Thermoelectric Cooling Using Peltier Cells in Cascade

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Introduction:

Thermoelectric devices are solid state devices that convert thermal energy from a temperature gradient into electrical energy (Seebeck effect) or convert electrical energy into a temperature gradient (Peltier effect). Seebeck first found that an electromotive force is generated by heating a junction between two dissimilar metals. The converse effect discovered by Peltier in 1834 and demonstrated beyond doubt by Lenz in 1838 when he successfully froze water at a bismuth-antimony junction. With the development of semiconductor compounds such as alloys of bismuth telluride or antimony telluride the pumping of substantial quantities of heat from one junction to another simply by passing an electric current is now possible and industrial and commercial applications are in process of development [1].

In this experiment a pair of commercial Peltier cells is used as a heat pump. The commercial Peltier coolers are used in small refrigerators, CPU coolers, electronic component cooler, etc.

Theory:

The thermoelectric effect is particularly interesting at metal-semiconductor junction, because it is much larger than in the case of a junction between two metals. Let us consider an *n*-type crystal with two ohmic contacts (Fig.1)

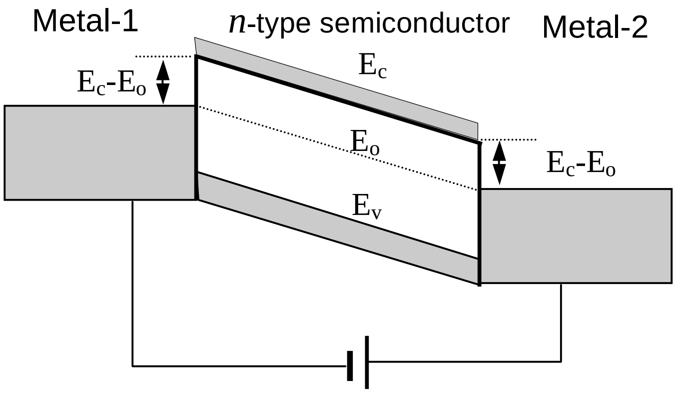


Fig. 1

E_c is the energy of the conduction band electrons of the semiconductor, E_o is the Fermi level. Shaded area represents electron-filled energy bands. As the contact is formed and upon reaching the equilibrium the Fermi levels of the metal and semiconductor merges. With a potential applied across the contacts, electrons with energies greater than E_c-E_o can get into the semiconductor. Consequently, metal-1 at the left-hand contact, loses the electrons occupying its highest energy states because electrons flow from left to right in the diagram in response to the applied potential. At the right hand contact, these electrons are deposited into metal-2, so that the hottest electrons are moved from metal-1 to metal-2 by virtue of the contact effects and the current flow. As a result, metal-1 is cooled and metal-2 is heated by the amount of energy transferred per electron, which clearly equals E_c-E_o plus the kinetic energy of the electrons moving from a hot to a cold region. This is expressed in terms of the thermoelectric power Q_n for a *n*-type semiconductor, which is defined as:

$$-Q_n T = (E_c - E_o) + 2K_B T \quad \text{………} \ (1)$$

Where, K_B is the Boltzmann Constant. Similarly the thermoelectric power for a *p*-type semiconductor is:

$$Q_p T = (E_o - E_v) + 2K_B T \ \text{…………} \ (2)$$

Equations (1) and (2) show that the large values of thermoelectric power found in semiconductors basically result from the fact that the average potential energy for conduction electrons (or holes) is larger than the Fermi energy, in contrast to the situation in metals. It is advantageous to use a *p*-type and an *n*-type element together, because the thermoelectric effects of the two are additive. If two contacts of a semiconductor are maintained at a different temperatures (T_h-T_c=ΔT) , a potential difference can be observed between them (V_S). This is called Seebeck voltage and arises from the more rapid diffusion of carriers at the hot junction. These carriers diffuse to the cold junction, so that such a contact acquires a potential having the same sign as the diffusing majority carriers. The seebeck coefficient, S is defined as

$$S = \frac{V_S}{\Delta T} \ \text{……..} \ (3)$$

Thermoelectric cooling: Current (I_P) flowing in a circuit containing a semiconductor-metal contact tends to pump heat from one electrode to the other because of the Peltier effect. The Thermoelectric power of semiconductor is large enough to make such electronic cooling of practical interest, particularly where small size and absence of mechanical movements are desired. A single cooling unit consisting of a *p*-type element and an *n*-type element joined with Ohmic contacts is sketched in Fig.2

