

SIMPLE MENSURATIONAL GUIDES

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STRATIFIED SAMPLING

This super-simple example shows calculations to use with this method which should be used if you know the acreage of strata in a forest (either before or after a cruise):

Stratum	No. acres (Ac)	Tenth-acre volumes (V)	ΣV	ΣV^2	n
A	30	5,6,10	21	161	3
B	70	2,4	6	20	2
Σ	100				5

$$\begin{aligned}\text{MeanA} &= (5+6+10)/3 \\ &= 7\end{aligned}$$

$$\begin{aligned}\text{MeanB} &= (2+4)/2 \\ &= 3\end{aligned}$$

$$\begin{aligned}\text{StratMean} &= \text{means weighted by acreages} \\ &= ((\text{AcA} * \text{MeanA}) + (\text{AcB} * \text{MeanB})) / \Sigma \text{Ac} \\ &= ((30*7) + (70*3)) / 100 \\ &= 4.2\end{aligned}$$

$$\text{Variance for a stratum} = (\Sigma V^2 - (\Sigma V)^2/n) / (n-1)$$

$$\begin{aligned}\text{VarianceA} &= (161 - 21^2/3) / (3-1) \\ &= 7.0\end{aligned}$$

$$\begin{aligned}\text{VarianceB} &= (20 - 6^2/2) / (2-1) \\ &= 2\end{aligned}$$

For each stratum we calculate $(\text{Ac}^2 * \text{Variance})/n$, which we'll call X

$$\begin{aligned}\text{for StratumA, } X_a &= (30^2 * 7) / 3 \\ &= 2100\end{aligned}$$

$$\begin{aligned}\text{for StratumB } X_b &= (70^2 * 2) / 2 \\ &= 4900\end{aligned}$$

$$\begin{aligned}\text{StratSE} &= (1/\Sigma \text{Ac}^2 * (X_a + X_b))^{0.5} \\ &= (1/100^2 * (2100 + 4900))^{0.5} \\ &= 0.8\end{aligned}$$

STRIP CRUISE

Strip cruises, with strips randomly or systematically located and running across contours, are often of unequal lengths. In that case, a ratio estimator is appropriate. Assume 3 chain-wide strips (n) are used on a 150-acre (A) tract which is 100 chains (N) wide:

Strip	Length(chains)	x=Area(ac)	y=volume(mbf)	x ²	y ²	xy
1	14	1.4	10	1.96	100	14.0
2	12	1.2	8	1.44	64	9.6
3	10	1.0	5	1.00	25	5.0
Σ	36	3.6	23	4.40	189	28.6

ratio (ratio is vol per acre):

$$\begin{aligned}
 r &= \Sigma y / \Sigma x \\
 &= 23 / 3.6 \\
 &= 6.39
 \end{aligned}$$

vol on forest:

$$\begin{aligned}
 \text{vol} &= r * A \\
 &= 6.39 * 150 \\
 &= 958.5
 \end{aligned}$$

variance (var) and standard error (SE) for total volume:

$$\begin{aligned}
 S_u^2 &= (\Sigma y^2 + r^2 * \Sigma x^2 - 2 * r * \Sigma xy) / (n-1) \\
 &= (189 + 6.39^2 * 4.40 - 2 * 6.39 * 28.6) / (3-1) \\
 &= 1.576
 \end{aligned}$$

$$\begin{aligned}
 \text{var} &= A^2 * 1 / (A/N)^2 * S_u^2 / n * ((N-n)/N) \\
 &= 150^2 * 1 / (150/100)^2 * 1.576 / 3 * ((100-3)/100) \\
 &= 5095
 \end{aligned}$$

$$\begin{aligned}
 \text{SE} &= 5095^{0.5} \\
 &= 71.4
 \end{aligned}$$

DOUBLE SAMPLING

With double sampling an easy-to-measure value (x) is obtained at each of n' points, such as number of "in-trees" at points, and at n randomly selected points the usual measurements necessary for volume estimates are made (y). The ratio estimator works well when the variation in y tends to increase with x (the usual case), although a linear regression estimator may be used if a linear trend with constant variation is indicated. Double sampling reduces the field time necessary to obtain a specified sampling error. A very simple example is used to illustrate calculations.

