Hasan G Pasha
ME06M045

DETERMINATION OF NATURAL FREQUENCY AND DAMPING RATIO

OBJECTIVE

- Determine the natural frequency and damping ration for an aluminum cantilever beam,
- Calculate the analytical value of the natural frequency and compare with the experimental value

APPARATUS

1. Test rig
2. Frequency analyzer
3. Function/Waveform generator

THEORY ON VIBRATION

Mechanical Vibration

Mechanical Vibration is defined as the motion of a system (a particle or a body) which oscillates about its stable equilibrium position.

Mechanical Vibration generally results when a system is displaced from a position of stable equilibrium. The system tends to return to its equilibrium position by virtue of restoring forces. However the system generally reaches its original position with certain acquired velocity that carries it beyond that position. Ideally this motion can repeat indefinitely.

Free Vibration

When the vibration motion is maintained by the restoring forces only, the vibration is termed as free vibration.

Natural frequency

Natural frequency is defined as the lowest inherent rate (cycles per second or radians per second) of free vibration of a vibrating system. Its unit is Hz or rad s^{-1} and it is designated by ω_n.

Damping

Damping is dissipation of energy in an oscillating system. It limits amplitude at resonance.

All vibrating systems are damped to some degree by friction forces. These forces can be caused by dry friction or Coulomb friction, between rigid bodies, by fluid friction when a rigid body moves in a fluid, or by internal friction between the molecules of a seemingly elastic body.

Viscous damping and Coefficient of viscous damping

Viscous damping is caused by fluid friction at low and moderate speeds. It is characterized by the fact that the friction force is directly proportional and opposite to the velocity of the moving body.

The magnitude of the friction force exerted on the plunger by the surrounding fluid is equal to cx'. Where c is known as the coefficient of viscous damping expressed in N s/m. It depends on the physical properties of the fluid and depends on the construction of the dashpot.

Critical damping coefficient

Assuming that the motion of the system is defined by the following differential equation:

$$mx'' + cx' + kx = 0$$

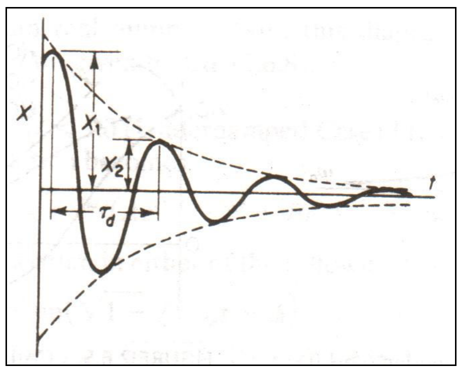
The motion is termed as critically damped when the coefficient of viscous damping equals 2 m ω_n and it is designated by c_c.

Damping ratio

Damping ratio is defined as the ratio of the coefficient of viscous damping to critical damping coefficient. It is designated by ζ.

Measurement of damping ratio experimentally - Logarithmic Decrement

A convenient way to measure the amount of damping present in a system is to measure the rate of decay of free oscillations. The larger the damping, the greater is the rate of decay.



Rate of decay of the oscillation

Considering a damped vibration expressed by the general equation:

$$x = X\, e^{-\varsigma \omega_n t} \sin(\sqrt{1-\varsigma^2}\, \omega_n t + \varphi)$$

Logarithmic decrement can be defined as the natural logarithm of the ration of any two successive amplitudes.

$$\delta = \ln \frac{x_{n-1}}{x_n} = \frac{1}{n} \ln \frac{x_0}{x_n}$$

$$\delta = \varsigma\, \omega_n\, \tau_d \cong 2\pi\varsigma$$

SCHEMATIC DIAGRAM

DESCRIPTION

The test rig consists of a rectangular cross-section, aluminum cantilever beam.

The free end of the beam is connected with a ferromagnetic disc with a pickup device below it. This mechanism serves to trace the vibration. It is based on the laws of electro-magnetic induction.

When the beam vibrates, the gap size changes and this causes the flux density to vary which is calibrated and read from a voltmeter and also fed to an oscilloscope.

The gap between the ferromagnetic disc and the pickup device is adjusted such that it is not less than 5 times the expected amplitude of vibration.

