The product rule is

$$\frac{d}{dx}[f(x).g(x)] = \frac{df(x)}{dx}.g(x) + f(x).\frac{dg(x)}{dx},$$

In other words,

The derivative of the product = (the derivative of the first).(second)+(first).(the derivative of the second).

Example (1) Find the derivative of $f(x) = 3x^3(x^2 - 2x + 2)$

Solution

$$\begin{aligned}\frac{df(x)}{dx} &= \frac{d3x^3}{dx}.(x^2 - 2x + 2) + 3x^3.\frac{d(x^2 - 2x + 2)}{dx} \\ &= 9x^2.(x^2 - 2x + 2) + 3x^3.(2x - 2) \\ &= 15x^4 - 24x^3 + 18x^2\end{aligned}$$

Example (2) If $y = (x-1)(x-2)(x-3)$, then find $\frac{dy}{dx}$

Solution

$$\begin{aligned}\frac{dy}{dx} &= \frac{d(x-1)}{dx}.(x-2)(x-3) + \frac{d(x-2)}{dx}.(x-1)(x-3) + \frac{d(x-3)}{dx}.(x-1)(x-2) \\ &= (1).(x-2)(x-3) + (1).(x-1)(x-3) + (1).(x-1)(x-2) \\ &= (x^2 - 5x + 6) + (x^2 - 4x + 3) + (x^2 - 3x + 2) \\ &= 3x^2 - 12x + 11\end{aligned}$$

The quotient rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\left[\frac{df(x)}{dx}\right].g(x) - f(x).\left[\frac{dg(x)}{dx}\right]}{[g(x)]^2},$$

In other words,

The derivative of the quotient

=

$$\frac{[(\text{the derivative of numerator}).(\text{the denominator}) - (\text{the numerator}).(\text{the derivative of denominator})]}{(\text{the denominator})^2}$$

Example (3) Find the derivative of $f(x)=\dfrac{2x-3}{4x+1}$

Solution

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{2x-3}{4x+1}\right)=\frac{\frac{d(2x-3)}{dx}.(4x+1)-(2x-3).\frac{d(4x+1)}{dx}}{(4x+1)^2}$$

$$=\frac{(2).(4x+1)-(2x-3).(4)}{(4x+1)^2}$$

$$=\frac{14}{(4x+1)^2}$$

Example (4) Differentiate $f(x)=\dfrac{x-5}{8\sqrt{x}}$

Solution

$$\frac{d}{dx}f(x)=\frac{1}{8}\frac{d}{dx}\left(\frac{x-5}{\sqrt{x}}\right)=\frac{1}{8}\left[\frac{\frac{d(x-5)}{dx}.\sqrt{x}-(x-5)\frac{d\sqrt{x}}{dx}}{(\sqrt{x})^2}\right]$$

$$=\frac{1}{8}\left[\frac{(1).\sqrt{x}-(x-5).\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}\right]$$

$$=\frac{1}{8}\left[\frac{\{\sqrt{x}-(x-5).\frac{1}{2\sqrt{x}}\}}{x}\right].\frac{\sqrt{x}}{\sqrt{x}}$$

$$=\frac{1}{8}\left[\frac{x-\frac{(x-5)}{2}}{x\sqrt{x}}\right].\frac{2}{2}$$

$$=\frac{2x-x+5}{16x\sqrt{x}}$$

$$=\frac{x+5}{16x\sqrt{x}}$$