	x'	x' ²	y	x	x ²	xy	y ²
	2	4	4	2	4	8	16
	3	9					
	6	36	10	6	36	60	100
Σ	11	49	14	8	40	68	116
Mean	3.67	16.3	7	4			

$$n'=3$$

$$n=2$$

r is the ratio of means:

$$r = \text{Mean}y / \text{Mean}x'$$

$$= 7/4$$

$$= 1.75$$

We can now estimate the mean y for the population (P_{pop}):

$$P_{\text{pop}} = r * \text{Mean}x'$$

$$= 1.75 * 3.67$$

$$= 6.42$$

$$S^2y = (\sum y^2 - (\sum y)^2/n) / (n-1)$$

$$= (116 - 14^2/2) / (2-1)$$

$$= 18$$

$$a = (\sum y^2 - 2*r*\sum xy + r^2*\sum x'^2) / (n-1)$$

$$= (116 - 2*1.75*68 + 1.75^2*40) / (2-1)$$

$$= 0.5$$

$$SE = (S^2y/n' + a/n*((n'-n)/n'))^{0.5}$$

$$= (18/3 + 0.5/2*((3-2)/3))^{0.5}$$

$$= 2.5$$

Note: Adjust stand and stock tables using measured points by percentage: (vol. per acre using double sampling - vol. per acre using only measured points) / (vol. per acre using only measured points) * 100

TWO-STAGE

A random sample of primaries (e.g., stands) of size n_p is selected from the total number of primaries (N_p). Sampling is done in selected primaries using n_s secondary sampling units (probably differing for different primaries) of the total number of secondary units in the primary (N_s). Here is a very simple example ($N_p=3$, $n_p=2$):

Primary	No. acres	Second. (N_s)	10th-acre vol. (V_s)	V_p	n_s
A	30	300	5,6,11	22	3
B	70	700	2,4	6	2
C	50	500			
Σ	150	1500			5

$$T_p = \text{estimated total volume in a primary}$$

$$= (V_p / n_s) * N_s: T_A = (22/3) * 300 = 2200: T_B = (6/2) * 700 = 2100$$

r_s is the ratio of volume per tenth acre for a primary:

$$r_s = T_p / N_s: r_A = 2200/300 = 7.3: r_B = 2100/700 = 3$$

r is the overall ratio estimator of volume per tenth acre using selected primaries:

$$r = \Sigma T_p / \Sigma N_s = (2200+2100)/(300+700) = 4.3$$

$$\text{estimated total volume of forest} = r * \Sigma N_s = 4.3 * 1500 = 6450$$

$$\text{Var}_p = ((\Sigma N_s^2 * r_s^2) - 2*r*(\Sigma N_s^2 * r_s) + r^2 * \Sigma N_s^2) / (n_p - 1)$$

$$= ((300^2 * 7.3^2 + 700^2 * 3^2) - 2 * 4.3 * (300^2 * 7.3 + 700^2 * 3) + 4.3^2 * (300^2 + 700^2)) / (2 - 1)$$

$$= 1638100$$

$$S^2 = (\Sigma V_s^2) - (\Sigma V_s)^2 / n_s / (n_s - 1)$$

$$S_A^2 = ((5^2 + 6^2 + 11^2) - 22^2 / 3) / (3 - 1) = 10.3:$$

$$S_B^2 = ((2^2 + 4^2) - 6^2 / 2) / (2 - 1) = 2$$

$$\text{Var}_s = \Sigma N_s^2 * (N_s - n_s) / N_s * S^2 / n_s$$

$$= 300^2 * (300 - 3) / 300 * 10.3 / 3 + 700^2 * (700 - 2) / 700 * 2 / 2 = 794510$$

$$\text{SE} = (N_p^2 / (\Sigma N_s * n_p) * (\text{Var}_p + \text{Var}_s))^{0.5}$$

$$= (3^2 / (1500^2 * 2) * (1638100 + 794510))^{0.5}$$

$$= 2.2 \quad (\text{for total volume} = 2.2 * 1500 = 3300)$$

BIG BAF

A small BAF of 20 and a large BAF of 60 are using at 3 points:

