# Distribution and growth in an economy with limited needs.

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# 1 Introduction

> "The whole system of capitalist production is based on the fact that the workman sells his labour-power as a commodity. Division of labour specialises this labour-power, by reducing it to skill in handling a particular tool. So soon as the handling of this tool becomes the work of a machine, then, with the use-value, the exchange-value too, of the workman's labour-power vanishes; the workman becomes unsaleable, like paper money thrown out of currency by legal enactment. That portion of the working-class, thus by machinery rendered superfluous, i.e., no longer immediately necessary for the self-expansion of capital, either goes to the wall in the unequal contest of the old handicrafts and manufactures with machinery, or else floods all the more easily accessible branches of industry, swamps the labour-market, and sinks the price of labour-power below its value. "— Karl Marx, *Das Kapital,* Vol. 1, Ch. 15.

Marx's vision of technical progress as an instrument of capitalist exploitation that raises profits at the expense of wages has been invalidated by more than a century of improvements in living standards. But in the last few decades inequality has risen in the United States, average real wages have stagnated, and have indeed fallen for the poorest. Furthermore, this process has not benefited profits so much as the wages of the most skilled. That is, whereas Marx insisted on conflicts of interests between capitalists and workers, education and human capital now appear as one of the key dimensions of inequality. One reason is that production is less "heavy" than in the past, so that knowledge and ideas, rather than the capacity to finance lumpy investments, have become key to economic success. Many new goods have been introduced that are very cheap to produce (media, software, records, etc....); inequality has increased perhaps because most of the growth permitted by such innovations has been appropriated by knowledge workers who benefit from the associated intellectual property rights. This has lead authors such as Robert Reich (1992) to envision a society with a widening cleavage between "symbol manipulators" and ordinary workers.

This paper develops a model where such a society may indeed arise, because productivity improvements may result in the stagnation or fall of wages—if properly measured as the return to production work—in a fashion echoing Marx's gloomy predictions about the pauperisation of proletariat. In contrast, such improvements boost the profits and market values of firms, as well as the incomes of knowledge workers. This paradox arises as the outcome of an interaction between imperfect competition and the finiteness of needs.

One characteristic of modern Western society is the saturation of many needs; households are loaded with consumer appliances, vehicles, clothes, and so forth. Indeed a visitor from a low income country would be struck by the proliferation of shops specialized in plainly useless goods, not to speak of the economic value he would ascribe to what can be found in our trash cans. Such bliss is permitted by the very low production costs that secular productivity growth has generated. This phenomenon is bound to accelerate with the growing share of intangible goods such as music, software, or video games in our consumption basket. When traded over say the internet, the production (or, rather, replication) cost of these goods is virtually zero. This has led some authors (Rifkin (1996)) to speculate about the end of work: if productivity goes on increasing and new activities are not invented, isn't labor going to be useless?

I show that this phenomenon of saturation of needs generates a productivity paradox, in that beyond some point increases in productivity result in falling wages. This is because when productivity is higher, people consume more of each good, and if consumption of a given good approaches the bliss point, its marginal utility and therefore its price must become very low, relative to income; but, typically, this means that *demand elasticity* is also very low. Consequently, if the good is produced by a monopolist, the markup over marginal cost will be very large. In general equilibrium, this means that real wages are very low. This implies that more than 100 % of the growth in GDP permitted by productivity growth is appropriated by profits, i.e. by those who own property rights over blueprints.

Thus, as productivity in the material goods sector goes up, the economy first goes through a "Solovian" zone where wages increase, and then reaches a "Marxian" zone where further productivity growth reduces wages. This fatality is inevitable unless new goods are being introduced. I assume (realistically, in my view) that while there exists a bliss point for any given good, new goods are always valued by consumers regardless of the initial number of goods. An increase in the number of varieties reduces the consumption of any given good, thus moving the economy away from the low-elasticity, saturation zone. Therefore, the question is: do productivity increases result in the introduction of a sufficient number of additional goods in order to prevent wages from falling? In my model, people can dedicate themselves either to production or to knowledge, i.e. the invention of new goods. By increasing profits, productivity improvements enhance the incentives for innovation: labor is moved away from production into knowledge, and wages in the knowledge sector rise relative to the production sector. Therefore, the number of goods indeed increases. However, as long as this labor reallocation process is not infinitely elastic, it is insufficient to prevent wages in the production sector from falling in response to large enough increases in productivity. Furthermore, if one assumes that creativity, which determines a person's productivity in the knowledge sector, is more unevenly distributed than physical productivity, then productivity growth always increases earnings inequality.

This picture of technical change which raises inequality and possibly lowers absolute wages at the bottom of the distribution of earnings, is reminiscent of the U.S. economy in the last three decades. The empirical literature has established that inequality has increased, that wages have stagnated overall, and that they have actually fallen for the least skilled. It has concluded that this is probably explained by a relative demand shift due to skill-biased technical progress.[1] One key contribution of the present paper is to show that such a bias inevitably results from the general equilibrium interaction between imperfect competition and bounded needs. General productivity growth is then skill-biased to the extent that it increases the (absolute and relative) returns to creativity, which is instrumental in producing new goods. If more skilled workers have a comparative advantage in these activities, then inequality will indeed increase.

This mechanism is quite different from the ones that have been discussed in the recent literature (Zeira (1998), Krusell et. al (2000), Caselli (2000), Acemoglu (1999,2000), or Beaudry and Green (2000)), which has emphasized that new technologies result in the allocation of more capital to skilled workers at the expense of unskilled ones. It is also different from the one studied by Cohen and Saint-Paul (1994), who, following Baumol (1967,1985), show that if sectors are complementary to each other and labor is imperfectly mobile, technical progress in a given sector may harm workers in that sector if productivity remains the same in the rest of the economy. Here productivity affects all sectors, except the knowledge sector, which fails to introduce enough varieties to prevent consumption of each good from reaching low-elasticity, high markup levels near the saturation point. As in Cohen and Saint-Paul, the harmful effects of technical change are due to some asymmetry; in this paper's model, 'balanced' technical change which reduces the cost of innovation proportionately to the cost of production does result in higher wages. Contrary to Cohen and Saint-Paul, imperfect competition, rather than complementarity, is the key ingredient driving the results.

The other key contribution is to show how, by assuming heterogeneous workers and distiguishing between creativity and productivity, growth models that integrate the innovation process can also be understood as a theory of the distribution of income based on workers' choice between specializing in

[1] See Katz and Murphy (1992), Juhn, Murphy and Pierce (1993) and Levy and Murnane (1992).

goods production vs. knowledge production.

The paper is organized as follows: in section 2, I study the impact of productivity growth on wages in a static general equilibrium model with a fixed number of goods each produced by a monopoly. I assume that there exists a saturation level of consumption of each good which gives maximum utility. I show that the response of wages to productivity is hump-shaped. At high productivity levels the economy eventually reaches a "Marxian" zone where wages fall in response to productivity improvements, that are more than entirely absorbed by increased markups. In section 3, the model is extended to endogenize the number of goods. As in Romer (1990) and Grossman and Helpman (1991), new varieties are produced by a "Research and Development" or "Knowledge" sector, which employs workers who have a comparative advantage in creativity. In section 4, we characterize this economy's balanced growth path. It is shown that a Marxian zone still exists unless labor is reallocated from material production to knowledge production in an infinitely elastic fashion; and that if creativity is more unevenly distributed than productivity, productivity growth always results in greater inequality. Section 5 shows that if productivity growth is unbalanced between the knowledge and production sectors, wages become equal to zero in finite time. Section 6 tackles some current policy debates by analyzing how the distribution of income is affected by (i) work rules that prevent productivity from increasing (such as mandatory working time reduction), (ii) redistribution in favor of workers in the production sector, and (iii) 'globalization', i.e. greater openness to internation al trade. Section 7 concludes.

# 2 The static model

Let us start with a static model with a fixed, exogenous number of goods. We will then extend it to take care of innovation, growth, and heterogeneity across workers.

Consider an economy with a continuum of goods of mass $N$ and a continuum of consumers of mass 1. Together, these consumers supply an inelastic quantity of labor equal to $L$. The utility of consumer $j$ is given by

$$u_j = \int_0^N v(c_{ij})di, \tag{1}$$

where $c_{ij}$ is agent $j$'s consumption of good $i$ and $v$ is given by

$$\begin{aligned} v(c) &= c_{ij}(\bar{c} - c_{ij}/2), \; c_{ij} \leq \bar{c} \\ &= \bar{c}^2/2, \; c_{ij} \geq \bar{c}. \end{aligned}$$

The quantity $\bar{c}$ defines the point where needs are fullfilled. Past this point further increases in consumption of good $i$ do not increase utility. Note that $v(0) = 0$. Concavity and positivity of $v(.)$ implies that there is a taste for diversity in the sense that if one consumes the same amount of all goods, then an increase in $N$ matched by an equiproportionate fall in consumption increases utility (it is then given by $u = C(\bar{c} - C/2N)$, where is total consumption, and therefore increases with $N$).

Let $R_j$ denote the income of consumer $j$, which is the sum of his labor income and his dividend income. People maximize their utility function (1) subject to their budget constraint

$$\int_0^N p_i c_{ij} = R_j,$$

where $p_i$ denotes the price of good $i$. The resulting demand function is:

$$c_{ij} = \bar{c} - p_i \frac{\bar{c}\int_0^N p_i di - R_j}{\int_0^N p_i^2 di}. \tag{2}$$

This formula holds provided $R_j < \bar{c}\int_0^N p_i di$. If that inequality is violated, the consumer can fulfill his needs in all goods. In this case his consumption is indeterminate and simply has to satisfy $c_{ij} \geq \bar{c}$. To simplify the analysis we shall assume that these satiated consumers also set their consumption according to (2). As (2) is linear in income, it can be aggregated across consumers to get the aggregate demand for good $i$ :

$$c_i = \bar{c} - p_i \frac{\bar{c}\int_0^N p_i di - R}{\int_0^N p_i^2 di}, \tag{3}$$

where $c_i = \int_0^1 c_{ij} dj$ is total consumption of good $i$ and $R = \int_0^1 R_{ij} dj$ is total income.

