## 4.4 Motion Coordination in 2-D

### 4.4.a Analysis of the kinematics of the motion:

The objective to analyze the kinematics of a positioning mechanism at which a point is going to be moved from an initial position ($x_0$,$y_0$) to a final position ($x_{final}$, $y_{final}$). This point is at fixed distance *h* from the point connecting the two actuators, and at a fixed angle $\theta_h$ with respect to the axis of actuator *a*. The system is shown in figure 4.6. The point is going to cut a distance S. the bases of actuators a and b are located at ($xa_{base}$ , $ya_{base}$)and ($xb_{base}$ ,$yb_{base}$), respectively.

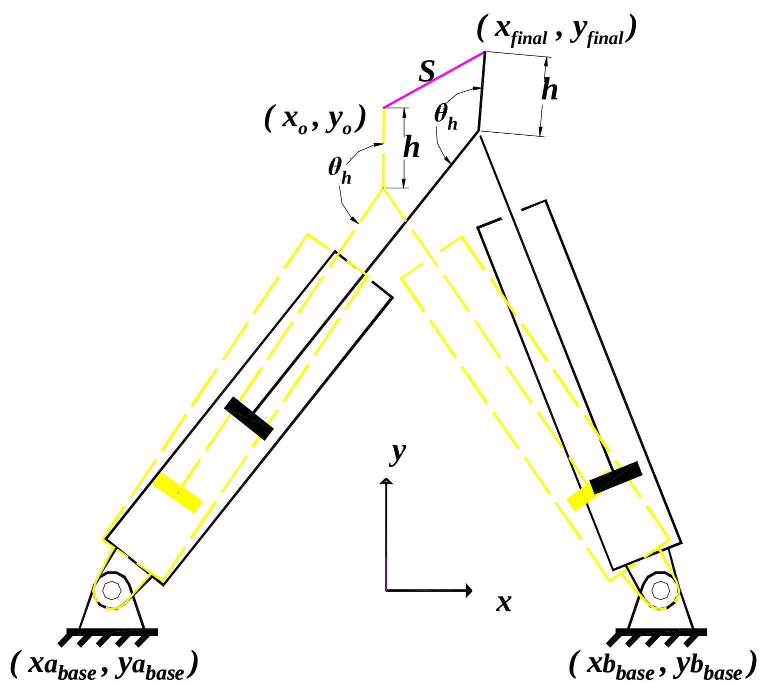


Figure 4.6: Positioning mechanism to move the linking points from ($x_0$ , $y_0$) to ($x_{final}$ , $y_{final}$)

The motion will be accomplished by lengthening/shortening actuators *a* and *b* simultaneously, every time step $\Delta t$. This will lead into a certain instantaneous position $s_i$.

Length of each actuator is function of the instantaneous angle rotated by each stepper motor attached to both actuators.

$$t_{i+1} = t_i + \Delta t, \quad i = 0, 1, 2, \ldots, i_{final} \tag{4.10}$$

where $t_{i=0} = 0$

So, if it is required to extend actuator *a* from its initial length $Lad_0$ to a desired instantaneous length $la\ d_I$ , we need to send

$$Nd_i = 200 \,.\, R \,.\, n \,.\, (lad_i - lad_0) \tag{4.11}$$

similarly for actuator *b*:

$$Nbd_i = 200 \,.\, R \,.\, n \,.\, (lbd_i - lbd_0) \tag{4.12}$$

The letter *d* denotes desired values and not actual values. Desired values *of N* can take any value, while actual values can only be integers because it represents number of pulses. Prediction of the actual values, denoted by *a*, will be shown in a reversed kinematics analysis coming later. The point will be following the same velocity and acceleration profiles discussed earlier. The instantaneous position $s_i$ in the acceleration region is

$$s_i = 0.5\, \ddot{s}_{\max}\ t_i^2 \tag{4.13}$$

In the constant speed region,

$$s_i = S_1 + \dot{s}_{\max}\,( t_i - T_1) \tag{4.14}$$

where

$$T_1 = \dot{s}_{\max} / \ddot{s}_{\max} \tag{4.15}$$

$$S_1 = 0.5\ \dot{s}_{\max}\ T_1 \tag{4.16}$$

The acceleration time $T_1$ is equal to the deceleration time $T_3$. $S_1$ must be less than or equal half $S_{total}$.