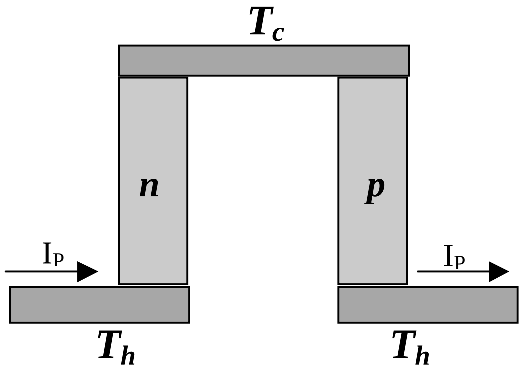


Fig. 2

The current I_P pumps heat from the common junction, cooling it an amount ΔT below the hot junction ($\Delta T = T_h - T_c$). The Peltier cooling effect is reduced by heat conducted down the elements normal thermal conductance, together with Joule heating in the elements due to the electric current. The heat removal rate at the cold junction is expressed as:

Commercial Peltier cells are characterized by the following parameters:
Maximum voltage, V_m, maximum current I_m, maximum cooling power Q_m, maximum obtainable temperature difference, ΔT under hot side temperature of T_h. S is the Seebeck coefficient, $\boldsymbol{V}_s$ is the Seebeck voltage. Heat flow rate from the cold junction to the hot junction is expressed by [2]:

$$Q_c = S\,I\,T_c - \frac{1}{2}I^2R - k\,\Delta T \qquad (1)$$

$$Q_h = S\,I\,T_h + \frac{1}{2}I^2R - k\,\Delta T \qquad (2)$$

$\Delta T = T_h - T_c$

k is the thermal conductance of the Peltier cell

R is the electrical resistance of the Peltier cell

Voltage applied to the Peltier cell is, ***V*** and the resulting current through the Peltier cell is ***I***. The Seebeck voltage ($\boldsymbol{V_s}$) developed across the Peltier cell is:

$$V_s = V - I\,R \qquad (3)$$

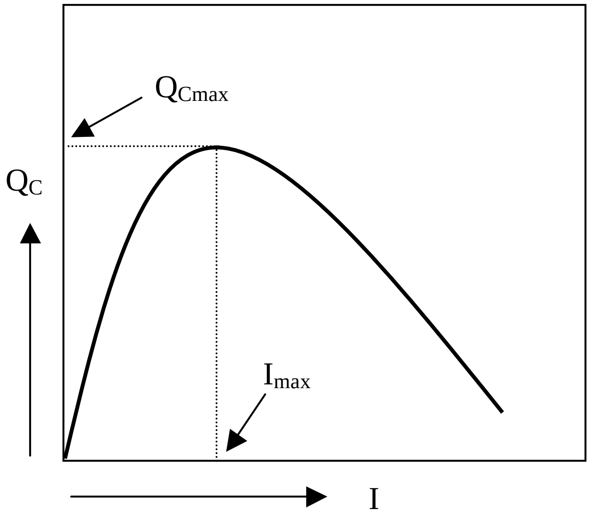
$$V_s = V_m - I_m\,R \qquad (4)$$

Where $V_s = S\Delta T$ (5)

Differentiating Q_c with respect to I and equating to 0, one can get the optimum current (I_m) for maximum cooling.

$$\frac{d\,Q_c}{dI} = S\,T_c - I\,R$$

Fig.3.



$$I_m = \frac{S\,T_c}{R} \qquad (6)$$

Combining eq (4), (5) and (6) we get Seebeck coefficient S,

$$S = \frac{V_m}{T_h} \qquad (7)$$

Equation (4) can be rewritten as:

$$R = \frac{V_m}{I_m} - \frac{V_s}{I_m} = \frac{V_m}{I_m} - \frac{S\Delta T}{I_m} = \frac{V_m}{I_m} - \frac{V_m}{T_h}\frac{(T_h - T_c)}{I_m}$$

$$R = \frac{V_m}{I_m}\frac{T_c}{T_h} \qquad (8)$$

Under thermal equilibrium condition equation (1) becomes:

$$Q_c = S\,I\,T_c - \frac{1}{2}I^2R - k\,\Delta T = 0$$

Thermal conductance, k is expressed as:

$$k\,\Delta T = S\,I\,T_c - \frac{1}{2}I^2R \qquad (9)$$

From Equation (7), (8) and (9):

$$k = V_m\,I_m\,\frac{T_c}{2\,T_h\,\Delta T} \qquad (10) \qquad (T_c = T_h - \Delta T)$$

Calculated parameters for a commercially available Peltier cells are given in Table-I

Table – 1

Peltier cell	V_m (V)	I_m (A)	Q_m (W)	T_h (^{o}C)	ΔT(^{o}C)	S (mV/K)	R (Ω)	K (W/K)
CUI Inc. CP85438	15.4	8.5	75.0	27	68	51.33	1.4	0.744

The performance characteristics of the Peltier cell (CP85438, Dimension: 40 x 40 x 4.8 mm) under various load condition is plotted in Fig. 4.

