Solution of Sheet # 3 (Shaft Design)

Example 1 A line shaft supporting two pulleys A and B is shown in Fig. 9.2 (a). Power is supplied to the shaft by means of a vertical belt on pulley A, which is then transmitted to pulley B carrying a horizontal belt. The ratio of belt tensions on tight and loose sides is 3 : 1 and the maximum tension in either belt is limited to 2.7 kN. The shaft is made of plain carbon steel 40C8 ($S_{ut} = 650$ N/mm^2 and $S_{yt} = 380$ N/mm^2). The pulleys are keyed to the shaft. Determine the shaft diameter according to the A.S.M.E. code if

$$k_b = 1.5 \quad \text{and} \quad k_t = 1.0$$

Solution

$$0.30 S_{yt} = (0.30)(380) = 114 \text{ N/mm}^2$$

$$0.18 S_{ut} = (0.18)(650) = 117 \text{ N/mm}^2$$

The lower of the two values is 114 N/mm^2 and there are keyways on shaft. Therefore

$$\tau_d = (0.75)(114) = 85.5 \text{ N/mm}^2 \qquad \text{(i)}$$

The maximum belt tension will occur in belt on pulley A due to smaller diameter. The tensions on the tight and loose sides of this belt are 2700 N and 900 N, respectively.

The torque supplied to the shaft is given by

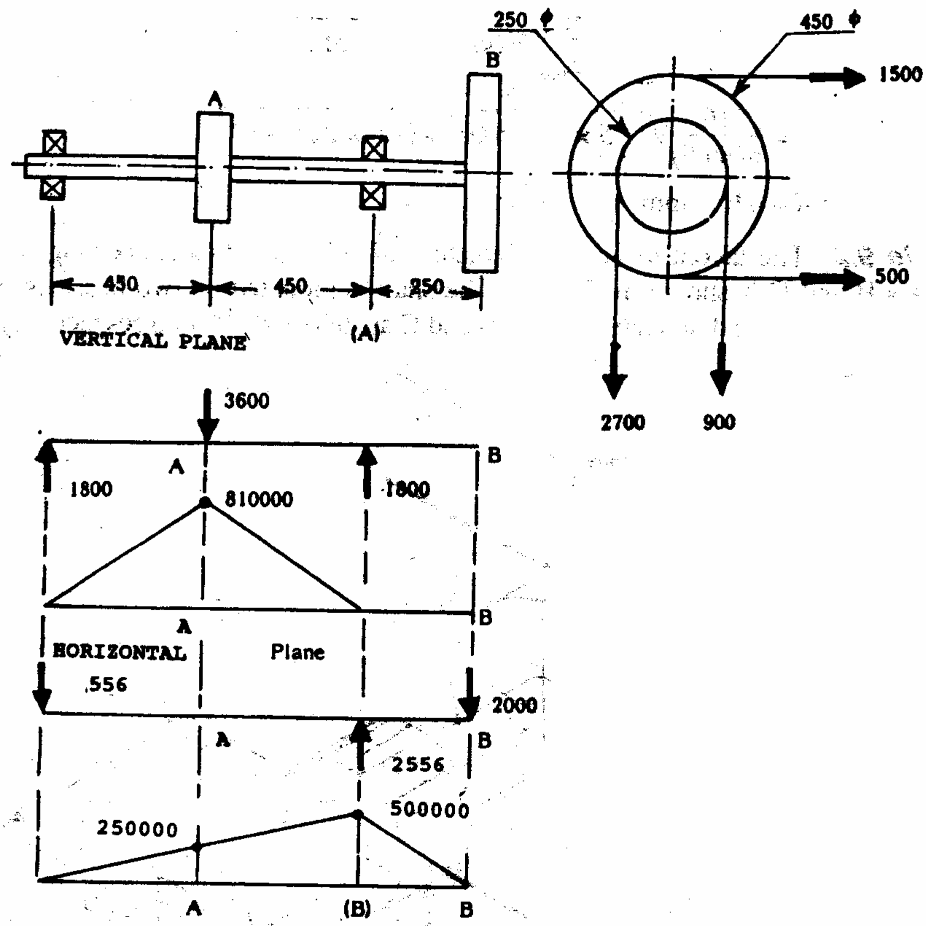


Fig. 9.2

$$M_t = (2700 - 900)\left(\frac{250}{2}\right) = 225\,000 \text{ N-mm}$$

The belt tensions for pulley B are P_{B1} and P_{B2}. Therefore,

$$\frac{P_{B1}}{P_{B2}} = 3 \qquad \text{(a)}$$

and

$$(P_{B1} - P_{B2})\left(\frac{450}{2}\right) = 225\,000 \qquad \text{(b)}$$

Solving Eqs (a) and (b),

$$P_{B1} = 1500 \text{ N} \quad \text{and} \quad P_{B2} = 500 \text{ N}$$

The forces and bending moments in vertical and horizontal planes are shown in Fig. 9.2 (b). The maximum bending moment is at A. The resultant bending moment is given by

