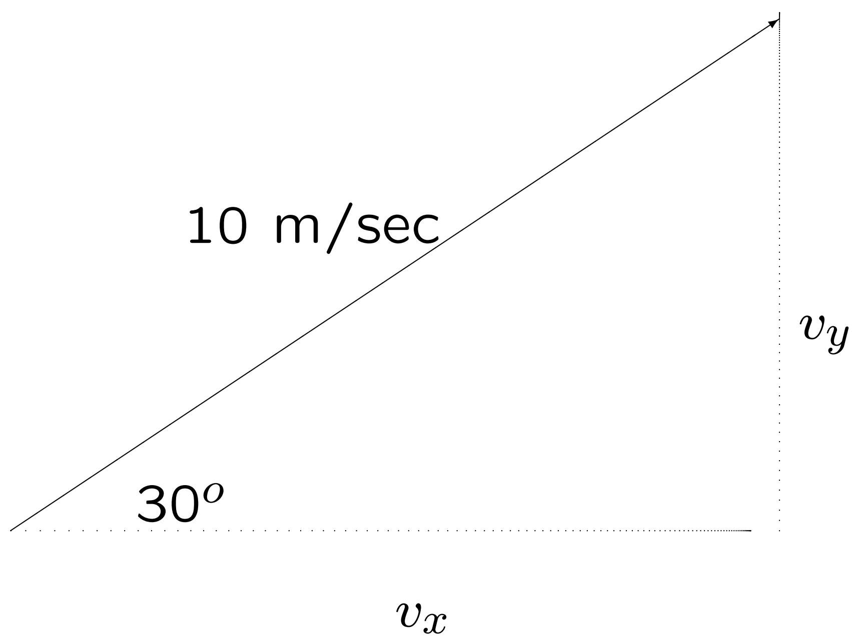
So far, we have only dealt with one dimensional problems in physics. Here, we will derive equations to use when an object moves in two dimensions.

Definition 1 *Horizontal motion is independent of vertical motion, and vice versa.*

- These equations will hold true for all types of motion. We have done a specific case (one dimensional kinematics), and now we will move on to a more general case (two dimensions).

- For a new theory to truly work, it should encompass previous theories as special cases of the new ones.

Practice



$$\sin\theta = \frac{v_y}{|v|}$$
$$v_y = |v|\sin\theta \quad (1)$$
$$v_y = (10m/sec)\sin(30)$$
$$v_y = 5.0\ m/sec \quad (2)$$

$$\cos\theta = \frac{v_x}{|v|}$$
$$v_x = |v|\cos\theta \quad (3)$$
$$v_y = (10m/sec)\cos(30)$$
$$v_y = 8.6\ m/sec \quad (4)$$

x Direction (Horizontal) Motion

Question 1 *Is there an acceleration in the horizontal direction?*

For a given velocity at some angle above the x axis, how much of it "lies" along the x axis, i.e. what is the x component of the velocity?

Question 2 *What is this process called?*

Remember from practice (3); this is what we use. Since we are moving at a constant velocity, we can use this:

$$\begin{aligned} v_x &= \frac{x_f - x_i}{t} \\ x_f &= x_i + v_x t \\ x_f &= x_i + (|v_i| \cos\theta) t \\ x_f &= x_i + v_i t \cos\theta \end{aligned} \qquad (5)$$

$$\boxed{x_f = x_i + v_i t \cos\theta}$$

y Direction (Vertical) Motion

Question 3 *Is there an acceleration in the vertical direction?*

First, we find the y component of the velocity, using (1). Since we are not moving with a constant velocity, we use our standard equation:

$$\begin{aligned} x_f &= x_i + vt + \frac{1}{2}at^2 \\ y_f &= y_i + v_y t + \frac{1}{2}at^2 \\ y_f &= y_i + (v_i \sin\theta)t + \frac{1}{2}at^2 \\ y_f &= y_i + v_i t \sin\theta + \frac{1}{2}at^2 \\ y_f &= y_i + v_i t \sin\theta - \frac{1}{2}gt^2 \qquad (6) \end{aligned}$$

$$\boxed{y_f = y_i + v_i t \sin\theta - \frac{1}{2}gt^2}$$

Trajectory

To follow the path of the particle, we need to find an equation independent of time. Luckily we have two equations from previous slides that we can use to eliminate time.

1. Use equation (5) and solve for time.

2. Plug into equation (6).

3. Simplify using trigonometric identities and other algebra techniques.

College prep should go through these steps. If we assume that the object starts with $y_i = 0$ and $x_i = 0$, then when you finish, you should find:

$$\boxed{y_f = v_i \tan\theta x_f - \frac{g}{2v_i^2 \cos^2\theta} x_f^2}$$

You should notice something very unique about this equation.

Question 4 *What is the relationship between y_f and x_f?*

Question 5 *What would the graph of y_f versus x_f look like?*

If you have a graphing calculator, you can use this equation to graph the trajectory of the object. Further, you can use the parametric mode to find the time an object is at a certain point, using equations (5) and (6). Please consult your manual or see me during office hours. Your graphing calculator is a very useful tool when solving these problems.

Question 6 *CP: Be able to use the quadratic formula to solve this for x_f.*

There are several terms that are helpful when dealing with projectile motion:

- time of flight

- range

- maximum height

Time of Flight

- time the object takes to return to its original height

- (sort of) the time the object is in the air

- If we use equation (6) when $y_f = y_i$, the quadratic formula, and trig identities, then we can get an expression for time of flight:

$$y_f = y_i + v_i t \sin\theta - \frac{1}{2} g t^2 \tag{7}$$

$$0 = v_i t \sin\theta - \frac{1}{2} g t^2 \tag{8}$$

$$t_f = \frac{2v_i^2}{g} \sin\theta \tag{9}$$

$$\boxed{t_f = \frac{2v_i}{g} \sin\theta}$$

Range

- the horizontal (x) distance traveled by the object during its time of flight

- projectiles fired at COMPLEMENTARY angles (why, thank you) at the same velocity have equal ranges

- to maximize range,

- Use the equation (9) and (5) to find this:

$$
\begin{aligned}
R \equiv x_f - x_i &= v_i t \cos\theta \\
R &= (2\cos\theta \sin\theta)\frac{v_i^2}{g} \\
R &= \frac{v_i^2}{g}\sin 2\theta \qquad (10)
\end{aligned}
$$

$$\boxed{R = \frac{v_i^2}{g}\sin 2\theta}$$

Maximum Height

- the maximum height an object reaches (kind of a misnomer)

- occurs when time derivative of equation (6) is zero

- (usually) occurs at half of the time of flight

- if you used equation (9) to find the time of flight to be 4 seconds, when does the maximum height occur?

- Use this time in equation (6) to find your max height

- This was for an object projected at any angle

- Objects can also be projected completely horizontal and completely vertical

Question 7 *What is the angle of an object that is horizontally projected?*

Question 8 *What is the angle of an object that is vertically projected?*

Using your answer to Question 8, figure out what equations (5) and (6) reduce to. Hint: It should be very familiar!