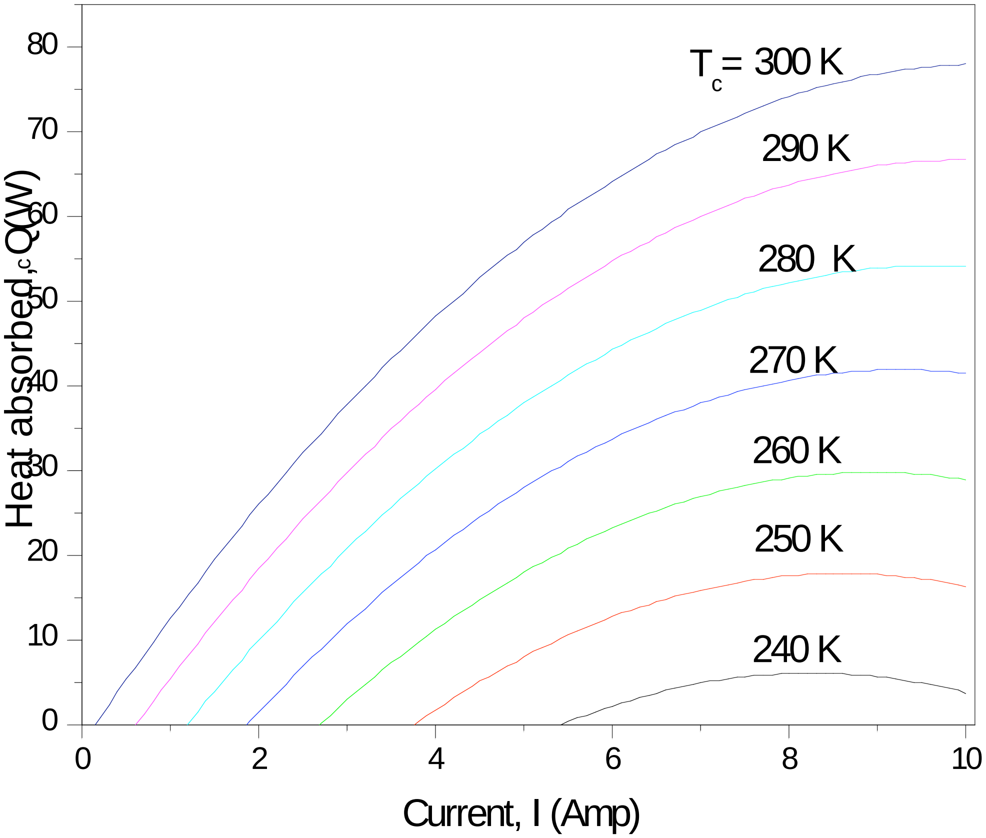


Fig.4. Performance (calculated) of Peltier cell (CP85438) under various heat load, producing varying temperature difference (ΔT).

Figure of Merit Z:

$$Q_c = S\,I\,T_c - \frac{1}{2} I^2 R - k\,\Delta T \qquad (1)$$

$$\frac{d\,Q_c}{dI} = S\,T_c - I\,R = 0 \text{ (For maximizing)}$$

$$I_m = \frac{S\,T_c}{R}$$

In equilibrium, $Q_c = 0$

Equation (1) becomes:

$$S\,I\,T_c - \frac{1}{2}I^2R - k\,\Delta T = 0$$

Inserting the value for I by I_m:

$$\Delta T_{\max} = \frac{S^2}{R\,k}\cdot\frac{T_C^2}{2}$$

The quantity $\frac{S^2}{Rk}$ is termed as the figure of merit

Table – 2

Peltier Cell		Experimental	Calculated
CP85438	S (mV/K)	53.5	51.33
	R (Ω)	1.6	1.4
	K (W/K)	0.75	0.744

Experiment:

Two experiments have been conducted to determine the performance of single-cell and a cascaded double-cell arrangement. Before using the Peltier cells, the thermal resistances of the heat load, thermal paste and the heat sink has to be determined. A solid aluminum block of dimension 7.5 x 6.5 x 2.5 cm^3, was used as a heat load. The thermal resistance of the heat was measured as, 4.25 K/W. A commercial CPU cooler for a dual-core processor was used as a heat sink for the cooler. This type of heat sinks are small in size and have low thermal resistances due to forced air-cooling. Thermal resistance of the heat sink used in this experiment was found in the manufacturer’s data sheet as, 0.35 K/W. It should mentioned that in our first attempt, we tried to use a very large aluminum plate as a heat sink, which was measured to have a thermal resistance of ~ 1 K/W. It was massive and did not remove enough heat, and was replaced by a light-weight CPU cooler. In the first experiment, a single cell was used. The hot side was placed on the heat sink. For good thermal contact, a thin layer of commercial thermal paste was applied. The heat load was placed on the cold side of the cell, similarly, thermal paste was used also to ensure good thermal contact between the load and the cold side. To measure the temperature of the heat load and the heat sink, two calibrated thermistors were used. Holes were drilled in the heat load and the heat sink and the thermistors were inserted in the holes. A variable DC power supply was used (12 V, 5 A) to power the Peltier cooler. Current was slowly increased in steps, and the corresponding temperatures (heat load and heat sink) were recorded. Ample time was given to settle the temperature before final measurements. In the second experiment, another Peltier cell of similar kind was placed on to the first cell, thermal paste was applied as us usual for good thermal contact. The first stage was supplied with the optimum power, a second variable DC power supply (12 V, 3.5 A) was used for the top stage. Current was varied and the temperatures (T_c and T_h) were recorded. The experimental schematic is shown in Fig. 5.

