Two kinds of potential difference for a capacitor

Hamid V. Ansari
Department of Physics, Isfahan University, Isfahan, IRAN
Personal address: No. 16, Salman-Farsi Lane, Zeinabieh Street, Isfahan, Postal Code 81986, IRAN

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Abstract

It is shown that contrary to this current belief that the electrostatic potential difference between the two conductors of a capacitor is the same potential difference between the two poles of the battery which has charged it, the first is twofold compared with the second. We see the influence of this in the experiments performed for determination of charge and mass of the electron.

1 Two kinds of potential difference for a capacitor

At present in all the textbooks of Electricity and Magnetism wherever electrostatic potential difference between the two conductors of a capacitor is concerned if to its producer source, ie the battery, has been pointed implicitly or explicitly, it is shown or stated implicitly or explicitly that this electrostatic potential difference is equal to the potential difference between the two poles of the battery that has charged the capacitor. But we now shall prove easily that the electrostatic potential difference between the two conductors of a capacitor is twofold compared with the potential difference between the two poles of the battery which has charged it.

Suppose that the potential difference between the two poles of the battery is $\Delta\phi$ and the electrostatic potential difference between the two conductors of the capacitor is $\Delta\phi'$. It is obvious that if the charge collected on the capacitor is Q, the battery has transmitted it through itself under the potential difference $\Delta\phi$ and then has given it an energy equal to $Q\Delta\phi$. But Eq. (3) of the paper "Independence of capacitance from dielectric" states that the electrostatic potential energy of the capacitor is $1/2Q\Delta\phi'$. According to the conservation law of energy then we must have $Q\Delta\phi = 1/2Q\Delta\phi'$ or $\Delta\phi' = 2\Delta\phi$.

A simple physical reasoning shows this fact too: When stating that the electrostatic potential energy between the two conductors of the capacitor is $\Delta\phi'$ we mean that supposing that all the capacitor charges are fixed, if supposedly a one-coulomb external point charge starts to move from one of the two conductors under the influence of the electrostatic force of the capacitor until it reaches the other conductor, the work performed on it by this force will be $\Delta\phi'$, without any change in the charges on the conductors. But if we suppose that the magnitude of the charge on each conductor of the above capacitor is one coulomb and it is possible that charges separate from a conductor and moving in the space between the two conductors reach the other conductor, then the total work performed on this one-coulomb charge by the electrostatic force of the capacitor will not be certainly equal to $\Delta\phi'$, because with each transmission of some part of the charge, magnitude of the charge on each conductor (and consequently the electrostatic field between the two conductors) is decreased and does not remain unchanged as before. The above argument shows that this work will be $1/2\Delta\phi'$, because this is in fact the same work done by the battery for charging the capacitor being conserved in the capacitor in the form of potential energy which is being released now. We show this matter in an analytical manner too: Suppose that our capacitor is a parallel-plate one and its charge is Q. If a separate Q-coulomb charge travels from a plate to the other one, the work performed on it will be

$$QEd = Q\frac{Q}{\epsilon A}d = \frac{d}{\epsilon A}Q^2, \tag{1}$$

while for calculating the work performed on the charge of the capacitor itself being plucked bit by bit traveling from a plate to the other one, we should say that the work performed on a differential charge $-dQ$ (note that dQ is negative), similar to Eq. (38) of the paper "Independence of capacitance from dielectric", is

$$(-dQ)Ed = -dQ\frac{Q+dQ}{\epsilon A}d = -\frac{d}{\epsilon A}(Q+dQ)dQ.$$

Sum of these differential works is

$$\int_{Q=Q}^{0} -\frac{d}{\epsilon A}(Q+dQ)dQ = \frac{1}{2}\frac{d}{\epsilon A}Q^2$$

which is half of the previous work (shown in Eq. (38) of the paper "Independence of capacitance from dielectric").