Each good is produced by a monopoly using labor as the only input. One unit of labor produces $a$ units of a good. The monopoly maximizes its profit $p_i c_i - w c_i / a$, where $w$ is the wage, subject to (3). As each monopoly is atomistic, it neglects the effect of its price on the integrals in the RHS of (2). Consequently, its optimal price is determined by

$$p_i = \frac{\bar{c} \int_0^N p_i^2 di + \frac{w}{a} \left( \bar{c} \int_0^N p_i di - R \right)}{2 \left( \bar{c} \int_0^N p_i di - R \right)}. \tag{4}$$

We have one degree of freedom in normalizing prices. Since all prices are equal by symmetry, it is natural to normalize them to one. Note, however, that as utility is not homothetic indirect utility cannot be expressed as a sole function of income and an aggregate price level. In particular, our normalization does *not* imply that the utility of a person with income $R_j$ only depends on $R_j$. Rather, for non satiated agents it is determined by

$$u(R_j, N) = R_j (\bar{c} - \frac{R_j}{2N}), \tag{5}$$

while we have $u(R_j, N) = N \bar{c}^2 / 2$ for satiated agents.

As long as $N$ is fixed, an increase in $R_j$ is indeed (weakly) equivalent to an increase in utility. But, if $N$ varies, looking at income changes is not enough to compute utility changes, one also has to take into account changes in $N$. Income still represents purchasing power in physical terms, but fails to capture the effects of greater diversity on utility.

Equation (4) then boils down to an equation linking wages with aggregate income:

$$w = a \frac{\bar{c} N - 2R}{\bar{c} N - R}. \tag{6}$$

How is then $R$ determined? Aggregating budget constraints, we get that the consumption of each good is given by

$$c_j = c = R/N, \ \forall j.$$

Finally, equilibrium in the labor market implies that $Nc = aL$, so that we simply have

$$R = aL. \tag{7}$$

Substituting into (6) we get the formula determining wages:

$$w = a\frac{\bar{c}N - 2aL}{\bar{c}N - aL} \tag{8}$$

The most interesting property of (8) is that $w$ is not monotonous in $a$. Rather, it is hump-shaped, as illustrated in Figure 1. As productivity increases, the economy gradually moves from a Solovian zone where productivity improvements are reflected in wages to a Marxian zone where they are more than entirely appropriated by profits and wages actually fall. There exists a critical level of productivity where wages are exactly equal to zero, given by $a = \bar{c}N/2L$.

This effect comes from the satiety and imperfect competition properties of the model. At the bliss point the price (i.e. marginal utility) of the good is equal to zero, and so is its elasticity, reflecting the fact that people "don't care". Consequently, as consumption increases the economy reaches a zone where the price elasticity of demand is smaller than 1. As the economy nears this zone, the markup goes to infinity, implying that, in general equilibrium, real wages tend to zero.

This property is not an artifact of my quadratic specification.[2] It is inherently related to the existence of a bliss point. To see this, consider a representative agent economy where utility is given by $\int_0^N u(c_i)di$, and where $u$ is concave, twice differentiable, and there exists $c^*$ such that $u'(c^*) = 0$. It is then easy to check that the equivalent of (8) is

$$w = a\left(1 + \frac{aLu''(aL/N)}{Nu'(aL/N)}\right).$$

Unless $u''(c^*)$ is equal to zero, which is a special case, the quantity $\frac{aLu''(aL/N)}{Nu'(aL/N)}$, which is nothing but the inverse elasticity of the demand for

[2] What the quadratic assumption buys us is linearity of demand with respect to $R_j$, which allows to aggregate demand across consumers.

goods, goes to $-\infty$ as $c$ goes to $c^*$, implying the existence of an interval of values of $c$ where it is decreasing. As productivity goes up, so does consumption, implying that this zone ends up being reached. As the inverse elasticity can be arbitrarily large in absolute value, wages eventually fall and become equal to zero. The point where this occurs is such that $cu''(c)/u'(c) = -1$, i.e. the point where the elasticity of demand is equal to -1, which implies an infinite markup. One can check that this is indeed the case in our quadratic specification, as this point is determined by $c = \bar{c}/2$.

Going back to our quadratic specification, the economy enters the Marxian zone for $a > \frac{\bar{c}N}{L}(1-\frac{\sqrt{2}}{2})$. This zone is more remote, the greater the number of goods $N$ and the greater the saturation point $\bar{c}$. In other words, "creating needs" is good for "social peace" because it makes it less likely that wages fall in response to productivity growth.

As productivity grows, wage-earners suffer from having to work for firms that sell their products to wealthy customers (here, the owners of firms) who are unresponsive to price; in other words, their impoverishment comes from the careless consumption behavior of the rich.

# 3 Introducing new blueprints: creativity vs. productivity and the distribution of income

The preceding model is obviously very incomplete, as it ignores the determination of the number of goods. In particular, one way for the economy to get out of the productivity dilemma is by the introduction of new goods, which moves consumption away from the bliss points by redistributing it over a larger number of varieties. In this section, I extend the model by allowing people to specialize in the development of new goods ("R and D") instead of working in the production sector. I do it in a now standard way, following Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992). Furthermore, in order to analyze the interpersonal distribution of income rather than the factor distribution of income, I assume that people differ in ability, which has two dimensions: creativity and productivity. Finally, I allow for balanced growth by assuming a productivity trend in both the production and knowledge sectors.

Thus, we now have a continuum of workers who differ by their skill level $b \in [0, \infty]$. $b$ is distributed with c.d.f. $F(b)$. This skill determines two abilities, productivity and creativity. Productivity is the number of efficiency units of labor that a worker has in the material goods sector, while creativity is the number of efficiency units of labor he would have in the knowledge sector. Furthermore, we assume that creativity is relatively more unequally distributed than productivity. More specifically, a worker of skill $b$ has a creativity given by $h = \alpha_h b + \beta_h$, and a productivity equal to $l = \alpha_l b + \beta_l$, with $\alpha_l, \alpha_h > 0, \beta_l, \beta_h \geq 0$, and $\beta_l/\beta_h > \alpha_l/\alpha_h$. This latter assumption implies that productivity is less elastic to skill than creativity.

The inflow of new blueprints is determined as follows. One efficiency unit of creativity dedicated to innovation during a small time interval $dt$ allows to produce $1/\gamma_t.dt$ new varieties. Consequently , the number of varieties $N_t$ evolves according to

$$\dot{N}_t = H_t/\gamma_t, \tag{9}$$

where $H_t$ is the aggregate creativity input in the R and D sector.[3]

The static model described in the previous section is still valid intratemporally, with the following modifications. First, total factor productivity $a$ is now time-varying, and thus denoted by $a_t$. Second, expenditure $R_t$ must now be consistent with intertemporal optimization by consumers. That is, we now assume that consumer $j$ maximizes

$$U_j = \int_0^{+\infty} u_{jt} e^{-\rho t} dt, \tag{10}$$

where $u_{jt}$ is defined by (1) (with a time subscript appended to $c_{ijt}$), and where

[3] Contrary to what is assumed in Grossman and Helpman (1991) , $\gamma_t$ is exogenous and there is no externality from the aggregate number of goods to $\gamma_t$. To get balanced exogenous growth paths, I shall assume that it falls at a constant rate $g$.

maximization takes place subject to the intertemporal budget constraint

$$\int_0^{+\infty} R_{jt} e^{-\int_0^t r_u du} \leq W_j, \tag{11}$$

where $W_j$ is the agent's total wealth and $r_u$ the instantaneous real interest rate at date $u$.

Our price normalization is unchanged and consumption of each good remains determined by $c_{ijt} = c_{jt} = R_{jt}/N_t$. Consequently (5) still holds for non satiated agents, implying that the Euler equation corresponding to (10)-(11) is

$$\dot{N}_t \bar{c} - \dot{R}_{jt} = \left[ \rho - r_t + \frac{\dot{N}_t}{N_t} \right] (N_t \bar{c} - R_{jt}). \tag{12}$$

Linearity of (12) in $R_{jt}$ allows again to aggregate across consumers,[4] getting:

$$\frac{\dot{N}_t \bar{c} - \dot{R}_t}{N_t \bar{c} - R_t} = \left[ \rho - r_t + \frac{\dot{N}_t}{N_t} \right]. \tag{13}$$

Within each period, labor market equilibrium is still determined by (7), and the intra-period pricing rules are unchanged. Consequently, the wage equation (8) still holds. We rewrite it for convenience, adding time indices:

$$w = a_t \frac{\bar{c} N_t - 2 a_t L_t}{\bar{c} N_t - a_t L_t}. \tag{14}$$

Finally, total efficiency labor supply $L$ is no longer fixed, as people arbitrate between innovation and production. More specifically, we assume that at each date $t$ people have to choose between specializing in the goods sector vs. the knowledge sector. There are no costs associated with changing one's specialization. Therefore, one will elect to be a pure production worker if and only if

$$\omega_t(\alpha_h b + \beta_h) < w_t(\alpha_l b + \beta_l),$$

[4]We can again assume that (12) holds for satiated agents as long as it yields an expenditure path such that $N_t \bar{c} < R_{jt}$ for all $t$. This will indeed be the case as such a differential equation implies that the sign of $N_t \bar{c} - R_{jt}$ cannot change along a trajectory.

where $\omega_t$ denotes the real price of creativity. There are three possible regimes. If $\omega_t < w_t\alpha_l/\alpha_h$, (regime I), then given our assumption that $\beta_l/\beta_h > \alpha_l/\alpha_h$, the preceding inequality holds for all $b$. In this case all agents specialize in pure production and no new blueprint is being produced. If $\omega_t > w_t\beta_l/\beta_h$, then all agents specialize in R & D (regime II). Finally, the most interesting case obtains if $w_t\alpha_l/\alpha_h < \omega_t < w_t\beta_l/\beta_h$ (regime III). In this case, agents below a critical skill level become producers, and those above that level specialize in invention. This critical level is equal to

$$b^*(\omega_t/w_t) = \frac{-\beta_l + \beta_h\omega_t/w_t}{\alpha_l - \alpha_h\omega_t/w_t}. \tag{15}$$

By extension, we can also define $b^*(.) = 0$ in regime II and $b^*(.) = +\infty$ in regime I.