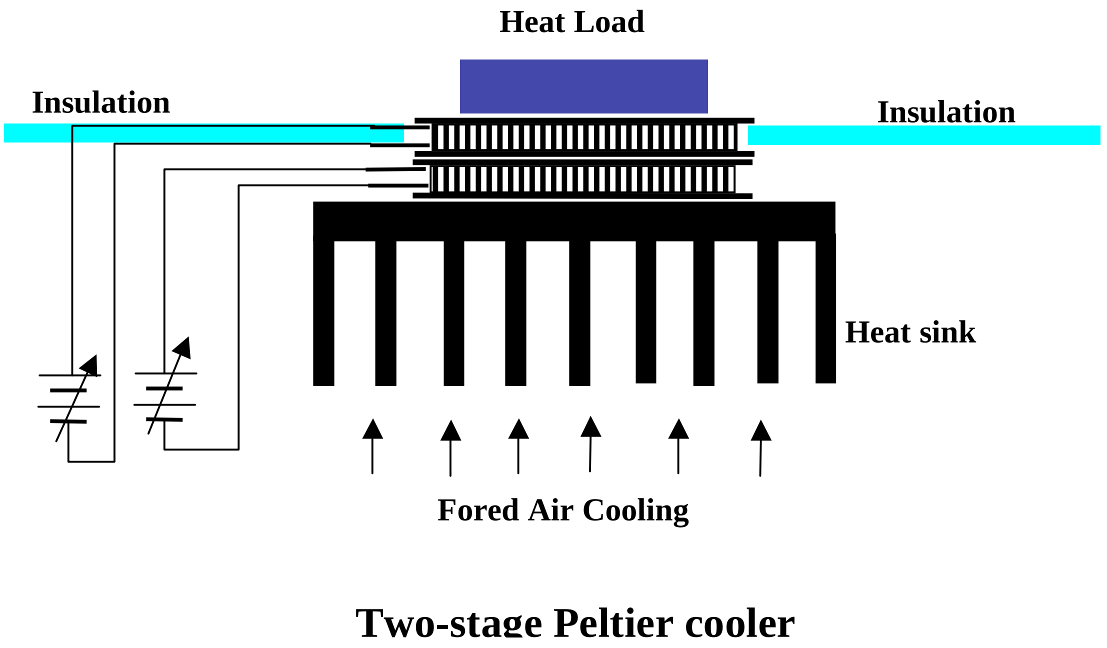


Fig. 5. Schematic diagram of the experimental setup.

To obtain the calculated data as well, several heat transfer equations have solved under different conditions. An alternative approach is to use electrical equivalent of the thermal parameters and then using software for analyzing electric circuits.

Equivalent circuit:

One approach for solving heat transfer problems is to apply an equivalent electrical circuit scheme [3]. All thermal processes are described in terms of electrical analogies, and dependent sources represent their interconnections. Table-3 shows the physical parameters of the thermal system and corresponding parameters of the Peltier cell.

Table – 3

Thermal quantities	**Units**	**Analogous electrical quantities**	**Units**
Heat, q	W	Current, I	A
Temperature, T	K	Voltage, V	V
Thermal resistance, θ	K/W	Resistance, R	Ω

Heat capacity, C	J/W	Capacitance, C	F
Absolute zero temperature	0 K	Ground	0 V

The electrical equivalent circuit of the single-stage and double-stage Peltier cooler is shown in Fig.5 and Fig.6, respectively. Simulations was carried out using well-known circuit simulation software, MULTISIM.