$$M_b = \sqrt{(810\,000)^2 + (250\,000)^2} = 847\,703 \text{ N-mm}$$

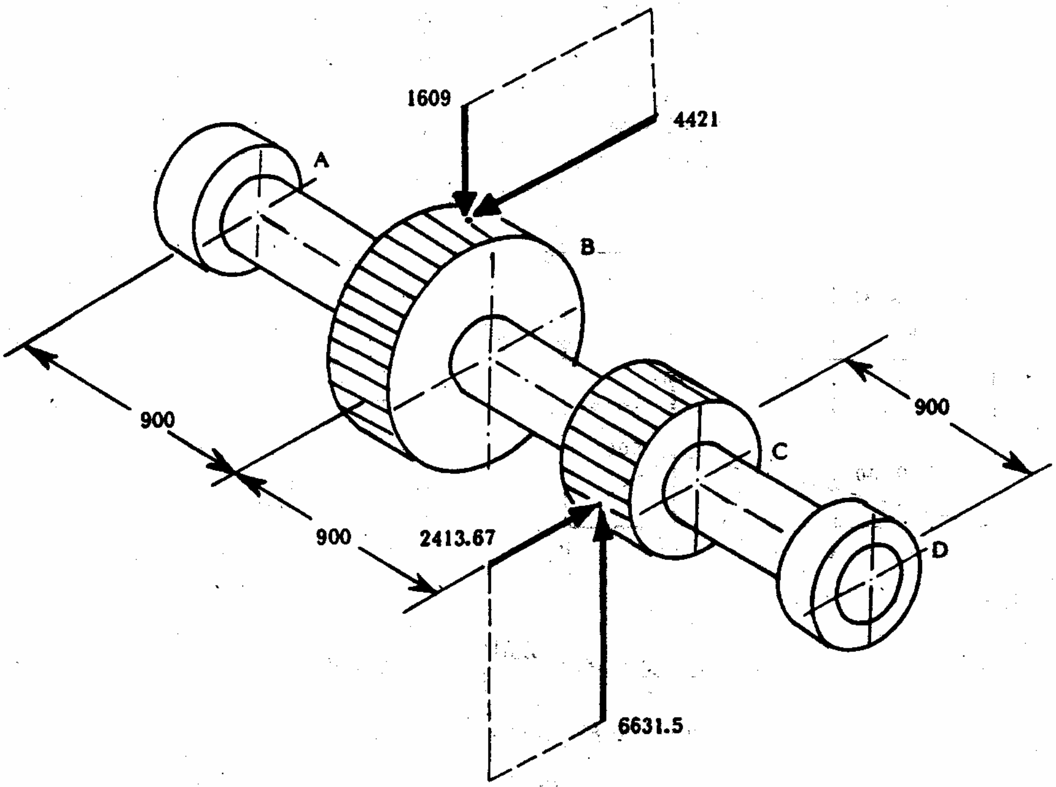
From Eq. (9.4),

$$d^3 = \frac{16}{\pi\tau_{max}} \sqrt{(k_b M_b)^2 + (k_t M_t)^2}$$

$$= \frac{16}{\pi(85.5)} \sqrt{(1.5 \times 847\,703)^2 + (1.0 \times 225\,000)^2}$$

$\therefore$ $d = 42.53$ mm

Example 2 The layout of an intermediate shaft of a gear box supporting two spur gears B and C is shown in Fig. 9.3. The shaft is mounted on two bearings A and D. The pitch circle diameters of gears B and C are 900 and 600 mm respectively.



The material of the shaft is steel ($S_{ut} = 770$ and $S_{yt} = 580$ N/mm^2). The factors k_b and k_t of A.S.M.E. code are 1.5 and 2.0 respectively. Determine the shaft diameter using the A.S.M.E. code.

Assume that the gears are keyed to the shaft.

Solution $0.30S_{yt} = 0.30(580) = 174$ N/mm^2

$0.18S_{ut} = 0.18(770) = 138.6$ N/mm^2

Since the gears are keyed to the shaft, therefore,

$$\tau_{max} = 0.75(138.6) = 103.95 \text{ N/mm}^2$$

The forces and bending moments in vertical and horizontal planes are shown in Fig. 9.4. The resultant bending moment is maximum at C.

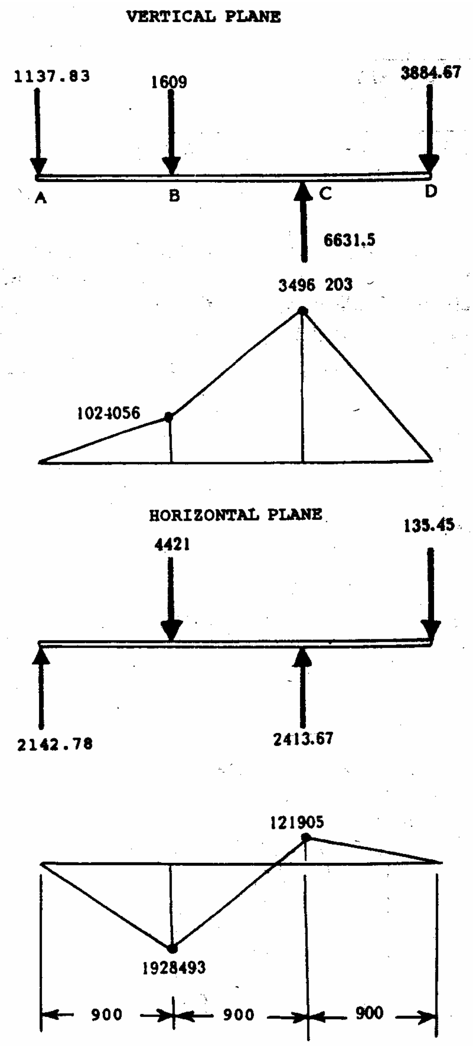


Fig. 9.4

At C,

$$M_b = \sqrt{(3\,496\,203)^2 + (121\,905)^2}$$
$$= 3\,498\,327.4 \text{ N-mm}$$

and

$$M_t = (4421)(450)$$
$$= 1\,989\,450 \text{ N-mm}$$

From Eq. (9.4)

$$d^3 = \frac{16}{\pi\tau_{max}}\sqrt{(k_b M_b)^2 + (k_t M_t)^2}$$

$$= \frac{16}{\pi(103.95)}\sqrt{(1.5 \times 3\,498\,327.4)^2 + (2 \times 1\,989\,450)^2}$$

$\therefore$ $$d = 68.59 \text{ mm}$$