	Using BAF=20		Using BAF=60			
Point	BA	BA ²	BA	Vol.	VBAR	VBAR ²
1	80	6400	5.2329	1027	196.258	38517.203
			2.1816	503	230.565	53160.219
2	100	10000	-	-	-	
3	30	900	-	-	-	
Σ	210	17300			426.823	91677.422

$$n = 2$$

$$m = 3$$

$$\begin{aligned}\text{Mean VBAR} &= 426.823/2 \\ &= 213.411\end{aligned}$$

$$\begin{aligned}\text{Mean BA} &= 210/3 \\ &= 70\end{aligned}$$

$$\begin{aligned}\text{Mean Vol./ac} &= 213.411*70 \\ &= 14939\end{aligned}$$

$$\begin{aligned}\text{VBAR SE} &= ((91677.422 - 426.823^2/2)/(2*(2-1)))^{0.5} \\ &= 17.154\end{aligned}$$

$$\begin{aligned}\text{VBAR SE\%} &= 17.154/213.411*100 \\ &= 8.04\end{aligned}$$

$$\begin{aligned}\text{BA SE} &= ((17300 - 210^2/3)/(3*(3-1)))^{0.5} \\ &= 20.817\end{aligned}$$

$$\begin{aligned}\text{BA SE\%} &= 20.817/70*100 \\ &= 29.74\end{aligned}$$

$$\begin{aligned}\text{Cruise SE\%} &= (8.04^2 + 29.74^2)^{0.5} \\ &= 30.81\end{aligned}$$

3P SAMPLING

3P sampling is similar to double sampling except Probability is Proportional to Prediction, thus PPP. To use 3P one must

- (1) Predict the total of the values to be estimated (as total volume of a hundred trees for which volumes are desired) or in this simulation the total square inches of images.
This estimated value is called KPI.
- (2) Determine the number off 3P samples needed. As you will see, sample size is a random variable (the number you get may be more or less than you want.
- (3) $KZ = KPI / (\text{no. 3P samples desired})$. After each tree is visited, the estimated volume is compared to a random number which can range from 1 to KZ. The volume of any tree for which the estimated volume equals or exceeds the paired random number is a 3P sample tree and must be measured.

Calculation are much simpler than for double sampling as shown, as shown below where x is the estimated value and y the measured value for a 3P sample:

x	y	r=y/x	r ²
2	4	2	4
3			
6	18	3	9
sum	11	5	13
mean		2.5	

$$\begin{aligned} \text{The 3P estimated volume} &= \text{sumx} * \text{average ratio:} \\ &= 11 * 2.5 \\ &= 27.5 \end{aligned}$$

$$\begin{aligned} S &= ((\text{sumr}^2 - (\text{sumr})^2/n) / (n-1))^{.5} \\ &= ((13 - 5^2/2) / (2-1))^{.5} \\ &= 0.707 \end{aligned}$$

$$\begin{aligned} \text{CV\%} &= (S/\text{mean}) * 100 \\ &= 28.3 \end{aligned}$$

$$\begin{aligned} \text{Sampling error \%} &= \text{CV}/n^{.5} \\ &= 28.3/2^{.5} \\ &= 20.0 \end{aligned}$$

LIST SAMPLING

List sampling is useful when a list for the population is available, which is rather rare. This technique is useful for very large timber holdings when all stands cannot be inventoried and when a list of primaries (stands) and their sizes (or better yet estimates of volumes) are available, as is often the case with our modern GPS/GIS technology. This is better than running a very low-intensity inventory on all stands. If the random number is from 1 to 20, Stand 1 is selected, and an appropriate inventory for total stand volume (V) is done, etc. Ratios (R) of (stand volume)/acres or previous volumes are used.

NOTE: A covariance term is ignored in this form of list sampling and also in 3P sampling.

Stand No.	Acres	Cumulative ac*	V	R	R2 = R^2
1	20	1-20	7	0.35	0.1225
2	10	21-30	not selected		
3	30	31-60	3	0.10	0.01
Sums	60			0.45	0.1325
Means				0.225	

$$n = 2$$

$$\begin{aligned}
 S &= \text{standard deviation for R} \\
 &= ((\text{SumR}^2 - \text{SumR}^2/n)/(n-1))^{0.5} \\
 &= ((0.1325 - 0.45^2/2)/(2-1))^{0.5} \\
 &= 0.1768
 \end{aligned}$$

$$\begin{aligned}
 \text{CV}\% &= \text{coefficient of variation \% for R} \\
 &= (S/\text{MeanR}) * 100 \\
 &= (0.1768/0.225) * 100 \\
 &= 78.58
 \end{aligned}$$

$$\begin{aligned}
 \text{Sampling error \%} &= \text{CV}\%/n^{0.5} \\
 &= 78.58/2^{0.5} \\
 &= 56
 \end{aligned}$$

$$\begin{aligned}
 \text{Estimated total volume} &= \text{MeanR} * \text{Acres} \\
 &= 0.225 * 60 \\
 &= 13.5
 \end{aligned}$$