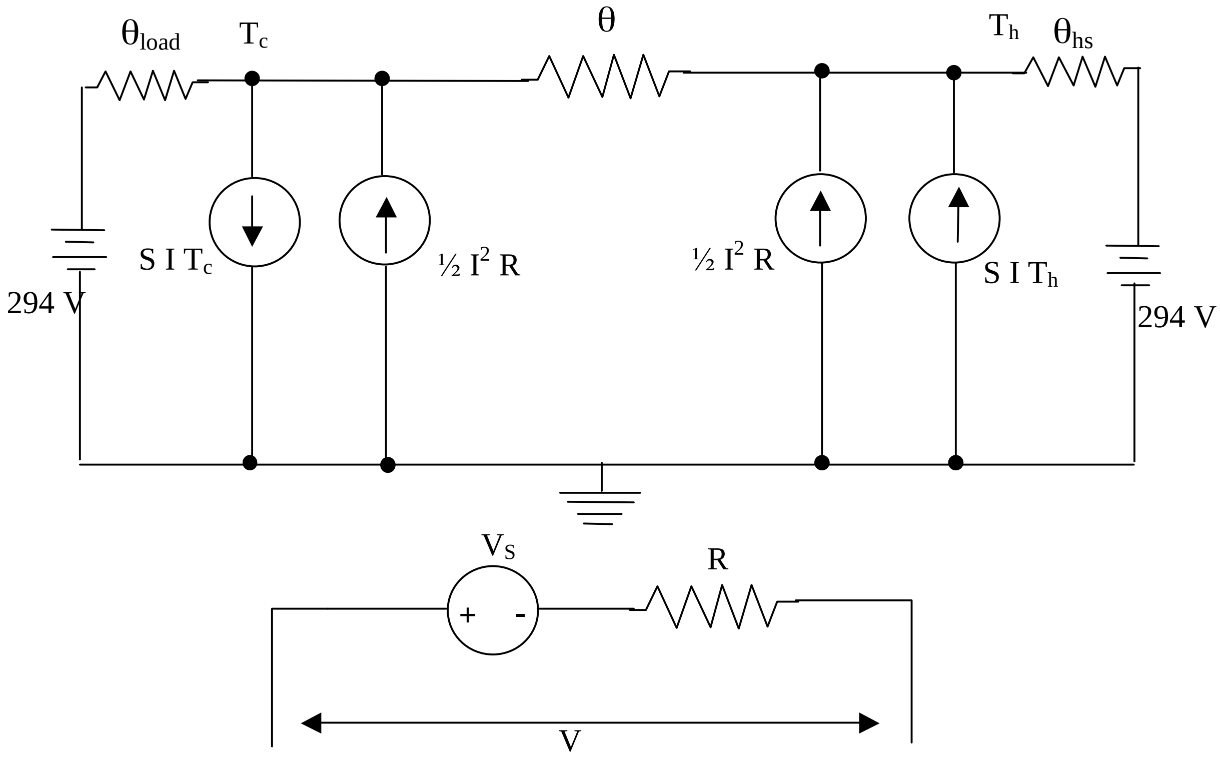


Fig. 5. Electrical equivalent of a single stage Peltier cooler

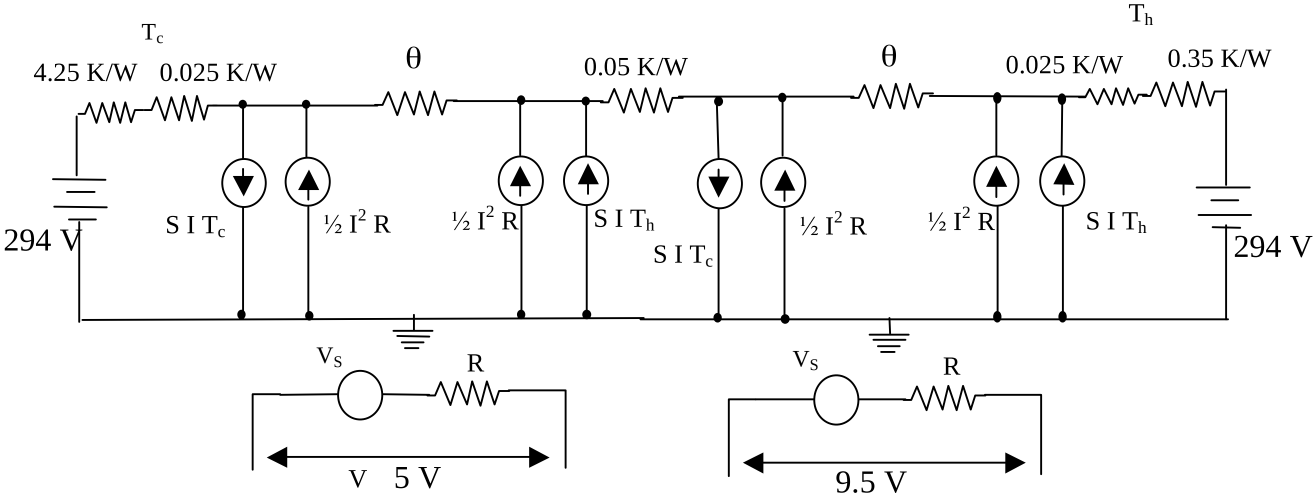


Fig. 6. Electrical equivalent of a double-stage Peltier cooler

The measured and estimated parameters used to analyze the electrical equivalent circuits are given in Table-4.

Table – 4

Parameters	Measured/Estimated Value
Thermal resistance: Heat sink, θ_{hs}	0.35 K/W
Thermal resistance: Heat load, θ_{Load}	4.25 K/W
Thermal resistance: Peltier cell, θ	1.34 K/W
Thermal resistance: Peltier ceramic plate	0.015 K/W
Thermal resistance: Thermal paste	0.01 K/W

Results and discussion:

For the single-cell cooler the experimental and calculated performance is shown in Fig. 7.

