \mu_2)$$

Chapter 8 TRANSMISSION LINES

First-order Differential Equations for Voltage and Current on a Transmission Line(Phasor form)

$$-\frac{dV(z)}{dz} = (R + j\omega L)I(z)$$

$$-\frac{dI(z)}{dz} = (G + j\omega C)V(z)$$

Second-order Transmission Line Equation for phasor voltage and Current

$$\frac{d^2V(z)}{dz^2} = \gamma^2 V(z)$$

$$\frac{d^2I(z)}{dz^2} = \gamma^2 I(z)$$

Propagation Constant

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

General Wave Solutions on a Transmission Line for Voltage and Current

$$V(z) = V^+(z) + V^-(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I(z) = I^+(z) + I^-(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

Characterstic Impedance of Transmission Line

$$Z_0 = \sqrt{R + j\omega L} / \sqrt{G + j\omega C}$$

Lossless Line(R=0, G=0)

$$\alpha = 0, \beta = \omega\sqrt{LC}$$

Phasor Velocity

$$u_p = \frac{1}{\sqrt{LC}}$$

Characterstic impedance of a Lossless Line

$$Z_0 = \sqrt{L/C}$$

Distortionless Line

$$\frac{R}{L} = \frac{G}{C}$$

Propagation constant

$$\gamma = \alpha + j\beta$$

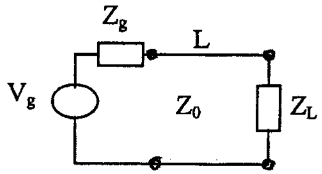
$$\alpha = R\frac{\sqrt{C}}{\sqrt{L}} \quad ,$$

$$\beta = \omega\sqrt{LC}$$

Characterstic impedance

$$Z_0 = \frac{\sqrt{L}}{\sqrt{C}}$$

Wave Characterstics on Finite Transmission Lines



General Wave Solution

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

General Formula for Input Impedance of a Line of Length ℓ

Lossy Line

$$Z_i = Z_0 \frac{Z_L + Z_0\tanh\gamma\ell}{Z_0 + Z_L\tanh\gamma\ell}$$

Lossless line

$$Z_i = Z_0 \frac{Z_L + jZ_0\tan\beta\ell}{Z_0 + jZ_L\tan\beta\ell}$$

Open-Circuited And Short-Circuited Lines

Input Impedance of a Lossless Open-Circuited Line

$$Z_{io} = -jZ_0\cot\beta\ell$$

Input Impedance of a Lossless Short-Circuited Line

$$Z_{is} = jZ_0\tan\beta\ell$$

Calculating Z_0 and γ of a Line from Input Impedances Measured Under Open- circuit and Short-circuit Conditions.

$Z_0 = \sqrt{Z_{i0} Z_{is}}$,

$\gamma = (1/l)\tanh^{-1}\sqrt{Z_{ic}/Z_{oc}}$

Definition of Voltage Reflection coefficient of Load Impedance Z_L

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta_\Gamma}$$

(Dimensionless)

Additional Formulas for Finite Transmission Lines with z = 0 at the Load Position.

$$V_0^- = \Gamma V_0^+$$

$$V_{in} = \frac{Z_{in}}{Z_g + Z_{in}} V_g$$

$$I_{in} = \frac{V_g}{Z_g + Z_{in}}$$

$$V_0^+ = \frac{V_{in}}{e^{j\beta l} + \Gamma e^{-j\beta l}}$$

Definition of Standing-wave Ratio

$$S = \frac{|V_{max}|}{|V_{min}|} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

(Dimensionless)

Slotted-Line Measurements (Determination of an Unknown Load)

$$|\Gamma| = \frac{S-1}{S+1} \qquad \text{(Dimensionless)}$$

$\theta_\Gamma = 2\beta l_{min} - \pi$ (Phase angle of the reflection Coefficient)