The total supply of creativity and labor in efficiency units is therefore determined by

$$H_t = \int_{b^*(\omega_t/w_t)}^{+\infty} (\alpha_h b + \beta_h) dF(b) = H(\omega_t/w_t) \tag{16}$$

$$L_t = \int_0^{b^*(\omega_t/w_t)} (\alpha_l b + \beta_l) dF(b) = L(\omega_t/w_t) \tag{17}$$

One clearly has $b^{*'} < 0$, $H' > 0$, and $L' < 0$. Dividing (16) by (17) we get a relationship which determines the *relative supply* for creativity:

$$H_t/L_t = \varphi(\omega_t/w_t), \tag{18}$$

where $\varphi' > 0$.

Inventors get a patent which gives them monopoly power over the production of the new variety for ever. The value of such a patent, $V_t$, evolves according to

$$r_t V_t = \pi_t + \dot{V}_t,$$

where $\pi_t$ is the profit of a monopoly at $t$, which we can compute as

$$\begin{aligned} \pi_t &= c_t(1 - w_t/a_t) \\ &= \frac{R_t}{N_t}(1 - \frac{w_t}{a_t}) \end{aligned} \tag{19}$$

Competition in the R and D sector guarantees that the cost of producing a variety equals the value of a patent:

$$\gamma_t \omega_t = V_t \tag{20}$$

To summarize, we have a fairly standard structure, as in the existing literature on innovation of growth. The two novelties are boundedness of needs, as captured by our quadratic specification, and heterogeneity across workers, which determines the supply response of the allocation of labor to the relative return to creativity, and allows to analyze the behavior of the distribution of income and its dependence on the model's parameters.

# 4 Balanced growth paths

We are now in a position to establish results regarding the properties of the model. We first characterize balanced exogenous growth paths. To do so we assume that total factor productivity grows at rate $g < \rho$, while the cost of producing blueprints shrinks at the same rate:

$$\begin{aligned} a_t &= a_0 e^{gt} \\ \gamma_t &= \gamma_0 e^{-gt}. \end{aligned}$$

We look for a balanced growth path where all variables grow at the same rate. In particular, this implies that total creativity and labor input $H$ and $L$ are constant, and so is the ratio between the return to each factor, $\omega/w$. Next, (9) implies that $N_t$ must grow at rate $g$, and that its equilibrium level must be related to $H$ according to

$$N_t = \frac{H}{g\gamma_0} e^{gt}. \tag{21}$$

Next, note that as (7) still holds, total expenditure $R_t$ also grows at rate $g$ :

$$R_t = a_0 L e^{gt}. \tag{22}$$

Consequently, the consumption level $c$ of each good, which equals $R_t/N_t$, is constant. Substituting these results into (13) shows that the real interest rate must be constant and equal to $\rho$ :

$$r = \rho.$$

Finally we can use equations (21) and (22) to express wages in terms of $H/L$, using (14):

$$w_t = a_0 e^{gt} \frac{\bar{c}(H/L) - 2a_0 g\gamma_0}{\bar{c}(H/L) - a_0 g\gamma_0} \tag{23}$$

Another equation can be obtained for $\omega$, using (19), (20), (21), (22) and (23):

$$\omega_t = e^{gt} \frac{g^2 a_0^2 \gamma_0}{\rho(H/L)(\bar{c}H/L - a_0 g\gamma_0)} \tag{24}$$

Dividing (24) by (23), we get an equation for the *relative demand* for creativity:

$$\frac{\omega}{w} = \frac{g^2 a_0 \gamma_0}{\rho(H/L)(\bar{c}H/L - 2a_0 g\gamma_0)} \tag{25}$$

The model is then solved as follows: First, (25) and (18) jointly determines the ratios $H/L$ and $\omega/w$. Once this is done, levels can be computed using (24), (23), (16) and (17).[5]

[5] At each instant of time, total human wealth is given by $W_{Ht} = (\omega_t H + w_t L)/(\rho - g)$, while total financial wealth is $W_{Ft} = \pi_t N_t/\rho$. Aggregate consumption must then be equal to $R_t = (W_{Ft} + W_{Ht})(\rho - g)$. Using (24), (23), (21), (19), and (20) we can check that this actually boils down to $R_t = a_t L$. As $\rho > g$, at each date consumption is the sum of labor income plus a fraction of capital income.

The structure of the growth process is captured by parameters $a_0, \gamma_0$, and the growth rate $g$. Parameters $\bar{c}$ and $\rho$ characterize the structure of preferences.

We are now in a position to discuss how the structure of preferences and of the growth process affect the distribution of income. It turns out that to understand the logic of the results it is best to deal with two special cases.

## 4.1 Two special cases

### 4.1.1 Perfect mobility: $\omega/w =$ Constant

We first consider the case where the supply response of the allocation of talent to the relative return to creativity is infinitely elastic. This will happen if creativity is exactly as concentrated as productivity, i.e. $\alpha_l/\alpha_h = \beta_l/\beta_h$. In such a case, in equilibrium the relative return to creativity must be equal to a constant exactly equal to this common ratio. As illustrated on figure 2, the relative supply of creativity is infinitely elastic, i.e. horizontal.

The equilibrium value of $H/L$ can then be computed by plugging the equilibrium ratio $\omega/w$ into (25). Normalizing it to 1 for simplicity, we get the following solution:

$$H/L = \frac{g a_0 \gamma_0}{\bar{c}}(1 + \sqrt{1 + \frac{\bar{c}}{\rho a_0 \gamma_0}}). \tag{26}$$

Plugging this into (23), this allows to compute wages:

$$w_t = \omega_t = a_0 \left( 1 - \frac{1}{\sqrt{1 + \frac{\bar{c}}{\rho a_0 \gamma_0}}} \right) e^{gt} \tag{27}$$

The question we are interested in is: how do wages depend on the demand and growth processes and is there a productivity paradox? This formula shows that wages are increasing in needs $\bar{c}$, as in the static model. They also increase when people are more impatient ($\rho$ falls), and innovations are cheaper ($\gamma_0$ falls). All these changes move the economy away from a Marxian zone by either directly increasing the elasticity of utility or by increasing the incentives to introduce new goods. What about $a_0$? The effect is not a priori obvious since it is the product of an increasing productivity time one minus an increasing mark-up. However, (27) reveals that $w$ is increasing in $a_0$[6] and converges from below to a level given by[7]

$$w_\infty = \frac{\bar{c}}{2\rho\gamma_0} e^{gt}.$$

This level grows with time because of balanced productivity growth, but at any given $t$ it remains finite despite that productivity $a_0$ tends to infinity. As (26) shows, $H/L$ increases and goes to infinity as $a_0$ rises. As productivity becomes infinite in the production sector, everybody specializes in the production of blueprints.

As productivity increases, new goods are being introduced, which prevents consumption from rising too much and hence the elasticity of demand from falling too much. As a result wages do not fall and the Marxian zone no longer exists (Figure 3). However, a flavor of it remains. Wages do not increase proportionally to productivity and the ratio between the two falls and goes to zero as productivity becomes infinite. Furthermore, as $a_0$ becomes infinite wages converge to a constant. Thus successive productivity improvements have ever an smaller impact on wages.

What about the distribution of earnings? As $\alpha_l/\alpha_h = \beta_l/\beta_h$, in equilibrium people are always indifferent between specializing in production or innovation. At any date $t$, the income of any agent $b$ is therefore given by $y_t(b) = w_t(\alpha_l b + \beta_l)$. Relative measures of inequality therefore only depend on $\alpha_l$ and $\beta_l$; they are affected neither by the growth process nor preferences. In the $(b, \ln y_t(b))$ plane, both balanced growth and changes in $a_0, \gamma_0, \bar{c}$,and $\rho$ translate the wage schedule vertically (Figure 4).

As we have argued above, purchasing power is different from utility. As $a_0$ becomes infinite, the number of goods increases, which in itself increases

[6] $dw/da_0$ is positive if and only if $(1+x)^{3/2} - (1+x) > x/2$, with $x = \bar{c}/(\rho a_0 \gamma_0)$. This inequality is always true.