In the deceleration region,

$$s_i = S_2 + \dot{s}_{max}\,[t_i - (T_1 + T_2)] - 0.5\,\ddot{s}_{max}\,[t_i - (T_1 + T_2)]^2 \qquad (4.17)$$

where

$$S_2 = S_{total} - 2\,S_1 \qquad (4.18)$$

$$S_{total} = sqrt\,[(x_{final} - x_0)^2 + (y_{final} - y_0)^2] \qquad (4.19)$$

$$T_2 = S_2 / \dot{s}_{max}$$

Notice from (4.10) that $t(i_{final}) = T_1 + T_2 + T_3$ , $s(i_{final}) = S_{total} = S_2 + S_2 + S_2$

The instantaneous desired coordinates $(xd_i\ yd_i)$ for the point is found from the following relationships:

$$xd_i = x_0 + s_i \cos(\alpha) \qquad (4.20)$$

$$yd_i = y_0 + s_i \sin(\alpha) \qquad (4.21)$$

$$: \alpha = \arctan\,[(y_{final} - y_0)/(x_{final} - x_0)] \qquad (4.22)$$

Now, its time to evaluate the instantaneous desired lengths of the actuators. The first step is to evaluate the distance between the point $(xd_i , yd_i)$ and the base of actuator *a*. This distance will be called $La'$ , see figure 4.7, where

$$la' = sqrt\,[(xd_i - xa_{base})^2 + (, yd_i - ya_{base})^2] \qquad (4.23)$$

From the sines law, the angle $\theta_{La'}$ that $la'$ makes with the axis of the actuator *a* is evaluated as

$$\theta_{La'} = \arcsin(h \,.\, \sin(\theta_h) / la') \qquad (4.24)$$

$\theta_{La'}$ is the angle that the axis of actuator *a* makes with the positive direction of the *x*-axis.

Adding $\theta$ to $\theta_{La'}$ will result into the angle γ that can be evaluated as

$$\gamma_i = \arctan [xd_i - xa_{base})/(yd_i - ya_{base})] \qquad (4.25)$$

so, $$\theta d_i = \gamma_i - \theta_{La'i} \qquad (4.26)$$

The variables needed to evaluated $lad_i$ *and* $lbd_i$ are all available now:

$$lad_i = la'_i \,.\, \sin(\pi - \theta_h - \theta_{La'i}) / \sin(\theta_h) - La \qquad (4.27)$$

$$lbd_i = sqrt[\{q - (lad_i + La) \,.\, \cos(\theta d_i)\}^2 + \{(lad_i + La) \,.\, \sin(\theta d_i)\}^2] - Lb \qquad (4.28)$$

Where *La* is the fixed length of actuator *a*, *Lb* is the fixed length of actuator *b*, *q* is the distance separating the centers of rotation of both actuators:

$$q = xa_{base} - xb_{base} \qquad (4.29)$$

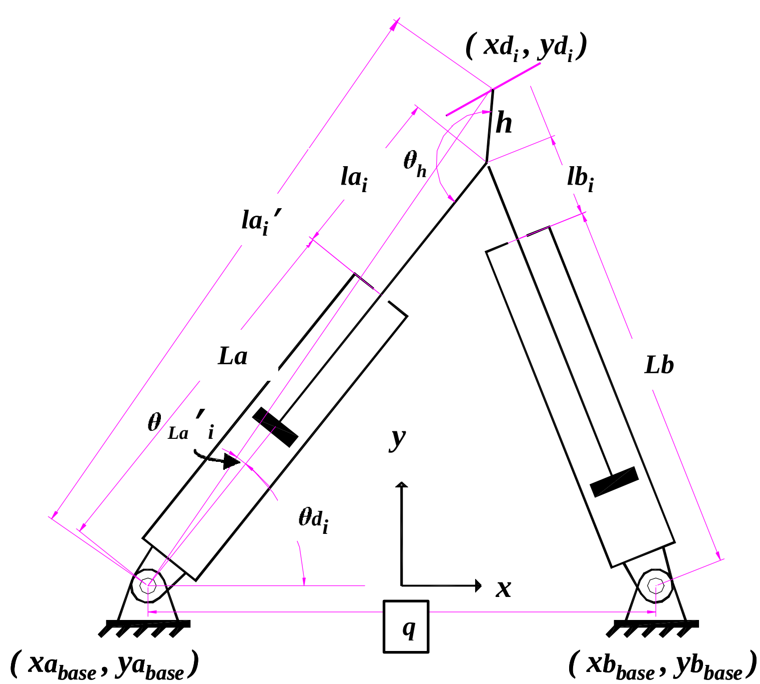

Figure 4.7: Process of evaluating the instantaneous desired lengths of the actuators

The instantaneous desired number of pulses send by motors *a* and *b*, $Nad_i$ and $Nbd_i$ respectively, are found then from (4.11) and (4.12). Notice that $lad_0$ and $lbd_0$ are evaluated using the same procedure discussed above, however, using $xd_i = x_0$ and $yd_i = y_0$.

**4.4.b Logic of pulse sending.**

As was mentioned earlier, actual values of $Na$ can only be integers because they represents number of pulses. Thus, firing of pulses will depend on the following logic:

if $abs[Nd_i - Na_{i-1}] > 0.5,$ (4.30)

$$Na_i = Na_{i-1} + 1.sign(Nd_i - Na_{i-1})$$

else

$$Na_i = Na_{i-1}$$

A pulse is not sent until the motor is desired to rotate an angle close to 1.8$^o$ after the previous actual position $Na_{i-1}$, and will remain at $Na_{i-1}$until that condition is fulfilled. The direction of rotation is dependent on the sign of difference between the desired angular position $Nd_i$ and the previous position $Na_{i-1}$.