Thus we should expect to have $2\Delta\phi = d/(\epsilon_0 A)Q$ when a battery with the potential difference $\Delta\phi$ has charged a parallel-plate capacitor, while hitherto it is thought that $\Delta\phi = d/(\epsilon_0 A)Q$. Since all the parameters of both the recent relations are measurable ($\Delta\phi$ by voltmeter), the truth or untruth of each can be tested practically.

We should notice a point. When connecting a voltmeter to the two conductors of a charged capacitor, it measures $\Delta\phi$ not $\Delta\phi'$, because its operation is based on passing a weak electric current through a circuit in the instrument and measuiring the potential difference between the two ends of the circuit; and passing of a current means in fact the same being plucked of the capacitor charge bit by bit from the conductors, and then the voltmeter measures $\Delta\phi$.

We should also say that there is no need that in the existent calculations of electrical cicuits the potential difference of each capacitor to be made double, because in these calculations the same $\Delta\phi$ has been in fact intended not $\Delta\phi'$, because the electric current passing through the circuit including the capacitor is the same process of gradual loading and unloading of the capacitor, not passing of charge through the space between the two conductors of the capacitor retaining the capacitor charge unchanged. Therefore, it is proper to give $\Delta\phi$ a name other than the electrostatic potential difference which is the name of $\Delta\phi'$. Let's call it (ie $\Delta\phi$) as circuital potential difference of the capacitor. In this manner when it is necessary to apply closed circuit law we must consider just this circuital potential difference when passing the capacitor not its electrostatic potential difference.

Now, again, consider a closed circuit of a battery, with the potential difference $\Delta\phi$, and a capacitor, with the capacitance C. Let's investigate the usual method of analysis of RC (or generally RLC) circuits and see what the difficulty is in it. Without missing anything we suppose that the circuit has no resistance (ie $R = 0$). When a differential electric charge dQ passes through the battery causes a differential change in the electrostatic energy of the capacitor. In the first instance it seems that when the differential charge dQ passes through the battery it gains the differential energy $\Delta\phi dQ$ which, as a rule according to the conservation law of energy, this same energy must be conserved in the capacitor in the form of $d(Q^2/(2C))$, and then

$$\Delta\phi dQ = d(\frac{Q^2}{2C}) \Rightarrow \Delta\phi dQ = \frac{Q}{C}dQ \Rightarrow \Delta\phi = \frac{Q}{C} \Rightarrow \Delta\phi - \frac{Q}{C} = 0$$

which is just the same result which we could obtain from the closed circuit law by traveling one time round the circuit if the potential difference between the two conductors of the capacitor was taken electrostatic potential difference, ie $\Delta\phi' = Q/C$, not circuital potential difference, ie $Q/(2C)$! The difficulty is that the relation $\Delta\phi dQ = d(Q^2/(2C))$ is not necessarily true, for this reason: If we had a mathematical relation, in the form of an equality, between the energy given by the battery and the electrostatic energy stored in the capacitor (ie $Q^2/(2C)$), we could differentiate from each side of the equality relation and understand that the change of energy in the capacitor in the form of $d(Q^2/(2C))(= Q/CdQ)$ is exactly arising from what the differential change in the battery. But since there is no such a relation, we cannot necessarily infer that change of energy in the capacitor in the form of Q/CdQ is arising from passing of the charge dQ through the battery and consequently from differential change of $\Delta\phi dQ$ in the energy given by the battery, because eg by writing $Q/(2C)(2dQ)$

instead of Q/CdQ we can claim that this change of energy in the capacitor is arising from passing of the charge $2dQ$ through the battery and consequently from differential change of $\Delta\phi(2dQ)$ in the energy given by the battery (ie $\Delta\phi(2dQ) = Q/(2C)(2dQ)$), and the previous reasonings showed that incidentally this is the case.