[7] This can be seen using a first-order taylor expansion of (27) in $1/a_0$.

welfare (see eq. (5)). Therefore, the changes in both wages and diversity tend to increase utility, so that there is no Marxian zone as well in terms of utility

The distribution of income labor and capital (which consists of intangible property rights on existing goods) is also interesting to look at. As profits must be equal to $\omega\rho\gamma_0 e^{gt}$, the ratio $\pi/w_t$ (which falls with $t$) does not depend on $a_0$. As $N_t$ increases with $a_0$, the ratio of total profits over total wages $\pi N_t/w_t$ (which is constant wit $t$) is increasing with $a_0$.

### 4.1.2 Zero mobility: $H/L =$ Constant

The other special case we consider is that of an inelastic relative supply curve of creativity (Figure 5). This case obtains if the distribution of skills has two masses, say if there are "creative" people unable to work in the production sector, and "productive" people unable to work in R and D.[8] Let us therefore assume $H = L = 1$. In this case, we can readily compute wages using (23), which now boils down to

$$w_t = a_0 e^{gt} \frac{\bar{c} - 2a_0 g\gamma_0}{\bar{c} - a_0 g\gamma_0}.$$

This equation is very similar to (8) and has the same analytical properties. Consequently we are back to the case where past a critical level of productivity, further increases in $a_0$ reduce wages in the production sector. In contrast, (24) imply that the wage of creative workers unambiguously increase, while (25) implies that inequality always increases provided $\omega \geq w$ initially, which would be the case if creative workers had the option of working in production with the same productivity as non creative ones, while the reverse would be impossible.

Therefore, the absence of mobility from production to innovation forces workers to get poorer as productivity in the output sector becomes too large relative to that in the R and D sector.

[8] It will also obtain if people don't have to specialize and get a wage equal to the sum of the returns to their creativity and productivity.

Creative workers always benefit from an increase in $a_0$, while productive workers may either benefit or lose. In contrast, a fall in the cost of R and D $\gamma_0$ unambiguously harms the former, while benefitting the latter. This is clear from (24). A fall in $\gamma_0$ increases the number of varieties. This reduces the quantity of each variety being consumed, which, given the structure of demand, triggers a sharp fall in markups. These two effects together are stronger than the increase in creative workers's productivity, so that on net their wages fall. Again, if $\omega/w > 1$ initially, inequality falls.

Faster growth, i.e. an increase in $g$, benefits creative workers while harming productive ones. This is because it boosts the value of R and D firms, which is reflected in the wages of creative workers. At the same time, the accumulated number of goods $N_t$ falls relative to the productivity growth trend (eq. (21)). This effect is similar to the fall in the capital/labor ratio that one gets in the Solow model when growth is faster. Here, it means that consumption of each good $c = R/N$ increases, which increases markups, thus adversely affecting wages in the output sector.

Finally, greater needs (a greater value of $\bar{c}$) increase wages for reasons already explained in section 2, while they reduce the return to creativity because of their adverse effects on markups.[9]

As in the previous section, the ratio $\pi/\omega$ does not depend on $a_0$. This implies that $\pi/w$ grows with $a_0$, as does $\pi N/w$. Contrary to the previous subsection, it increases because each firm makes more profits, not because there are more firms. In both cases the model predicts that productivity increases result in an increase in the ratio between stock market capitalization and the wage bill.

## 4.2 Some more general results

We now study the more general case where the relative supply of creativity has an arbitrary elasticity. The equilibrium values of $H/L$ and $\omega/w$ are then

[9] Here, analysis in terms of wages coincides with welfare analysis, as the number of goods does not change.

determined by the intersection of the relative supply and relative demand curve, as illustrated on Figure 6. Let us again discuss the impact of an increase in productivity $a_0$. As (25) makes clear, the relative demand for creativity shifts up, while the relative supply schedule is unaffected. The relative return to creativity $\omega/w$ necessarily rises, as does the $H/L$ ratio.

What happens to the distribution of income? Our assumptions imply that the most skilled will specialize in knowledge production, and that the wage schedule is steeper for these workers than for those who specialize in the output sector. This is illustrated on Figure 7. Figure 8 depicts the impact of an increase in $a$. Inequality increases for two reasons. First, the critical skill level $b^*$ falls, so that more people have their earnings determined by the steeper schedule which rewards creativity. Second, as $\omega/w$ rise, the gap between the two slopes rises, thus augmenting inequality. The Solovian case where there is no absolute fall in wages is represented on figure 8a, while the Marxian case is represented on figure 8b.

Therefore, general technical progress in the production sector inherently increases inequality through its effect on markups, which harms workers specialized in that sector, who, under the assumption that creativity is more spread than productivity, turn out to be the least rich. In contrast, if markups were constant, wages would be proportional to productivity, $w_t = \varphi a_t$, and it is easy to check that $\omega_t$ would be given by $\omega_t = a_t g(1-\varphi)L/(H\rho)$. Therefore, the relative demand for creativity would be unaffected by technical progress. As the relative supply is also unaffected, technical progress would be plainly neutral, increasing all incomes proportionally without any effect on the allocation of labor.

What about the *absolute* wages of workers in the output sector? They may fall if there exists a Marxian zone. We have seen that such a zone does not exist if relative supply is infinitely elastic, while it does if elasticity is zero. Intuitively, therefore, wages may fall if the relative supply curve is steep enough.

What can be established is that if the elasticity of relative supply is bounded, a Marxian zone necessarily exists, and that it includes all productivity levels beyond a critical one.

*PROPOSITION 1 — Assume there exists* $\eta > 0$ *such that* $x\varphi'(x)/\varphi < \eta, \forall x > 0$. *Then one of the two following statements must hold:*

*(i) There exists a value of* $a_0$ *such that* $w_t = 0$ *for all* $t$ *in the associated balanced growth path,*

*OR:*

*(ii) for all* $a^*$ *there exists* $\tilde{a} > a^*$ *such that*

$$\frac{\partial w_t}{\partial a_0} < 0 \text{ at } a_0 = \tilde{a}$$

PROOF — See Appendix

Therefore, Marxian zones are the rule rather than the exception. Only if relative supply is infinitely elastic asymptotically can they fail to arise.

Again, the loss of purchasing power may be compensated, in terms of welfare, by the increase in $N$ triggered by a rise in productivity. However, if there exist workers with an arbitrarily low labor endowment, their welfare necessarily falls. This is because such workers value diversity very little: the effect of $N$ on their utility is of second-order relative to that of $w$.[10] We shall return to this property in section 6 when we discuss redistributive policies.

# 5 Unbalanced growth

Note that the previous exercises compare wage levels across trajectories where all variables grow at rate $g$. While an increase in productivity may have an adverse effect on wages, they nevertheless keep growing so that there is no secular decline in wages. Such a decline, following a jump in $a_0$, is at best transitory. Things are radically different, however, if growth is unbalanced, i.e. if $\gamma$ does not fall at the same rate as $a$ increases. Assume for example

[10] This is again implied by the formula for the utility flow: $u = wl(\bar{c} - wl/2N)$. The effect of $w$ is of order $l$, while that of $N$ is of order $l^2$.

that $\gamma$ falls at a rate $h$ lower than $g$. It is easy to see that in the long run, the number of goods cannot growth at a rate faster than $h$, as the total input in the production process is bounded above. Assume it does: $\dot{N}/N = h$. As implied by eq. (14), for wages not to become equal to zero in finite time, total employment in efficiency units in the production sector, $L$, must fall at a rate at least equal to $h-g$. Using (8), this implies that in the long run wages grow at rate $g$. Assuming an isoelastic relative supply curve, $\varphi(\omega/w) = (\omega/w)^{\eta}$, we can also see that in the long run the rate of growth of $\omega$ must exceed that of $w$ by at least $(g-h)/\eta$. Furthermore, (19) implies that profits cannot grow faster than $g_R - g_N = g + g_L - h \leq 0$. This in turn implies that $\omega$ grows at most at rate $h$, which is lower than $g$. This contradicts the claim that $\omega$ must grow faster than $g$.

Therefore, such unbalanced growth necessarily drives the economy to a point where wages are equal to zero. That is, because ideas cannot keep up with productivity gains in the material sector, the demand for each good tends to be saturated and the economy ends up in the marxian zone until wage actually become equal to zero. In the end, everybody works in the knowledge sector, introducing new goods sold at an infinite markup. Everybody has become a 'symbol manipulator' and inequality has increased as creativity is more unevenly distributed than productivity.

# 6 Policy implications

## 6.1 Malthusian policies vs. redistribution

The existence of Marxian zones seem to vindicate "Malthusian" policies that oppose the adoption of new technologies by requiring work rules,[11] or prevent

[11] In *Das Kapital,* Chapter 15, Marx gives some interesting historical examples of such policies:

"In the 17th century nearly all Europe experienced revolts of the workpeople against the ribbon-loom, a machine for weaving ribbons and trimmings, called in Germany Bandmühle, Schnurmühle, and Mühlenstuhl. These machines were invented in Germany. Abbé Lancellotti, in a work that appeared in Venice in 1636, but which was written in 1579, productivity from rising by imposing working time reductions. Such policies are often advocated by people whose view of the world is similar to the "end of work" arguments. Indeed, the above analysis suggests that if reducing inequality and increasing the wages of the poorest is a political or social goal, such policies will indeed work in the desired direction.