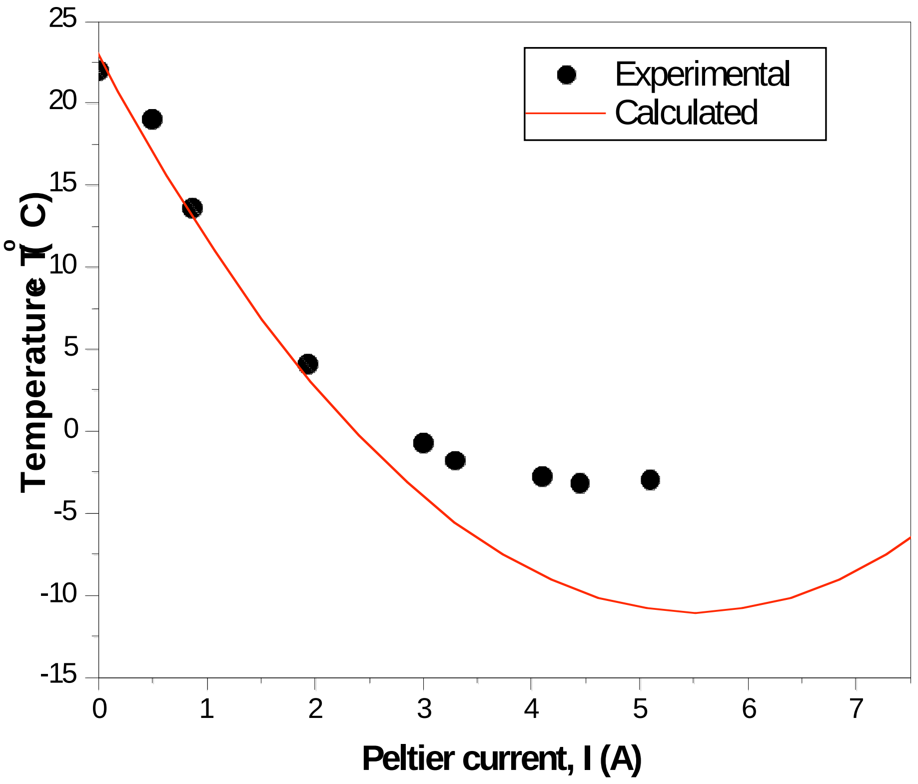


Fig. 7. Cooling effect of a single-stage Peltier cooler.

The minimum temperature achieved was recorded as -3.1 ^{0}C; however the simulated result shows that for the parameters used in this experiment, the minimum achievable temperature should be -11 ^{0}C. The reasons for this discrepancy could be due to the estimation of the thermal resistances of the contacts, where thermal paste is used. As the thermal paste is very dense it was difficult to apply it uniformly. Slight variation of layer thickness and formation of air bubble or gap can cause deviation from the estimated value.

For the cascaded Peltier cooler, the cooling performance is shown in Fig. 8. The minimum temperature achieved for the heat load was -8.0 ^{0}C. The simulated result shows the minimum achievable temperature should be -17 ^{0}C, under similar condition. The reason for this discrepancy is the same as that has been discussed.

