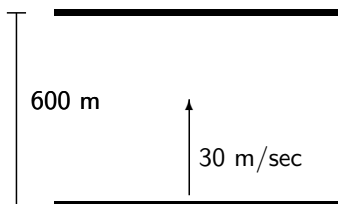


1 Introduction

One of the common misconceptions concerning physics is the independence of two dimensional motion. Generally speaking, a change in horizontal (or east/west) motion of an object will not affect the vertical motion (or north/south). This also works vice-versa. Let's see this in action by examining a boat crossing a river in two different situations.

2 Crossing a River - No Current

Our first situation involves a boat crossing a still river, i.e. no current. The river is 600 meters wide, and the boat's engine speed is 30 m/sec. If the boat sets out perpendicular to the river, how long will it take to reach the other side?



$$t = ?$$

$$x_f = 600m$$

$$x_i = 0$$

$$v = 30m/sec$$

$$v = \frac{x_f - x_i}{t} \quad (1)$$

$$t = \frac{x_f}{v} \quad (2)$$

$$= \frac{600m}{30m/sec} \quad (3)$$

$$t = 20sec \quad (4)$$

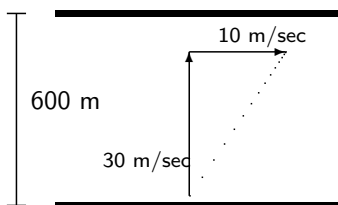
So in the absence of a current, the boat will take 20 seconds to cross the river.

3 Crossing a River - Current = 10 m/sec

This leads us to our second scenario: how will a current affect the time to cross? That is, if there is a (completely) downstream current of 10 m/sec, now how long will it take to cross the river?¹

Question 1 Will it take more time, less time, or the same amount of time?

We can all agree the boat will no longer go straight across the river; it will follow a path that is somewhat northeast. This leads us to two questions: (1) How does the current affect the boat's resultant velocity? and (2) What direction will the boat travel? We will take the current to be horizontal (x-direction), and the boat to launch vertically (y-direction).



To find the magnitude of resultant:

$$|v|^2 = v_x^2 + v_y^2$$

$$|v| = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(10m/sec)^2 + (30m/sec)^2}$$

$$= \sqrt{1000m^2/sec^2}$$

$$|v| = 31.623m/sec$$

To find the direction:

$$(5) \tan \theta = \frac{v_y}{v_x}$$

$$(6) \tan \theta = \frac{30}{10}$$

$$(7) \tan \theta = 3$$

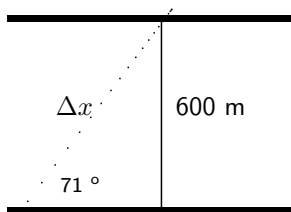
$$(8) \theta = \tan^{-1}(3)$$

$$(9) \theta = 71.565^\circ$$

¹Assume the boat's speed is still 30 m/sec, and it launches perpendicular to the river; the river is still 600 m wide.

Notice that when I go perpendicular to the current, I am able to go faster than my engine allows me to. Everyone can agree that if I go with the current, I will move faster, but this also applies if I am moving perpendicular to the current.

So far, we have only used our knowledge of trig to figure this out. How does this relate to physics? We want to know how long it will take now to reach the other shore. Before, we used the boat's engine speed (30 m/sec) and the displacement of the boat (600 m). What should we use this time? Now the boat does not travel directly across the river; it travels at a angle of 71.565° with the horizontal and with a speed of 31.623 m/sec. Let's now look at our **displacement** vectors:



$$\begin{aligned}\sin \theta &= \frac{OPP}{HYP} \\ \sin \theta &= \frac{y}{\Delta x} \\ \Delta x &= \frac{y}{\sin \theta} \\ &= \frac{600m}{\sin 71.565} \\ \Delta x &= 632.456m\end{aligned}\tag{10}$$

(11)

Now that we have the velocity from (9) and the displacement from (10), we can use our knowledge of physics. Using the same equation as (1), we see:

$$v = \frac{x_f - x_i}{t}\tag{12}$$

$$t = \frac{x_f}{v}\tag{13}$$

$$= \frac{632.456m}{31.623m/sec}\tag{14}$$

$$t = 20sec\tag{15}$$

We see that from (15) and (4) the time to cross, 20 seconds, is the same. This means that the time to cross the river is the same!

4 Conclusions

The point of this is to illustrate the independence of horizontal and vertical motion. Regardless of the current (horizontal motion), the time to cross (vertical motion) the river is not affected. In general, we say,

Definition 1 *Horizontal motion is independent of vertical motion, i.e. a change in one³ does not affect the other.*

²We don't have the adjacent side!

³especially, say, an acceleration (specifically gravity)