Comment on "A note on the construction of the Ermakov-Lewis invariant" [J. Phys. A: Math. Gen. **35** (2002) 5333-5345]

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Abstract

We show that the basic results on the paper referred in the title, concerning the derivation of the Ermakov invariant from Noether symmetry methods, are not new.

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The purpose of this comment is to point out that the main results presented in a recently published paper [1], are not new. At the end of the introduction of this paper, the authors claims that "...*this is the first time the Noether symmetries are being considered to discuss the source of the Ermakov-Lewis* *invariant.*" Unfortunately, the authors miss the reference *Dynamical symmetries and the Ermakov invariant*, by Haas and Goedert [2]. In this paper, the Ermakov invariant is, apparently for the first time, deduced as a consequence of a dynamical Noether symmetry. To make this point clear, it is enough to compare equation (29) in the paper by Haas and Goedert with Proposition 1 and equation (4.17) in the paper by Moyo and Leach. Both equations present the dynamical symmetry associated to the Ermakov invariant, for the case of Lagrangian Ermakov systems, as the result from a straightforward application of the converse of Noether's theorem.

We further notice that the Lagrangian formulation for Ermakov systems in the referred publication [1] is by no means new. This can be seen by comparing the potential functions from equations (11) in the work by Haas and Goedert with the potential given by equation (3.18) in the work by Moyo and Leach. This later work also ignores the Hamiltonian descriptions for Ermakov systems developed earlier in [3], for the case of frequency functions depending on time only, and in [4], for the case of frequency functions depending also on dynamical variables.

As a final remark, the work by Moyo and Leach also ignores the papers [5]-[8], dedicated to the analysis of uncoupled Ermakov systems in the light of Noether's theorem. These works, however, do not deal with truly two-dimensional, coupled Ermakov systems, as in the case of references [1, 2]. Rather, these papers [5]-[8] deal with Ermakov systems in which one of the equations plays the principal role, while the other, decoupled from the first, is treated as an auxiliary equation. In these cases, the Lagrangian description is effectively one-dimensional and the Ermakov invariant cannot be obtained as a result from an associated dynamical Noether symmetry.

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