At face value, such productivity-reducing measures are likely to be quite inefficient, i.e. dominated by policies that simply redistribute from firms or creative workers in favor of productive workers. Such redistribution, however, is not without distortions, as it induces a movement of labor away from innovation into the output sector. This has two implications. First, more of each good is being consumed, which tends to boost markups and results in lower wages; this is avoided by malthusian policies which push the economy into the low markup zone. The question is: can the effect be strong enough to overturn our a priori presumption that redistribution dominates malthusian policies. Second, there is less diversity, which tends to reduce welfare. The question is: can this effect be strong enough so as to make redistribution ineffective, in the sense that it does not manage to increase the welfare of

says as follows: "Anthony Müller of Danzig saw about 50 years ago in that town, a very ingenious machine, which weaves 4 to 6 pieces at once. But the Mayor being apprehensive that this invention might throw a large number of workmen on the streets, caused the inventor to be secretly strangled or drowned." In Leyden, this machine was not used till 1629; there the riots of the ribbon-weavers at length compelled the Town Council to prohibit it. "In hac urbe," says Boxhorn (Inst. Pol., 1663), referring to the introduction of this machine into Leyden, "ante hos viginti circiter annos instrumentum quidam invenerunt textorium, quo solus plus panni et facilius conficere poterat, quan plures aequali tempore. Hinc turbae ortae et querulae textorum, tandemque usus hujus instrumenti a magistratu prohibitus est." After making various decrees more or less prohibitive against this loom in 1632, 1639, &c., the States General of Holland at length permitted it to be used, under certain conditions, by the decree of the 15th December, 1661. It was also prohibited in Cologne in 1676, at the same time that its introduction into England was causing disturbances among the workpeople. By an imperial Edict of 19th Feb., 1685, its use was forbidden throughout all Germany. In Hamburg it was burnt in public by order of the Senate. The Emperor Charles VI., on 9th Feb., 1719, renewed the edict of 1685, and not till 1765 was its use openly allowed in the Electorate of Saxony. This machine, which shook Europe to its foundations, was in fact the precursor of the mule and the power-loom, and of the industrial revolution of the 18th century. It enabled a totally inexperienced boy, to set the whole loom with all its shuttles in motion, by simply moving a rod backwards and forwards, and in its improved form produced from 40 to 50 pieces at once. "

the poorest?

As for the first question, one can clearly show that redistribution dominates. A simple graphical argument is depicted on figure 9. Redistribution allows to move the economy along a trade-off between creative and productive income. Restricting productivity moves this trade-off inwards, so that to achieve the same level of wages one gets a lower return to creativity. While restricting productivity can move the economy from point A to point C where wages are higher, redistributing up to the point where $H/L$ (and therefore $\omega/w$) is the same as at C will achieve higher wages and a higher return to creativity.[12] This result is far from surprising, but worth proving given the popularity of malthusian policies in some European countries.

Let us now deal with the other question, namely the extent to which reductions in diversity may end up harming those people who are supposed to benefit from redistribution. To analyze that, let us consider a scheme that taxes profits (as in Weitzman (1985)) and uses the proceeds to subsidize wages. One gets a situation where productive labor is paid $\bar{w}_t > w_t$. Next, take equation (5), which gives the utility flow of a worker with income $R_j$. This equation still applies to our balanced growth path. A worker with labor endowment $l$, specialized in the productive sector, will get a utility flow of $\bar{w}_t l(\bar{c} - l\bar{w}_t/2N_t)$. As was already pointed out, for $l$ arbitrarily small, the increase in $\bar{w}_t$ dominates over the fall in $N_t$, which only has a second order effect on welfare. In other words, diversity benefits the rich. The poor are too far from their bliss point on each good to benefit from the introduction of new varieties. At low consumption level utility is near linear, and it is as if the physical quantities of each good could simply be added to compute one's utility.

Consequently, redistribution could only harm arbitrarily poor people if it reduced their post-subsidy wage. But this cannot happen, as Proposition 2 makes clear.

*PROPOSITION 2 – Consider the introduction of a profit-sharing scheme*

[12] A more formal argument is presented in the Appendix.

*such that a fraction $\theta$ of profits is taxed to finance a proportional wage subsidy to productive work. Then, if $\bar{w}_t$ denotes the wage, inclusive of the subsidy:*

$$\frac{\partial \bar{w}_t}{\partial \theta} > 0,$$

*across steady states.*

PROOF – See Appendix.

Of course, redistribution may well harm some productive workers whose labor endowment is not too low so that they suffer more from the lower diversity than they gain from greater wages. That is, there exists a cut-off level of skill above which people may be harmed even though they work in the productive sector; this level is lower, the greater the fall in the number of goods associated with a given increase in the post-subsidy wage. Only a pure Rawlsian in a world where arbitrarily poor people can be found would want to engage in such a profit-sharing scheme regardless of its effect on diversity.[13]

Conceivably, redistribution can also increase the wages (i.e. material purchasing power) of creative workers, as more labor is dedicated to material production—which may result in greater material consumption for everybody. This is indeed what happens, as a corollary of proposition 2, in the infinite elasticity case where the $\omega/\bar{w}$ is fixed. More surprisingly, it is even possible that redistribution be Pareto-improving, because it can be shown that, given our specification, there are two many varieties relative to the utilitarian first best (see the Appendix for a formal result). However, just as the diversity-reducing effect of redistribution is negligible relative to their wage effect for arbitarily poor people, just the opposite holds for arbitrarily rich people, i.e. people whose consumption level is arbitrarily close to the bliss point. They cannot be made happier by an improvement in material

[13]Note that in this case the proportional wage subsidy that we consider is far from appropriate.

living standards; only greater diversity can please them. Formally, the derivative of the utility flow $R_j(\bar{c} - R_j/2N_t)$ with respect to income can be made arbitrarily small if $R_j$ is close enough to the bliss point $\bar{c}N_t$. Consequently, redistribution cannot be Pareto-improving if there exists such people in the economy.

The preceding discussion suggests that there are non trivial redistributive conflicts associated with innovation. In our world with horizontal innovation, moving resources toward the "blueprint" sector creates more varieties but reduces physical output, which is what the poor really care about. That is, policies that favor knowledge producers such as tougher enforcement of intellectual property rights or subsidies to R & D may have undesirable distributive effects.

## 6.2 Unions

It is interesting to consider the role of a specific redistributive institution, namely labor unions. Going back to the simple static model of section 1, let us assume that each firm is unionized, and that the wage within each firm is determined by the union which maximizes of the total wage bill. Substituting the pricing rule into (3) and using the price normalization and the production function, we get the labor demand curve faced by the union in sector $i$ :

$$l_i(w) = \frac{\bar{c}(1 - w/a) + Rw/aN}{2a} \tag{28}$$

Maximization of $wl_i(w)$ gives the following wage-setting rule:

$$w_i = w = \frac{aN\bar{c}}{2(N\bar{c} - R)} \tag{29}$$

Substituting this into (4) and using the normalization $p_i = 1$, we get the determination of total expenditure

$$R = N\bar{c}/4, \tag{30}$$

which, given (29) implies that in general equilibrium wages are given by

$$w = \frac{2a}{3}. \tag{31}$$

Consequently wages are proportional to productivity and there is no Marxian zone. Insiders appropriate two-thirds of productivity gains via their wage-setting behaviour. On the other hand, using (31) and (30) into (28), we get that total employment is

$$l = \frac{N\bar{c}}{4a}.$$

Thus productivity growth unambiguously reduces employment! That is the only way proportional increases in wage demands can be reconciled with the incipient tendency to increase markups. By generating unemployment, this unionized economy prevents expenditure from approaching the saturation level as productivity increases: indeed total expenditure is maintained constant (eq. (30)), so that markups do not increase and the Marxian zone is not reached, at the cost of higher unemployment.

In the more general dynamic model of section 3, rather than unemployment, we would have involuntary work in the creative sector, that is production of knowledge that is not very useful and generates little income. This reminds street entertainers, freelance writers, small theater companies, and so forth. Paradoxically, this increases the number of varieties, which, as we have seen, benefits the richest but does little to help the poorest.

## 6.3 Globalization

The model may also have striking implications regarding the impact of 'globalization', i.e. international trade, on wages. While the empirical literature has somewhat discredited the idea that trade is responsible for the observed rise in inequality (see e.g. Lawrence and Slaughter (1993)), the typical prediction is that trade should exert a downward pressure on wages of the unskilled in rich countries, because this factor is more abundant in poor countries. This is the Stolper-Samuelson theorem.

We show that this prediction may well be reversed, i.e. unskilled wages may rise in rich countries and fall in poor countries, so that globalization may lead in falling inequality in rich countries and rising inequality in poor countries. Before discussing the economic mechanism, let us show how the model can be extended to a two-country world.

We do it in the case where $H$ and $L$ are constant in each country, which is equivalent to using the static model with a fixed number of goods. There are two countries, Home (1) and Foreign (2). We assume that they differ in their productivities: $a_1 \neq a_2$. Country specific variables are denoted by a subscript $k = 1, 2$, while world variables are denoted by a tilda ($\tilde{}$). When these two countries trade with each other, the demand function and price rules (3) and (4) still hold, with $R$ and $N$ replaced by $\tilde{R}$ and $\tilde{N}$, the world expenditure level and number of goods; and $w$ and $a$ replaced by their levels specific to the country where the good is produced. Hence, applying (4), if $q$ denotes the fraction of goods produced at home, $q = N_1/\tilde{N}$, a monopoly producing in country $k$ will charge a price given by

$$p_k = \frac{\bar{c}\left[\tilde{N}qp_1^2 + \tilde{N}(1-q)p_2^2\right] + \frac{w_k}{a_k}\left(\bar{c}\left[\tilde{N}qp_1 + \tilde{N}(1-q)p_2\right] - \tilde{R}\right)}{2\left(\bar{c}\left[\tilde{N}qp_1 + \tilde{N}(1-q)p_2\right] - \tilde{R}\right)}; k = 1, 2.$$

Similarly the output level of such a producers is given by

$$c_k = \bar{c} - p_k \frac{\bar{c}\left[\tilde{N}qp_H + \tilde{N}(1-q)p_F\right] - \tilde{R}}{\tilde{N}qp_1^2 + \tilde{N}(1-q)p_2^2}; k = 1, 2$$

Equilibrium in the labor market of each country implies

$$\begin{aligned} L_1 a_1 &= \tilde{N}qc_1 \\ L_2 a_2 &= \tilde{N}(1-q)c_2 \end{aligned}$$

Finally, we assume that monopolies can freely choose the localisation of their production activity. This implies that profits must be equal across the two countries, i.e.

$$(p_1 - \frac{w_1}{a_1})c_1 = (p_2 - \frac{w_2}{a_2})c_2.$$

There is a fixed number of firms $N_k^A$ based in each country $k$, which can be interpreted as proportional to the fixed number of creative workers in that country. But these firms can produce in the other country if they want to, i.e. $N_k^A$ may be different from $N_k$. Thus factories as well as goods are perfectly mobile.

