1 General Housekeeping

So far our knowledge of vectors has been pretty sophomoric. We now start a more formal knowledge and use of vectors. The vectors we have talked about so far were really vector quantities. A vector is defined with a formal set of mathematical rules, which we will go into here. We still have 3 basic physics vectors: displacement, velocity, and acceleration. Keep this in mind as we start a general description of vectors.

Vectors are denoted in print by bold print, **A** , or with an arrow on top $\vec{A}$.

The magnitude, or length of a vector, is given by $|\vec{A}|$. It is simply the vector surrounded by absolute value bars.

2 Hat Notation

First define a two dimensional[1] vector $\vec{A}$ in the x-y plane. We first define a set of axis. We do this by defining unit vectors. Unit vectors have a length of one[2] and indicate the direction of the axis. They are symbolized by a hat: $\hat{x}$, $\hat{y}$.

First resolve this vector into its x and y components:

$$\begin{aligned}
\sin\theta &= \frac{A_y}{|A|} \Rightarrow A_y = A\sin\theta \\
\cos\theta &= \frac{A_x}{|A|} \Rightarrow A_x = A\cos\theta \\
\tan\theta &= \frac{A_y}{A_x} \qquad \text{and so on}
\end{aligned}$$

We can then write a vector $\vec{A}$ in terms of its x and y components:

$$\vec{A} = A_x\hat{x} + A_y\hat{y} \tag{1}$$

$$\vec{A} = (A\cos\theta)\hat{x} + (A\sin\theta)\hat{y} \tag{2}$$

This is called hat notation[3].

Question 1 *A person walks 15 m east and then 45 m north. Express the displacement in hat notation.*

When adding two vectors, $\vec{A}$ and $\vec{B}$, simply add their x and y components:

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{x} + (A_x + B_y)\hat{y} \tag{3}$$

[1] In college, Physics for science majors deal with three dimensions. Our description is an incomplete analysis; however, this should help further your understanding. Two dimensions is a special case of three dimensions. In relativity, we extend this to four dimensions; in more advanced physics theories, 11 and more dimensions are being developed.

[2] Remember in trig, the unit circle which has a radius of 1.

[3] I don't know if there is a formal name for this; I simply call it hat notation. I know, I know, that is a real creative name

Vectors have several mathematical addition rules they follow. They are commutative

$$\vec{A} + \vec{B} = \vec{B} + \vec{A} \tag{4}$$

they are associative

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C}) \tag{5}$$

and they can be subtracted

$$\vec{A} - \vec{B} = \vec{A} + (\vec{-B}) \tag{6}$$

Also, when multiplying a vector and a scalar together, you simply multiply the scalar by each component:

$$a\vec{A} = aA_x\hat{x} + aA_y\hat{y} \tag{7}$$

Question 2 *What does $-\vec{A}$ look like?*

3 Practice Questions With Hat Notation

For $\vec{A} = \hat{x} + 2\hat{y}$ and $\vec{B} = -2\hat{x} + 3\hat{y}$, find the following:

1. $\vec{A} + \vec{B}$
2. $\vec{A} - \vec{B}$
3. $\vec{B} - \vec{A}$
4. $|\vec{A}|$
5. $|\vec{B}|$
6. $|\vec{A} + \vec{B}|$

4 Dot Product

There are a few ways to multiply vectors together. The one that is most important to us is the dot[4] product. Given two vectors, $\vec{A}$ and $\vec{B}$ is defined as:

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y \tag{8}$$

It is read as "A dot B."

This is especially useful when finding the magnitude of a vector.

Question 3 *Find $\vec{A} \cdot \vec{A}$*

[4] AKA scalar product or inner product

The answer you get should remind you of the Pythagorean Theorem. What does $\vec{A} \cdot \vec{A}$ equal? You should see it is related to the magnitude of the vector, i.e.

$$\vec{A} \cdot \vec{A} = |\vec{A}|^2 \tag{9}$$

Therefore we can say

$$|\vec{A}| = \sqrt{\vec{A} \cdot \vec{A}} \tag{10}$$

In class, we used the Pythagorean Theorem to find the magnitude. In more than two dimensions, we can not use the theorem. We must use the dot product. Luckily in our class, we can use either method.

5 Practice With Dot Products

For these vectors, $\vec{a} = 4\hat{x}$, $\vec{b} = 3\hat{y}$, $\vec{c} = 3\hat{x} + 4\hat{y}$, find the following:

1. $\vec{a} \cdot \vec{b}$
2. $\vec{a} \cdot \vec{c}$
3. $\vec{b} \cdot \vec{c}$
4. $\vec{c} \cdot \vec{b}$
5. $|\vec{a}|$
6. $|\vec{b}|$
7. $|\vec{c}|$
8. $|-\vec{a}|$
9. $|-\vec{b}|$
10. $|\vec{a} + \vec{b}|$

6 Dot Product Revisited

There is another definition of the dot product:

$$\vec{A} \cdot \vec{B} = |A||B| \cos\theta \tag{11}$$

Here θ is the angle between the two vectors. This may be helpful when finding angles between axis.

Question 4 *For $|C| = 4$ and $|D| = 3$, find θ if $\vec{C} \cdot \vec{D}$ = A) 0 B) 12 C) -12*

Question 5 *Find the angle between the vector $\vec{d} = 3\hat{x} + 4\hat{y}$ and the x axis.*

7 General Vector Problems

1. For $\vec{a} = 3.0\hat{x} - 4.0\hat{y}$ and $\vec{b} = -2.0\hat{x} + 3\hat{y}$, find the following:
   (a) $\vec{a} + \vec{b}$
   (b) $\vec{a} - \vec{b}$
   (c) $\vec{b} - \vec{a}$
   (d) $|\vec{b}|$
   (e) $|\vec{a}|$
   (f) $|\vec{a} + \vec{b}|$
   (g) The direction of $\vec{a}$
   (h) The direction of $\vec{b}$
   (i) The direction of $\vec{a} + \vec{b}$
2. Two vectors are given by $\vec{a} = 3.0\hat{x} + 5.0\hat{y}$ and $\vec{b} = 2.0\hat{x} + 4\hat{y}$. Find A) $\vec{a} \cdot \vec{b}$ B) $(\vec{a} + \vec{b}) \cdot \vec{b}$

8 Physics Problems

1. A person has a velocity of $5\hat{x} + 12\hat{y}$ m/sec and walks for 30 seconds. What is his displacement?
2. A person has a velocity of 5 m/sec in a direction of 53° and walks for 30 seconds What is his displacement?
3. A boat travels at 30° north of east for 150 m and then 120 m at 0 °. What is the total displacement?
4. A boat travels at 30° north of east for 150 m and then 120 m at 75 °north of east. What is the total displacement?
5. A boat travels at 30° north of east for 150 m and then 120 m at 75 °south of east. What is the total displacement?
6. A boat travels at 30° north of east with a velocity of 10 m/sec for 30 seconds and then at 75 °north of east with a velocity of 4 m/sec for 80 seconds. What is the total displacement of the boat?
7. A golfer takes three putts to get the ball in the hole. The first putt displaces the ball 12 ft north, the second 6.0 ft -45° and the third 3.0 ft 225°What displacement was needed to put the ball in the hole on the first put.*****Problem Set
8. A radar station detects an airplane approaching directly from the east. At first observation, the range to the plane is 1200 ft at 40° above the horizon. The airplane is tracked for another 123° in the vertical east-west plane. The range at final contact is 2580 ft. Find the displacement of the plane during this period.