**4.4.c Inverse kinematics to obtain actual path of the point:**

The actual values for the variable actuator lengths can now be obtained from actual number of pulses sent to the actuators as follows:

$$la a_i = lad_0 + Naa_i /200R.n \quad (4.31)$$

$$lba_i = lbd_0 + Nba_i /200R.n \quad (4.326)$$

The actual value for the angle between actuator *a* and the positive x axis $\theta a_i$ is

$$\theta a_i = acos[\{ q^2 + ( laa_i + La)^2 - (lba_i + La)^2\} / \{2 * q * ( laa_i + La)\}] \quad (4.337)$$

Actual coordinates of the controlled point can now be evaluated:

$$xa_i = xa_{base} + ( la a_i + La ) \, . \, cos(\theta a_i) + h \, . \, cos \, ( \pi + \theta a_i + \theta_h) \quad (4.34)$$

$$ya_i = ya_{base} + ( la a_i + La ) \, . \, sin(\theta a_i) + h \, . \, cos \, ( \pi + \theta a_i + \theta_h) \quad (4.35)$$

**4.4.d Motion along a circular path centered around ($x_c$, $y_c$):**

The very same to the analysis conducted for the motion on a straight path. The variables $s_i$ , $S_1$ , $S_2$ , $S_3$ , $\dot{s}_{max}$ and $\ddot{s}_{max}$ *are replaced by* $\Phi_i$ , $\Phi_1$ , $\Phi_2$ , $\Phi_3$ , $\Phi_{total}$ , $\dot{\phi}_{max}$ and $\ddot{\phi}_{max}$ . The analysis starts with computing the radius of the circular path as

$$r = sqrt \, [(x_c - x_0)^2 + ( y_c - y_0)^2] \quad (4.36)$$

The maximum angular speed is computed as

$$\dot{\phi}_{max} = \dot{s}_{max} / r \quad (4.37)$$

The initial angle of rotation $\Phi_0$ is angle made by the line passing through the center of rotation ($x_c$, $y_c$) and the initial position ($x_0$ , $y_0$), with the positive $x$-axis as shown in figure 4.8. It is computed as

$$\Phi_0 = arctan \, [(y_c - y_0) / (x_c - x_0)] \quad (4.38)$$

*And the final postion is obtained as*

$$x_{final} = x_c + r \, cos(\Phi_{total} + \Phi_0) \quad (4.39)$$

$$y_{final} = y_c + r \, sin(\Phi_{total} + \Phi_0) \quad (4.40)$$

*The instantaneous desired position is evaluated as*

$$xd_i = x_c + r \, cos(\Phi_i + \Phi_0) \quad (4.41)$$

$$yd_i = y_c + r \, sin(\Phi_+ \Phi_0) \quad (4.42)$$

The rest is exactly the same as the analysis of the motion on a straight path. Figure 4.8

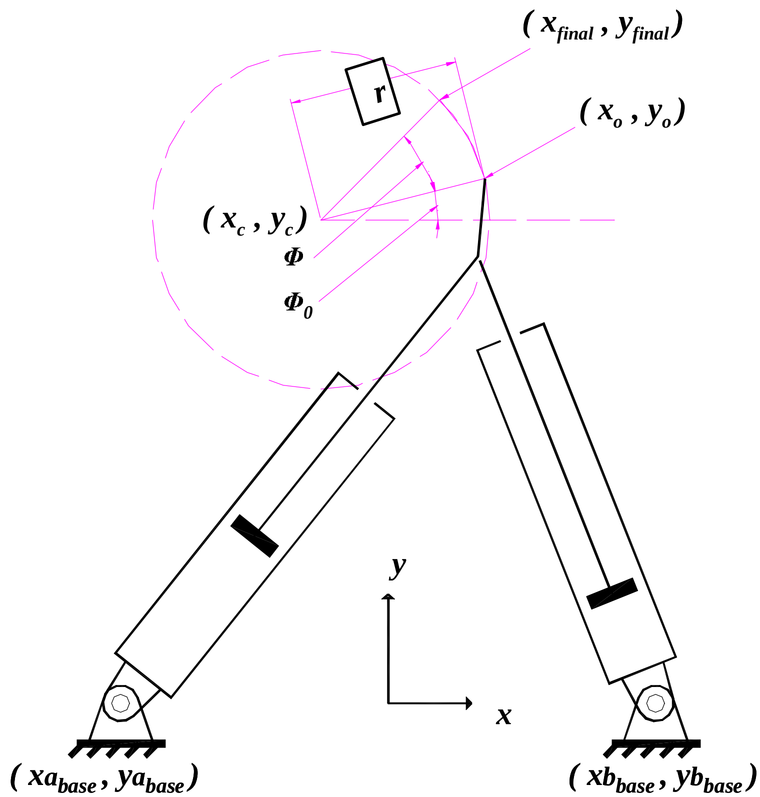


Figure 4.8: Analysis of motion on a circular path.