Given that $\tilde{N} = N_1^A + N_2^A$ is exogenous, the above equations define 7 relationships in 8 variables: $p_k, c_k, w_k, \tilde{R}$ and $q$. The model can be solved by picking up any price normalization. It is easy to see that the solution is given by:

$$p_1 = p_2 = 1$$

$$\frac{w_1}{a_1} = \frac{w_2}{a_2} = \frac{\tilde{N}\bar{c} - 2(a_1 L_1 + a_2 L_2)}{\tilde{N}\bar{c} - (a_1 L_1 + a_2 L_2)}. \tag{32}$$

$$\tilde{R} = a_1 L_1 + a_2 L_2$$

$$q = \frac{a_1 L_1}{a_1 L_1 + a_2 L_2}$$

$$c_1 = c_2 = \frac{a_1 L_1 + a_2 L_2}{\tilde{N}}$$

How does globalization affect wages? To know that we should compare wages as defined by the above formula to autarkic wages. For country 1, they are equal to

$$w_1^A = a_1 \frac{N_1^A \bar{c} - 2a_1 L_1}{N_1^A \bar{c} - a_1 L_1} \tag{33}$$

Straightworward comparison of (33) and (15) shows that $w_1^A > w_1$ if and only if

$$\frac{L_1 a_1}{N_1^A} < \frac{L_2 a_2}{N_2^A},$$

or equivalently

$$c_1^A < c_2^A,$$

where $c_k^A$ is the consumption level of each good, in autarky, in country $k$.

Consequently the country where wages fall is the country where consumption of each good is initially lower, i.e. the country where the number of firms is higher relative to total productive capacity. By increasing the output of the firms based in that country, globalization moves this country closer to the Marxian zone, and its workers suffer from the higher markups applied by its firms to new customers who care less about the good. From a factual point of view, this may well be the poorest country. [14]

As in the previous discussion, the same caveats regarding the effect of greater diversity apply: in the country were wages fall, some workers may still gain in welfare terms because of greater variety, but this is not the case of arbitrarily poor ones.

# 7 Conclusion and assessment

The idea that general technical progress is harmful to labor and may lead to "The end of work" is typically inconsistent with general equilibrium analysis. General technical progress makes one unit of labor worth more in terms of consumption goods. It is very hard to escape this conclusion in a well-specified model. One can get transitory negative effects on labor if retraining is needed (as in Aghion and Howitt, 1994), or if technical change is asymmetrical and labor reallocation is costly (as in Cohen and Saint-Paul, 1994), but these effects are unlikely to be very long lived. In this paper I have shown, in the context of a standard model of monopolistic competition between differentiated goods, that if needs are finite, productivity will affect markups in a way that is systematically detrimental to wages in the tangible goods sector.

[14] Another interesting implication of (15) is that productivity growth in one country unambiguously reduces wages in the other one.

Before concluding, it is worth repeating that wages have increased by several orders of magnitude over the very long run. This is consistent with the basic Solow model, and it is what happens along the balanced growth paths that we have studied. The Marxian productivity paradox highlighted in this paper certainly does not apply to the experience of modern economies over the last two centuries. However, what is more plausible is that an economy experiences stagnation or decline in wages for several decades as the outcome of productivity growth and the "banalization" of consumption. This is what has been observed recently and what this model is aimed at explaining. In the longer run, such a decline will be reversed, because productivity will also increase in the knowledge sector, and also because public authorities are likely to force a reduction in markups by competition policies such as anti-trust laws, a mechanism we have ignored in our analysis by assuming that monopoly power is intact in response to changes in productivity.

APPENDIX

PROOF OF PROPOSITION 1 — Let $h = H/L$ and $x = h\bar{c}/(ga_0\gamma_0)$. In any equilibrium one must have $x > 2$. Differentiating (23) we see that wages are locally falling with an increase in $a_0$ if and only if

$$\frac{a_0}{h}\frac{dh}{da_0} < 4 - x - 2/x \tag{34}$$

Furthermore, using (25) together with (18) and our assumption that $x\varphi'(x)/\varphi < \eta$ we get that

$$\frac{a_0}{h}\frac{dh}{da_0} < \frac{\frac{g}{\rho h} + 2\varphi^{-1}(h)}{\varphi^{-1}(h)\left[(x-2)/\eta + 2(x-1)\right]}. \tag{35}$$

First, note that given our formula for wages, for wages to remain positive with $a_0$ it must be that $h$ goes to infinity with $a_0$. Otherwise (i) holds. Let us therefore assume that (i) does not hold. Then $h$ becomes infinite with $a_0$.

Second, when $h$ becomes infinite so must $\varphi^{-1}(h)$. Otherwise it would converge to a constant and the condition $x\varphi'(x)/\varphi < \eta$ would be violated as $x$ converges to that constant from below.

Third, the following statement cannot be true:

$$\exists A, \tilde{a} > 0, \forall a_0 > \tilde{a}, x > 2(1+A) \tag{36}$$

where $x$ refers to the value of $x$ in the balanced growth path corresponding to this value of $a_0$.

If (36) were to hold, then (35) would imply

$$\frac{a_0}{h}\frac{dh}{da_0} < \frac{\frac{g}{\rho h} + 2\varphi^{-1}(h)}{\varphi^{-1}(h)[2A/\eta + 2(1+2A)]}.$$

As $h$ and $\varphi^{-1}(h)$ go to infinity with $a_0$, the RHS converges to $\frac{2}{2A/\eta+2(1+2A)}$, which is strictly below one. But this implies that $h/a_0$ eventually goes to zero, and so does $x$, which contradicts (36).

Therefore, the contrary of (36) holds, i.e.

$$\forall A, \tilde{a} > 0, \exists a_0 > \tilde{a}, x < 2(1+A)$$

In words, there exist steady states with $a_0$ arbitarily large and $x$ arbitrarily close to 2.

Let us write $x = 2(1+\varepsilon)$, with $\varepsilon \ll 1$. Then (25) and (18) imply that

$$\frac{1}{h} = \varphi^{-1}(h)\frac{2\rho}{g}\varepsilon.$$

Plugging this into (35) we get that

$$\frac{a_0}{h}\frac{dh}{da_0} < \frac{(1+\varepsilon)}{[\varepsilon/\eta + 1 + 2\varepsilon]} \tag{37}$$

Now, for $\varepsilon$ small enough, the RHS of (37) is strictly smaller than the RHS of (34). To see this, apply a first-order Taylor expansion on both expressions and note that the RHS of (37) is then given by $1 - \varepsilon - \varepsilon/\eta$, while the RHS of (34) is given by $1 - \varepsilon$. Therefore, wages are locally falling with $a_0$ in all the steady states such that $\varepsilon$ is small enough. Since such steady states can be constructed for arbitrarily large values of $a_0$, (ii) must hold. Q.E.D.

PROOF OF PROPOSITION 2 — As in Weitzman (1984), firms maximize $(1-\theta)(p_i c_i - w c_i/a)$, which leads to the same pricing rule. The post-subsidy wage is then given by, using again our price normalization $p_i = 1$ :

$$\bar{w} = w + \theta\frac{p_i c_i - w c_i/a}{c_i/a} = (1-\theta)w + \theta a.$$

People get a weighted average of the marginal revenue of labor and the productivity level. From there, it is easy to see that pre-subsidy wages are still given by (23), so that the post-subsidy wage is

$$\bar{w}_t = a_0 e^{gt}\left[(1-\theta)\frac{\bar{c}(H/L) - 2a_0 g\gamma_0}{\bar{c}(H/L) - a_0 g\gamma_0} + \theta\right]. \tag{38}$$

Differentiating and denoting by $x = \bar{c}(H/L)/(a_0 g\gamma_0) > 2$, we get that this increases with $\theta$ iff:

$$1 + \frac{1-\theta}{x-1}\frac{dx}{d\theta} > 0. \tag{39}$$

The supply of creativity now depends on the post-subsidy wage ratio $\omega/\bar{w}$. Therefore, the relative supply curve can be expressed as

$$x = \frac{\bar{c}}{ga_0\gamma_0}\varphi(\frac{\omega}{\bar{w}}). \tag{40}$$

As for the return to creativity $\omega$, it can be computed by noting that pre-subsidy wages are unaffected by $\theta$, while firms only get a fraction $1-\theta$ of profits. Thus $\omega$, which is proportional to the present discounted value of profits, is simply given by the RHS of (24) multiplied by $1-\theta$ :

$$\omega_t = e^{gt}(1-\theta)\frac{g^2 a_0^2 \gamma_0}{\rho(H/L)(\bar{c}H/L - a_0 g\gamma_0)}. \tag{41}$$

Confronting (38), (40) and (41) and differentiating, we can compute $dx/d\theta$ and substitute into (39) to get the following equivalent condition:

$$1 > \frac{1-\theta}{x-1}\frac{\bar{c} + \frac{\omega}{\bar{w}}\rho x a_0\gamma_0}{\rho a_0\gamma_0\left[\frac{\omega}{\bar{w}}(2(x-1)+\theta) + \frac{x(x-2+\theta)ga_0\gamma_0}{\bar{c}\varphi'}\right]}$$

Now, the RHS is always smaller than $(1-\theta)/(x-1)/[2(x-1)+\theta]$. When $x$ moves from 2 to infinity, this quantity varies between $(1-\theta)/(2+\theta)$ and 0. Therefore this inequality is always satisfied. Q.E.D.