TABULATION

Beam Length 350 mm

Sl No	No of cycles	Scaled Amplitude		Scaled Time		Natural Frequency	Logarithmic decrement	Damping ratio
		Initial mV	Final mV	Initial ms	Final ms	Hz		
1	10	79.71	73.4	-288	-51.6	42.301	0.0082	0.0013
2	10	142.7	126.6	-367.2	-101.4	37.622	0.0120	0.0019
3	10	87.5	78.1	-340.4	-100.8	41.736	0.0114	0.0018
						40.553		0.0017

Beam Length 450 mm

Sl No	No of cycles	Scaled Amplitude		Scaled Time		Natural Frequency	Logarithmic decrement	Damping ratio
		Initial mV	Final mV	Initial ms	Final ms	Hz		
1	10	100	90.6	894	1331	22.883	0.0099	0.0016
2	10	71.9	62.5	402	839.6	22.852	0.0140	0.0022
3	10	65.6	59.4	-475.6	-36.8	22.789	0.0099	0.0016
						22.842		0.0018

Comparison of standard values with experimental values

Sl No	Beam length m	Natural Frequency Hz		Relative error %
		Standard	Experimental	
1	0.35	41.00465	40.553	1.10
2	0.45	24.80528	22.842	7.91

PROCEDURE

1. Set the beam length to 350 mm
2. Excite the aluminum cantilever beam
3. Record the output wave
4. Observe and tabulate the scaled initial and final values (of a set of 10 successive oscillations) of the amplitude and time period
5. Repeat steps 2 through 4 for a beam length of 450 mm
6. Calculate the natural frequency and damping ratio
7. Calculate the standard value of natural frequency and compare it with the experimental values

FORMULAE

$$\delta = \ln\frac{x_{n-1}}{x_n} = \frac{1}{n}\ln\frac{x_0}{x_n}$$

$$\delta = \varsigma\,\omega_n\,\tau_d \cong 2\pi\varsigma$$

$$\varsigma =$$

		$\frac{\delta}{2\pi}$	
f_n	≈	$\frac{n}{\tau_d}$	Hz
	≈	$\frac{n \cdot 1000}{\tau_{final} - \tau_{initial}}$	Hz
f_n	=	$\frac{\omega_n}{2\pi} = \frac{3.52}{2\pi l^2}\sqrt{\frac{EI}{m}}$	Hz
I	=	$\frac{bh^3}{12}$	m^4
m	=	$b\,h\,\rho$	kg m^{-1}

δ	→	Logarithmic decrement	
X_0	→	Amplitude of the first cycle	M
x_n	→	Amplitude of the n^{th} cycle	M
N	→	Number of cycles	
ζ	→	Damping ratio	
τ_d	→	Damped vibration time period	S
ω_n	→	Natural frequency	rad s^{-1}
f_n	→	Natural frequency	Hz
E	→	Modulus of Elasticity/Young's modulus	Pa
I	→	Moment of area about central axis parallel to width	m^{-4}
B	→	Breadth of the beam 0.076 m	M
H	→	Thickness of the beam 0.0061 m	M
ρ	→	Density of the beam 2700 kg m^{-3} for Aluminum	kg m^{-3}

SAMPLE CALCULATION

δ	=	$\frac{1}{n}\ln\frac{x_0}{x_n}$	
	=	$\frac{1}{10}\ln\frac{100}{90.6}$	
	=	0.0099	
ς	=	$\frac{\delta}{2\pi}$	
	=	$\frac{0.0099}{2\pi}$	
	=	0.0016	
f_n	≈	$\frac{n}{\tau_d}$	Hz
	≈	$\frac{n \cdot 1000}{\tau_{final} - \tau_{initial}}$	Hz
	≈	$\frac{10 * 1000}{1331 - 894}$	Hz
	=	22.883	Hz
I	=	$\frac{bh^3}{12}$	m^4
	=	$\frac{(0.076) \cdot (0.0061)^3}{12}$	m^4
	=	1.43755×10^{-9}	m^4

m	=	$b\,h\,\rho$	kg m^{-1}
	=	$(0.076)\,(0.0061)\,(2700)$	
	=	1.25172	
f_n	=	$\dfrac{\omega_n}{2\pi} = \dfrac{3.52}{2\pi\,l^2}\sqrt{\dfrac{EI}{m}}$	Hz
	=	$\dfrac{3.52}{2\pi\,(0.35)^2}\sqrt{\dfrac{(7\text{x}\,10^{10})\cdot(1.43755\text{ x }10^{-9})}{(1.25172)}}$	Hz
	=	41.004645	Hz

SOURCES OF ERROR

The error calculated by comparing the experimental value of the natural frequency with the standard value is as a result of the fact that any vibration is damped to some extent. In this case the Coulomb damping caused due to air was neglected.

Error can also be attributed to the fact that the material in the cantilever might not be uniformly distributed in the material continuum as assumed.

RESULT

- The natural frequency and damping ratio for the aluminum cantilever beam were found experimentally. The results are tabulated below:

Beam length m	**Natural Frequency** Hz	**Damping Ratio**
0.35	40.553	0.0017
0.45	22.842	0.0018

- The standard value of the natural frequency was calculated and compared to the experimental value. The % of relative error was calculated as 1.10 % and 7.91 % beam lengths of 0.35 m and 0.45 m.