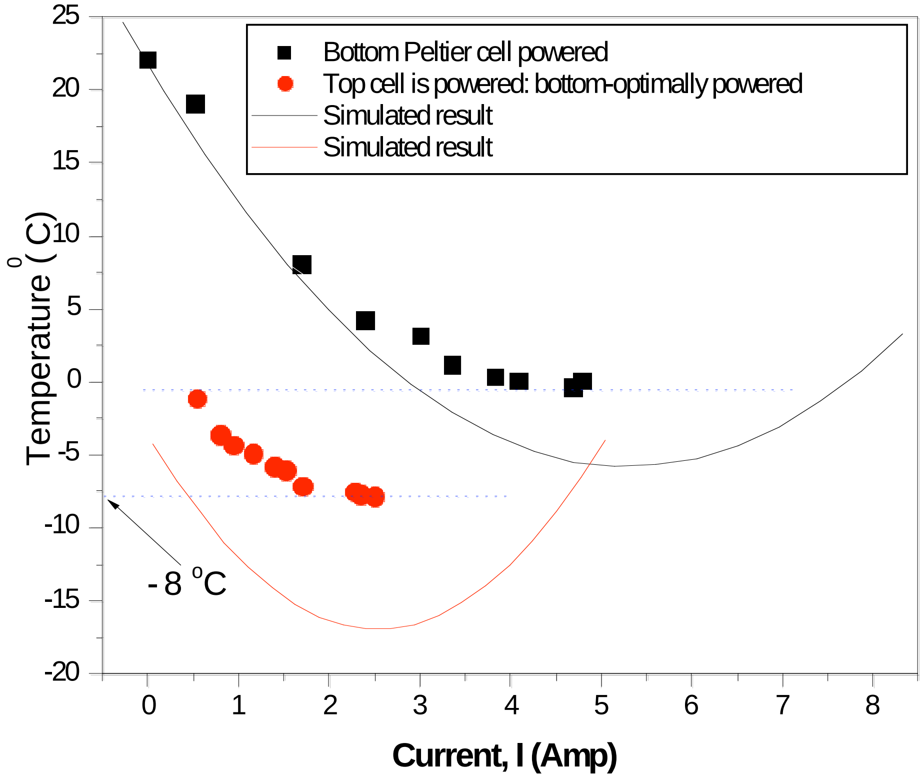


Fig. 8. Cooling effect of a double-stage Peltier cooler.

In another experiment, the forced air cooled heat sink was removed and a single Peltier cell was firmly positioned on a small aluminum heat sink immersed in a water tank. The water tank was supplied with running water to keep the water temperature constant at ~21 ^{0}C. With this arrangement minimum temperature was achieved to -12.2 ^{0}C at the Peltier current of 5.6 A. As it was difficult to measure the temperature of the hot side of the Peltier cell, we could not use the parameters to obtain simulated results

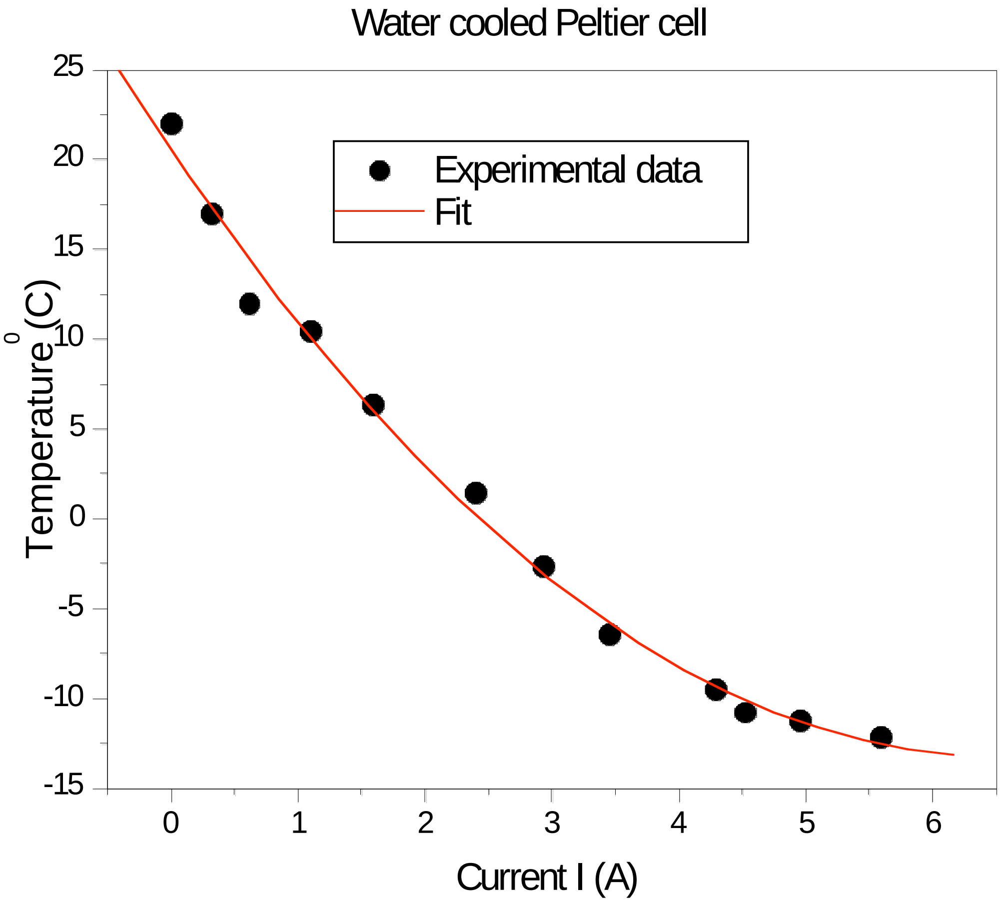


Fig. 9. Cooling performance of a single-stage Peltier cooler, with a running water cooled heat sink.

References:

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2. Y. Kraftmakher, “Simple experiments with a thermoelectric module” , Eur. J. Phys, 26, 959-967, (2005).

3. S. Lineykin and S.B. Yaakov, “Modeling and analysis of Thermoelectric Modules” IEEE transactions on industry application, Vol. 43, No. 2, 505 – 512, (2007).