STAND TABLE PROJECTION

Points to remember:

- Obtain increment boring at dbh for all trees at or approaching merchantable size on small fixed-area plots (even if point sampling is used for the main cruise). Measure bark thickness and inches of wood grown (I_i) during the interval for which you are predicting growth (which we'll assume is 10 years).
- Mortality estimates may be available from historical data, but can often be ignored for short time intervals.
- A local volume table often can be made using the data you collect on the cruise.
- As trees increase in girth, diameter increments tend to decrease but basal area increment will tend to be more constant.

For our simple example of bored-tree data, with D = dbh outside bark now, D_{10} = dbh 10 years ago, d = dbh inside bark now, B = basal area now, B_{10} = basal area 10 years ago:

D	B	d	I_i	I_o	D_{10}	B_{10}	B_i
10.0	0.54	8.2	2.1	2.48	7.52	0.31	0.23
11.2	0.68	9.8	1.8	2.13	9.07	0.45	0.23
11.9	0.77	10.0	1.3	1.54	10.36	0.59	0.18
Σ 33.1		28.0					

note:

$$k = \Sigma D / \Sigma d = 33.1 / 28.0 = 1.182 \text{ (outside/inside bark ratio)}$$

$$I_o = k * I_i \text{ (10-year increment outside bark)}$$

$$D_{10} = D - I_o$$

$$B_i = B - B_{10} \text{ (basal area increment)}$$

ok to here

Using a linear regression of the form $B_i = a + bB$, $B_i = 0.3450 - 0.1985 * B$.

Using the stand and stock table for the tract, with D = dbh class, N = number of trees,

V_p = present volume, d_i = dbh increment, and no expected mortality:

D	B	N	V_p	B_i	d_i	N_s	N_U	N in 10 yrs.
10	0.54	21	6	0.24	1.959	0.4	20.6	0.4
12	0.79	23	11	0.19	1.405	6.8	16.2	27.4
14	1.07	16	9	0.13	0.833	9.3	6.7	25.5
16								6.7

$$\text{note: } d_i = ((B + B_i) / 0.005454)^{0.5} - D$$

N_s = number of trees staying in same dbh class = $d_i / \text{dbh class} = d_i / 2$ as %

N_U = number of trees moving one dbh class = 100 - above answer

Can estimate volume in 10 years using local volume table

For 10-inch class (9.0 to 10.9), simply assume 1 tree in each

0.1 class and grow them 1.959 inches to predict dbh in 10 years.

For 1-inch class, a d_i of 1.10 indicates all trees move up 1 class

and 10 % up 2 classes; a d_i of 0.80 means 80% move 1 class and 20% stay in the same class.

LOCAL VOLUME TABLE CONSTRUCTION

To illustrate the local volume table method, assume you measure the diameter at 1-foot stump heights for trees removed in a trespass, as follows:

Stump diam. (in.)

14

23

17

20

In an adjacent, apparently similar stand, you measure stump diameters and obtain volumes, using the volume tables or equations appropriate in that area and for that species, with the following results:

D = Stump diam. (in.) V = Volume (bf)

10 50

16 200

24 600

Using CurveExpert and the quadratic fit of the form $y = a + bx + cx^2$, which generally works well for this type data, we find:

$$V = -42.857 - 16.071 * D + 1.7857 * D^2$$

With this equation, volume is easily calculated for any stump diameter.

MensiCard Approximations H. Wiant / J. Brooks

tree: D =dbh , L =16-ft logs, H =tot ht ft, BA =basal area
stand: L =avgL, H =avgH, BA =BA / ac (trees of interest)

$bf=f*BA*L$ (f: Int =67, Scrib =61, Doyle=51)

