Example (1) If $y = f(x)$, and $y' = 3x - 4$, $y(-1) = \frac{13}{2}$. Find y?

Solution:

$$y = \int (3x-4)dx = 3.\frac{x^2}{2} - 4x + c$$

At $x = -1$ then $y = \frac{13}{2}$, this implies that $\frac{13}{2} = \frac{3}{2}(-1)^2 - 4(-1) + c \quad \Rightarrow \quad c = 1$

Now, $y = 3.\frac{x^2}{2} - 4x + 1$.

Example (2) Find y subject to the given conditions: $y'' = -3x^2 + 4x$, $y'(1) = 2$ and $y(1) = 3$.

Solution:

Since, $y' = \int y'' dx \quad \Rightarrow \quad y' = \int (-3x^2 + 4x)dx$

$$\Rightarrow \quad y' = -3\frac{x^3}{3} + 4\frac{x^2}{2} + c_1 = -x^3 + 2x^2 + c_1.$$

$\because \; y' = 2$ at x = 1:

$$\Rightarrow \quad 2 = -(1)^3 + 2(1)^2 + c_1 \quad \Rightarrow \quad c_1 = 1$$

Then $y' = -x^3 + 2x^2 + 1$

Also,

$$y = \int y' dx = \int (-x^3 + 2x^2 + 1)dx$$

$$= -\frac{x^4}{4} + 2\frac{x^3}{3} + x + c_2$$

At x = 1, y = 3 then,

$$3 = -\frac{1}{4} + \frac{2}{3} + 1 + c_2 \quad \Rightarrow \quad c_2 = \frac{19}{12}.$$

Now, $y = -\frac{x^4}{4} + 2\frac{x^3}{3} + x + \frac{19}{12}$.

Example (3) Find y subject to the given conditions: $y''' = 2x$, $y''(-1) = 3$, $y'(3) = 10$ and $y(0) = 13$.

Solution:

$y'' = \int y''' dx = \int 2x dx = x^2 + c_1$.

At $x = -1 \Rightarrow y'' = 3$

Thus, $3 = (-1)^2 + c_1 \quad \Rightarrow \quad c_1 = 3 - 1 = 2$.

Then $y'' = x^2 + 2$.

Similarly, $y' = \int y'' dx = \int (x^2 + 2) dx = \frac{x^3}{3} + 2x + c_2$.

At $x = 3 \Rightarrow y' = 10$

Thus, $10 = \frac{3^3}{3} + 2(3) + c_2$ this implies that $c_2 = -5$.

$\Rightarrow \qquad y' = \frac{x^3}{3} + 2x - 5.$

Now, $y = \int y' dx = \int (\frac{x^3}{3} + 2x - 5) dx = \frac{x^4}{12} + x^2 - 5x + c_3$.

At $x = 0 \Rightarrow y = 13$

Thus, $13 = 0 + 0 - 0 + c_3$ this implies that $c_3 = 13$.

Finally, $y = \frac{x^4}{12} + x^2 - 5x + 13$.

Example (4) If the marginal-revenue function for a manufacturer's product is $r' = \frac{dr}{dq} = 275 - q - 0.3q^2$. Find the demand function?

Solution

Since the marginal-revenue function r' is the derivative of the total revenue r, then

$$r = \int r' dq = \int (275 - q - 0.3q^2) dq = 275q - \frac{q^2}{2} - 0.3\frac{q^3}{3} + c$$

When $q = 0$ (no production = no money received), and therefore the revenue $r = 0$.

$\Rightarrow\ 0 = 275(0) - \frac{(0)^2}{2} - 0.3\frac{(0)^3}{3} + c$, this implies that $c = 0$.

Now, $r = 275q - \frac{q^2}{2} - \frac{q^3}{10}$

But, $r = p.q \quad \Rightarrow \quad p = \frac{r}{q}$, where p is the demand function.

Finally, $p = \frac{275q - \frac{q^2}{2} - \frac{q^3}{10}}{q} = 275 - \frac{q}{2} - \frac{q^2}{10}$.

Example (5) In the manufacture of a product, fixed costs per week are $2000. If the marginal cost function is $\frac{dc}{dq} = 3q^2 + 10$. Find the total cost function, and then find the total cost of producing 20 units.

Solution

Since the marginal cost c' is the derivative of the total cost function c, then $c = \int c'dq$

$\Rightarrow\ c = \int (3q^2 + 10)dq = q^3 + 10q + k$

When q = 0, the total cost c equal the fixed cost only. i.e., when $q = 0 \Rightarrow c = 2000$

This implies that $2000 = (0)^3 + 10(0) + k$ consequently, $k = 2000$.

Now, the total cost function is:

$c = q^3 + 10q + 2000$.

When q = 20, the total cost function is:

$$c = (20)^3 + 10(20) + 2000$$
$$= 8000 + 200 + 2000$$
$$= 10200$$

Thus, as q = 20, the total cost is $10200.

Home Work: solve the book exercises 14.3 problems 6, 8, 10, 12, 14.