DERIVATION OF FIGURE 9 — Using (41), (40), and (38) we can derive the frontier between $\omega$ and $\bar{w}$ when $\theta$ varies, getting

$$\bar{\omega}_t = \frac{(a_t - \bar{w}_t)g}{\rho\varphi(\bar{\omega}_t/w_t)}.$$

The negative slope of this locus is a corrolary of prop. 2, while the preceding formula makes it evident that it shifts inwards when $a_t$ falls.

THE SOCIAL OPTIMUM — Let us consider a social planner who ascribes a weight $\lambda_b$ to type $b$ in its social welfare function. Thus it faces the following maximization problem:

$$\max_{\{c_{ibt}, N_t, b_t^*\}} \int_{b=0}^{\bar{b}} \lambda_b \int_{t=0}^{+\infty} \int_{i=0}^{N_t} c_{ibt}(\bar{c} - c_{ibt}/2)e^{-\rho t} didtdF(b),$$

subject to the following constraints

$$
\begin{aligned}
H_t &= \int_{b_t^*}^{\bar{b}} (\alpha_h b + \beta_h) dF(b) \qquad (42) \\
L_t &= \int_0^{b_t^*} (\alpha_l b + \beta_l) dF(b) \\
\dot{N}_t &= \frac{H_t}{\gamma_0} e^{gt} \\
L_t &= \int_0^{N_t} \frac{c_{it}}{a_0 e^{gt}} di. \qquad (43) \\
c_{it} &= \int_0^{\bar{b}} c_{ibt} dF(b) \qquad (44)
\end{aligned}
$$

In these constraints, we have already embodied the fact that it is optimal to assign people above some critical skill level $b_t^*$ to the knowledge sector, and people below that critical level to the material sector. This is an implication of our assumption that $\alpha_h/\alpha_l > \beta_h/\beta_l$.

$\lambda_b$ is the weight ascribed to type $b$. We assume the $\lambda$'s satisfy the following normalization:

$$\int_0^{\bar{b}} \frac{1}{\lambda_b} dF(b) = 1. \qquad (45)$$

A benchmark case is $\lambda_b = 1$, i.e. the utilitarian case.

The problem can then be solved in two steps. First, one computes the optimal intra-temporal allocation of consumption given $H_t$ and $L_t$. This is done by maximizing $\int_{b=0}^{\bar{b}} \lambda_b \int_{i=0}^{N_t} c_{ibt}(\bar{c} - c_{ibt}/2) di dF(b)$ under (43) and (44), taking $N_t$ as given. The solution is

$$c_{ibt} = c_{bt} = \bar{c} - \frac{1}{\lambda_b} \left[ \bar{c} - \frac{a_0 L_t e^{gt}}{N_t} \right].$$

Second, plugging this into the objective function, we see that the social planner's problem is reduced to the following:

$$\max_{\{N_t, b_t^*\}} \int_{t=0}^{+\infty} N_t \left[ I\bar{c}^2 - \left( \bar{c} - \frac{a_0 L_t e^{gt}}{N_t} \right)^2 \right] e^{-\rho t} dt,$$

subject to

$$
\begin{aligned}
H_t &= \int_{b_t^*}^{\bar{b}} (\alpha_h b + \beta_h) dF(b) = H(b^*) \\
L_t &= \int_0^{b_t^*} (\alpha_l b + \beta_l) dF(b) = L(b^*) \\
\dot{N}_t &= \frac{H_t}{\gamma_0} e^{gt},
\end{aligned}
$$

where $I = \int \lambda_b dF(b)$ is a constant, which, given (45) and Jensen's inequality, is greater than 1, equal to 1 in the utilitarian case, and increasing with the dispersion of $\lambda_b$.

The Hamiltonian is

$$\mathcal{H} = \frac{N_t}{2} \left[ I\bar{c}^2 - \left( \bar{c} - \frac{a_0 L_t e^{gt}}{N_t} \right)^2 \right] e^{-\rho t} + \nu_t e^{-\rho t} \frac{H_t}{\gamma_0} e^{gt},$$

where $\nu_t$ is the co-state variable, i.e. the marginal social value of an extra variety at $t$.

The first-order condition with respect to $b_t^*$, the control variable, is

$$a_0 \left( \bar{c} - \frac{a_0 L_t e^{gt}}{N_t} \right) \frac{dL}{db^*} + \frac{\nu_t}{\gamma_0} \frac{dH}{db^*} = 0$$

We know that $\frac{dL}{db^*} = \alpha_l b^* + \beta_l$ and $\frac{dL}{db^*} = -\alpha_h b^* + \beta_h$. Substituting, and noting that (15) can be written as $w \frac{dL}{db^*} + \omega \frac{dL}{db^*} = 0$, this equation can be written in a fashion similar to (18):

$$\frac{H_t}{L_t} = \varphi \left( \frac{\nu_t}{\gamma_0 a_0 \left( \bar{c} - \frac{a_0 L_t e^{gt}}{N_t} \right)} \right), \tag{46}$$

where $\varphi$ is the same function as in the text and $\frac{\nu_t}{\gamma_0 a_0 \left( \bar{c} - \frac{a_0 L_t e^{gt}}{N_t} \right)} = (\omega / w)_S$ is properly interpreted as the *marginal relative social return to creativity.*

The first order condition with respect to the state variable $N_t$ is:

$$\rho \nu - \dot{\nu} = \frac{1}{2} \left[ (I - 1)\bar{c}^2 + \left( \frac{a_0 L_t e^{gt}}{N_t} \right)^2 \right].$$

In a balanced growth path, one must have $\nu =$ Constant, and this allows to compute the relative social return to creativity. Using the relationships between $N$ and $H$, we get:

$$
\begin{aligned}
(\omega/w)_S &= \frac{\nu_t}{\gamma_0 a_0 \left(\bar{c} - \frac{a_0 L_t e^{gt}}{N_t}\right)} \\
&= \frac{\left[(I-1)\bar{c}^2 + \left(\frac{a_0\gamma_0 g L}{H}\right)^2\right]}{2\rho a_0 \gamma_0 \left(\bar{c} - \frac{a_0\gamma_0 L}{H}\right)}
\end{aligned}
\tag{47}
$$

As illustrated on figure A1, the social optimum is determined by the intersection of the relative social demand for creativity, given by (47), and the relative supply curve, given by (46), while equilibrium is determined by the intersection of the same supply curve with the private relative demand curve given by (25). It is straightforward to check that in the utilitarian case where $\lambda_b = I = 1$, the former is above the latter, so that there are too many goods, i.e. too many people specialized in knowledge production, in the equilibrium relative to the optimum. As $I$ goes up—meaning that there is a greater dispersion of weights in the welfare function—, however, so does the optimal number of goods, and for large enough $I$ the optimum entails more goods than the equilibrium. The explanation is as follows. The more the social planner favors specific groups, the more likely it will hit the satiety constraint when redistributing income in their favor. The only way to further increase their welfare is then by increasing the number of goods. We are back to the basic idea that diversity favors the "rich", i.e. here those who have the greatest weight in the welfare function.

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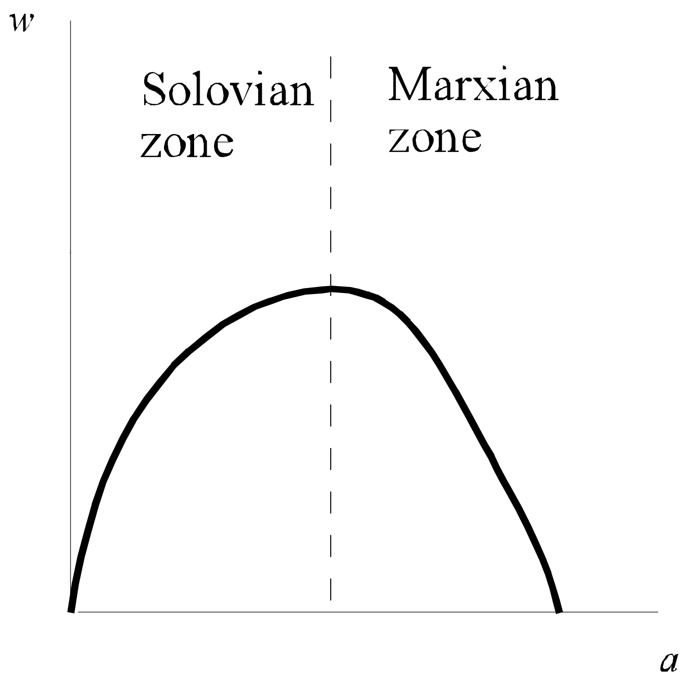