$cords=0.0046*BA*H$

$tons=0.013*BA*H$

$hwd: ft^3=0.42*BA*H$

$conifer: ft^3=0.44*BA*H$

assuming 5bf / ft³, $bf=2*BA*H$

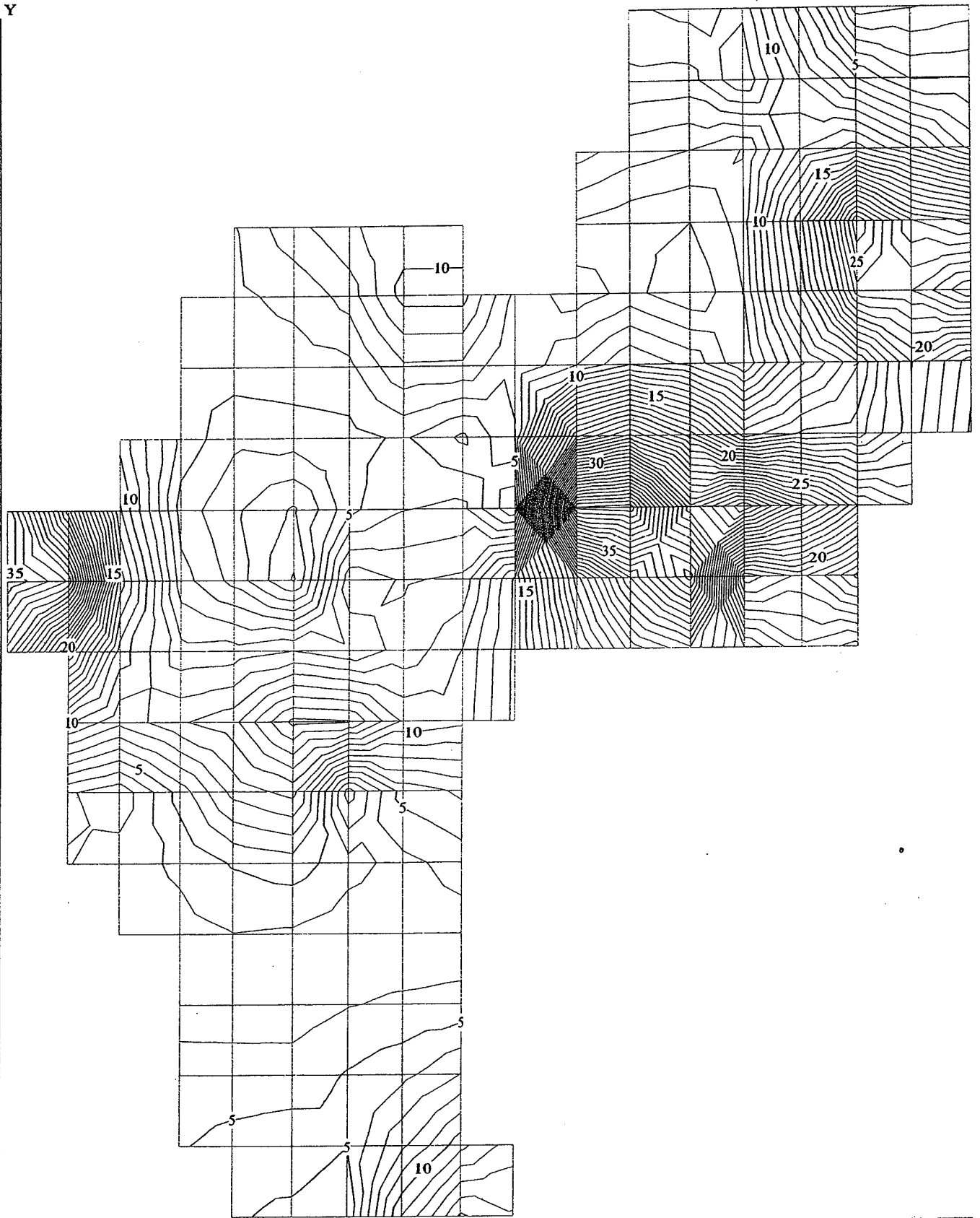
INCLUDE VOLUME CONTOURS ON YOUR CRUISE REPORTS

by Harry V. Wiant, Jr.

Foresters often include maps when providing clients with cruise reports. A map with contours indicating volumes is very helpful, especially to potential buyers of timber who may want to concentrate their field examination on the portions of the tract with greater volumes. This is easily done if the cruise program you use provides per-acre estimates for each point or plot. A version of my Cruise99 inventory program, called CruzPlot, does this. It can be downloaded at <http://www.geocities.com/harryvwiant/>

The contour software, which I highly recommend, is simple to use. Go to <http://www.perspectiveedge.com/availability.html> and download the program, QuikGrid. The program is freeware!

17.1 Y



1.8

0.7

12.6 X