Property of Quarter-wave Transformers

$$Z_i = \frac{Z_0^2}{Z_L} \quad \textbf{(Quarter wave line)}$$

Chapter 9 Waveguides and Cavity Resonators

Wave Equations for Waveguides

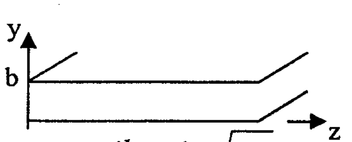
$$\nabla_{xy}^2 \vec{E} + h^2 \vec{E} = 0 \quad h^2 = \gamma^2 + \kappa^2$$

$$\nabla_{xy}^2 \vec{H} + h^2 \vec{H} = 0$$

$$h^2 = \gamma^2 + \kappa^2$$

Transverse ElectroMagnetic Waves

Parallel Plate Wave Guide (TEM)



$$\gamma_{TEM} = jk = j\omega\sqrt{\mu\varepsilon}$$

Phase Velocity for TEM Waves

$$u_{p(TEM)} = \frac{\omega}{\kappa} = \frac{1}{\sqrt{\mu\varepsilon}}$$

Wave impedance for TEM waves

$$Z_{TEM} = \frac{\sqrt{\mu}}{\sqrt{\varepsilon}} = \eta$$

Transverse Magnetic Waves (TM_n)

$\nabla_{xy}^2 E_z^0 + h^2 E_z^0 = 0$ (the wave Eq.)

Cut off Frequency for parallel-plate waveguides

$$f_c = \frac{n}{2b\sqrt{\mu\varepsilon}} \qquad \lambda_c = \frac{u}{f_c}$$

Waves with $f > f_c$ are propagating modes $\gamma = j\beta$

$$\beta = \kappa\sqrt{1 - (\frac{f_c}{f})^2}$$

Evanescent wave

Waves with $f < f_c$ are evanescent (non-propagating) modes

γ is real

$$\gamma = \alpha' = h\sqrt{1 - (\frac{f}{f_c})^2} \qquad f < f_c$$

Guide Wavelength

Relationship between the three wavelengths

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$$

The Wave impedance for TM modes

$$Z_{TM} = \eta\sqrt{1 - (\frac{f_c}{f})^2}$$

Transverse Electric Waves (TE_n)

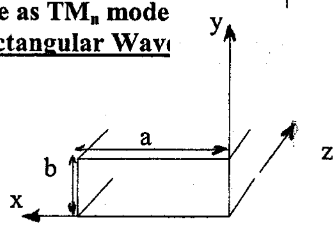
$$\nabla_{xy}^2 H_z^0 + h^2 H_z^0 = 0$$

The wave impedance for propagating TE modes

$$Z_{TE} = \frac{\eta}{\sqrt{1 - (\frac{f_c}{f})^2}}$$

Equations for $f_c, \alpha', \beta, \lambda_g$ are the same as the same as TM_n mode

TM waves in Rectangular Wave Guides



$$E_z^0(x,y) = E_0 \sin(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)$$

$$, \quad H_z^0 = 0$$

$$h^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2$$

$$m = (1,2,3...) \quad , \quad n = (1,2,3......)$$

$$\gamma = j\beta = j\sqrt{k^2 - h^2} = j\sqrt{\omega^2\mu\varepsilon - (\frac{m\pi}{a})^2 - (\frac{n\pi}{b})^2}$$

Cutoff frequency of TM_{mn} modes

$$(f_c)_{mn} = \frac{h}{2\pi\sqrt{\mu\varepsilon}} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}$$

Cutoff and guide wavelength of TM_{mn} modes

$$(\lambda_c)_{mn} = \frac{2}{\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}}$$

$$\lambda_g = \frac{2\pi}{\beta}$$

Wave Impedance

$$Z_{TM} = \eta\sqrt{1 - (\frac{f_c}{f})^2}$$

TE Waves in Rectangular Waveguides

$$H_z^0(x,y) = H_0 \cos(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$$

$$, \quad E_z^0 = 0$$

$$Z_{TE} = \frac{\eta}{\sqrt{1 - (\frac{f_c}{f})^2}}$$

Equations for $f_c, \lambda_c, \beta, \lambda_g$ are the same as the TM case

Power Tansmission in Rectangular Waveguide

$$P(Total) = \int \vec{p}_{av} . d\vec{s}$$

$$\vec{p}_{av} = \frac{|E_x|^2 + |E_y|^2}{2Z}\hat{z} \qquad \text{where}$$

$$Z = Z_{TM} \text{ or } Z = Z_{TE}$$