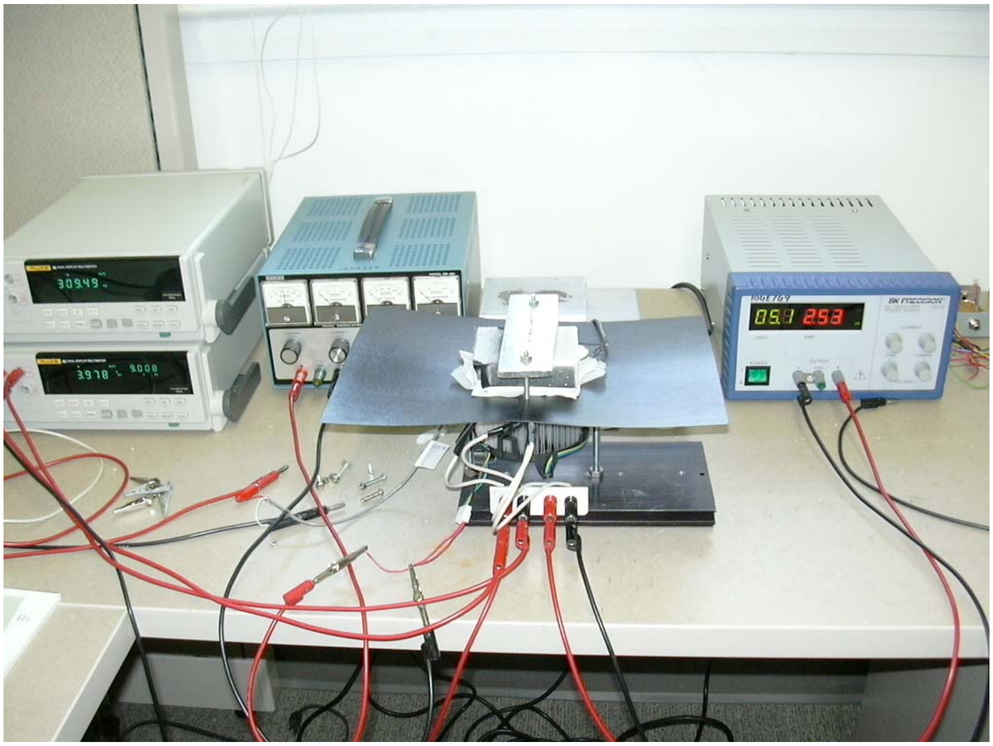

Photo – 1: Experimental setup

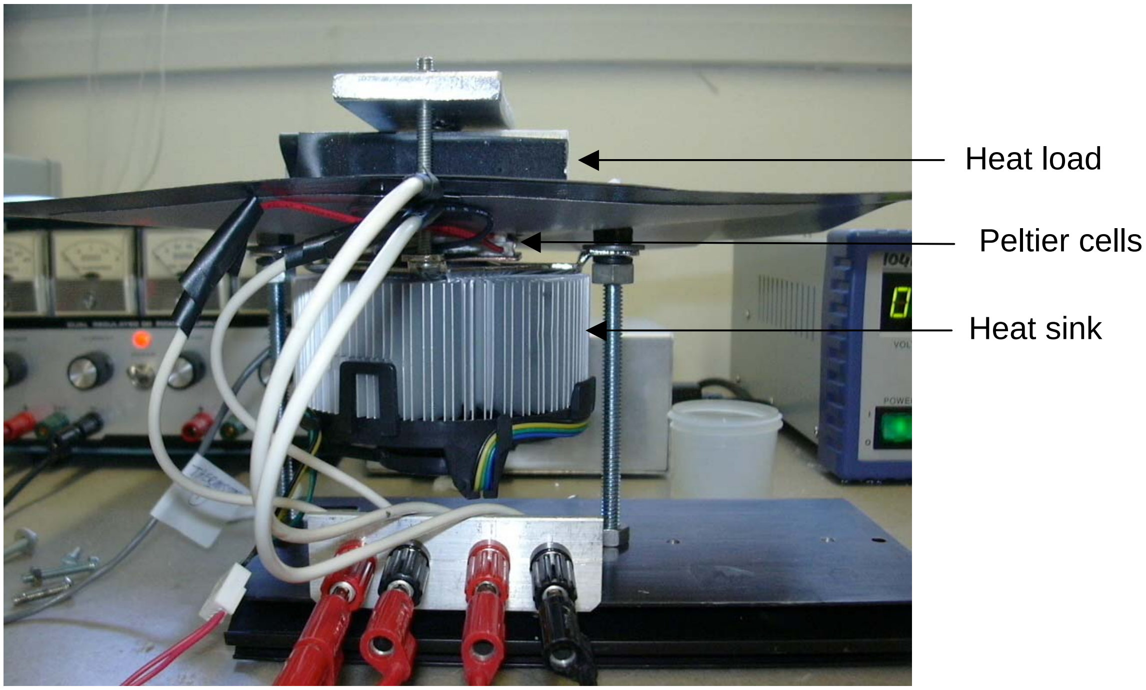

Photo – 2: Peltier cell, heat sink and heat load.