Thus, we should bear in mind that in the analysis of RLC circuits we must attribute only the circuital potential difference, ie $Q/(2C)$, not the electrostatic potential difference, ie Q/C, to the capacitor of the circuit. (Refer to the discussion of RLC circuit in the paper "Independence of capacitance from dielectric".) Also it is notable that since current instruments indeed measure capacitance of a capacitor using the formula $C = Q/\Delta\phi'$ while taking $\Delta\phi$ instead of $\Delta\phi'$, they give us in fact $Q/\Delta\phi = Q/(\Delta\phi'/2) = 2(Q/\Delta\phi') = 2C$ as the capacitance; in other words what they measure as capacitance is in fact double the capacitance. In this manner what we see as $2C$ in the equations (29) and (30), for example, is the same amount our current instruments give as the capacitance.

It is necessary to note the influence that inattention to the abovementioned problem (ie difference between $\Delta\phi$ and $\Delta\phi'$) has on the results of the experiments of Millikan and Thomson for determining charge and mass of the electron (and similarly positive ions).

In the experiment of Millikan the electric charge of each charged oil droplet is proportional to k/E in which k is the coefficient of proportion of Stokes and E is the electrostatic field between the two plates of the parallel-plate capacitor used in the experiment. As we know E between the two plates of a parallel-plate capacitor is equal to the electrostatic potential difference $\Delta\phi'$ divided by the distance d between the two plates. So the charge of each droplet is proportional to $k/\Delta\phi'$. But for practical determination of $\Delta\phi'$ the potential difference read by the voltmeter connected to the plates of the capacitor is considered erroneously, while as we said this potential difference, $\Delta\phi$, which we called it as circuital potential difference, is half of $\Delta\phi'$. In other words as a rule the quantity so far recognized as the charge of a droplet should be two times larger than the real charge of the droplet and then the electron's charge obtained from the numerous repetitions of the experiment of Millikan should be really half of what is at present accepted as the charge of electron.

But this is not the case because the experiment of Millikan plainly lacks sufficient accuracy (and a tolerance up to half of the real amount seems natural for it because certainly it is unlikely that the electrons are added or deducted only one by one). In fact it seems that the results of this experiment have been adapted in some manner for being in conformity with the results of the exact experiment of determination of electric charge of electron by X-ray. (As we know in this experiment the wavelength of X-ray can be determined by its diffraction via a diffraction grating with quite known specifications, and then having this wavelength and Bragg's equation and analyzing the diffraction of the ray via a crystal lattice the lattice spacing, d, of the crystal can be determined; thereupon considering the molecular mass and crystal density Avogadro's number N_0 can be calculated with sufficient accuracy and using it in the formula $F = N_0 e$, in which F is the Faraday constant and e is the charge of electron, e can be obtained which is the same that has been accepted at present as the charge of the electron.)

In the experiment of Thomson too, for evaluation of q/m related to the charge and mass of the electron in the cathodic ray, this quantity, ie q/m, is obtained proportional to the electrostatic field E between the two plates of the parallel-plate capacitor through which the cathodic ray passes. But again for practical determination of E the above-mentioned error is repeated and while E is really equal to $\Delta\phi'/d$ the amount read on the voltmeter, $\Delta\phi$, (which is in fact equal to $1/2\Delta\phi'$) is set instead of $\Delta\phi'$. In other words as a rule the quantity hitherto considered as q/m of the electron (in the experiment of Thomson) should be half of its real amount. Then, to obtain the real value of q/m we must multiply the value accepted presently as q/m by 2.

But here we should say that it seems that this experiment (or any other similar one) is not accurate in determining q/m of electron or positive ions since in it a shooting motion has been assumed for the electron in the cathodic ray (or for the positive ion in the positive ray), while as explained in detail in the paper "Independence of capacitance from dielectric" we must consider for it a longitudinal wave motion in the gas medium existent in the tube without any charge transferring, and it seems that such a wave motion, although has many similarities with the shooting motion, is not exactly the same shooting motion and has difference with it. Thus, it is necessary to doubt what has been accepted as the mass of electron.