Figure 1: Effect of productivity on wages

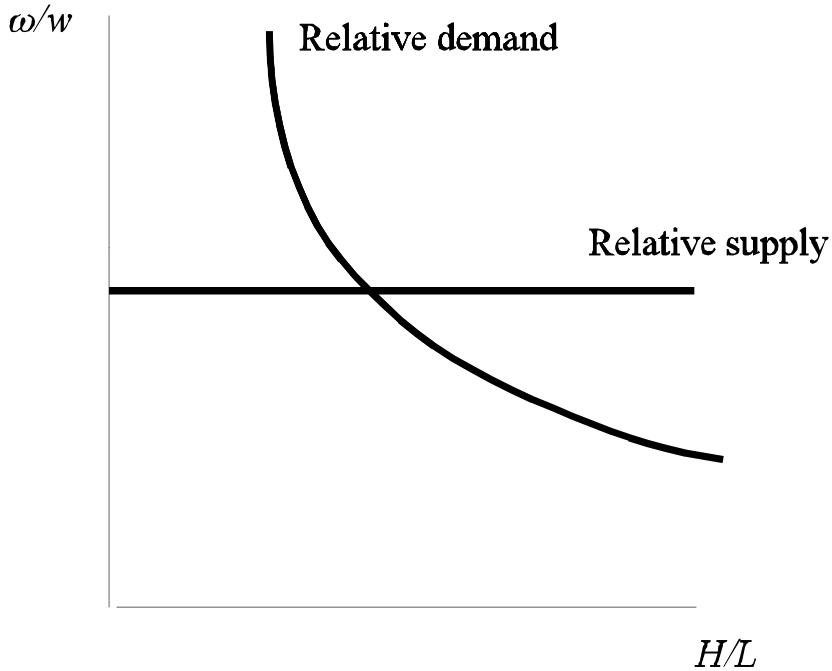


Figure 2: Equilibrium determination in the perfect mobility case

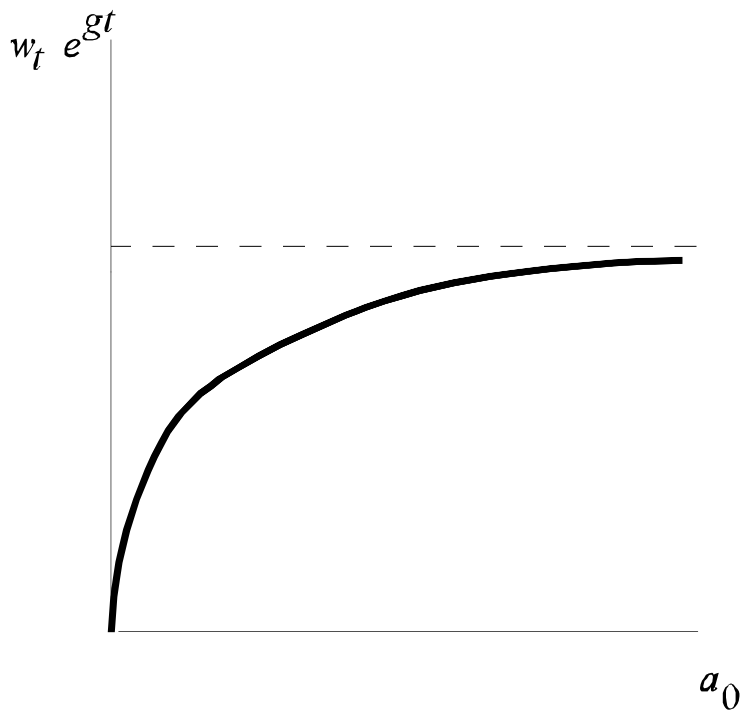


Figure 3: Effect of productivity on wages in the perfect mobility case

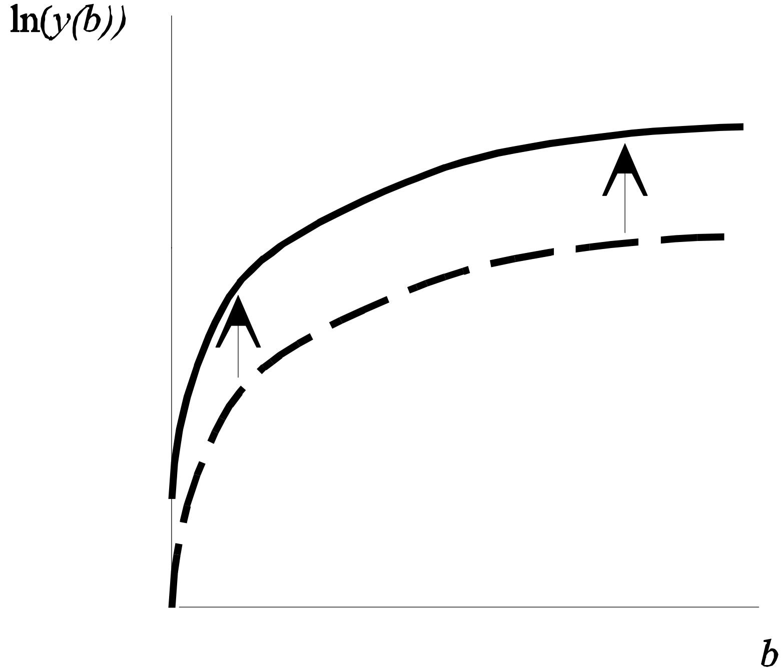


Figure 4: Effect of a productivity increase on the distribution of income; perfect mobility case

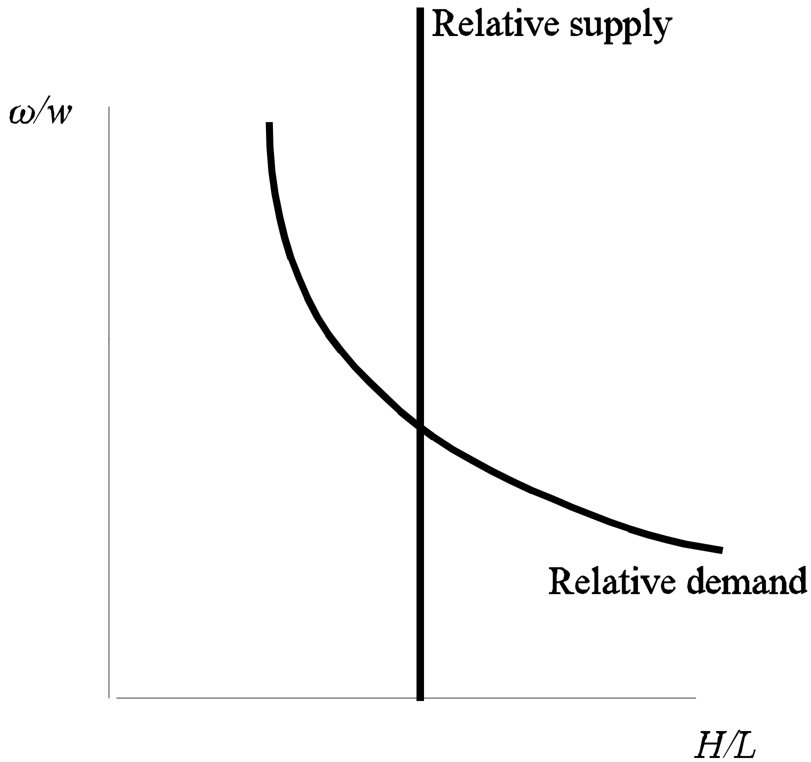


Figure 5: Equilibrium determination in the zero mobility case

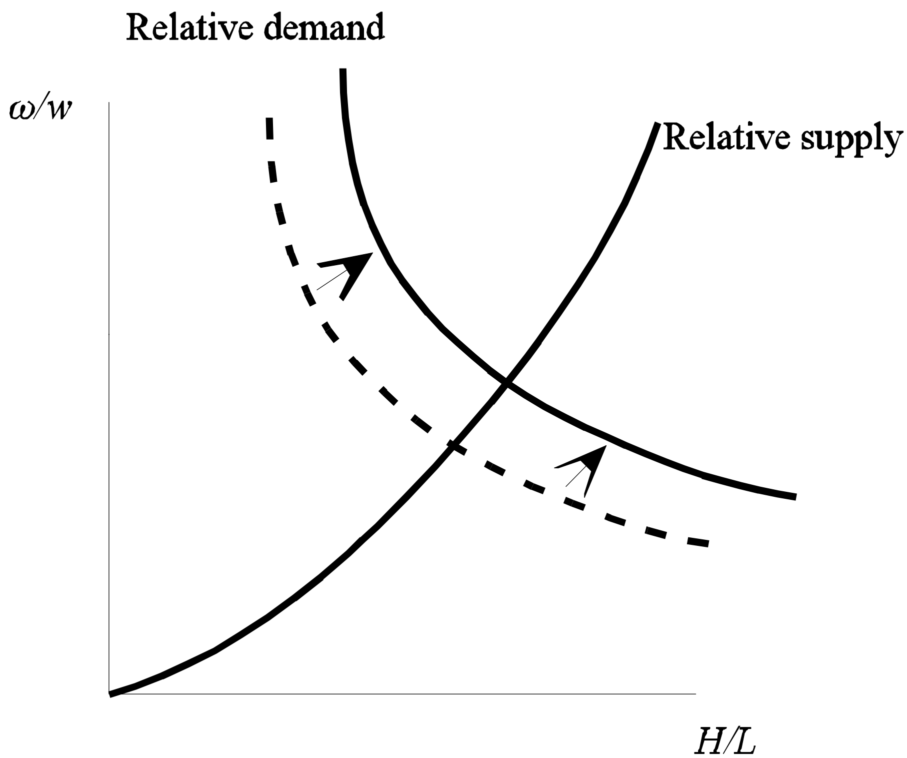


Figure 6: Impact of an increase in productivity in the general case

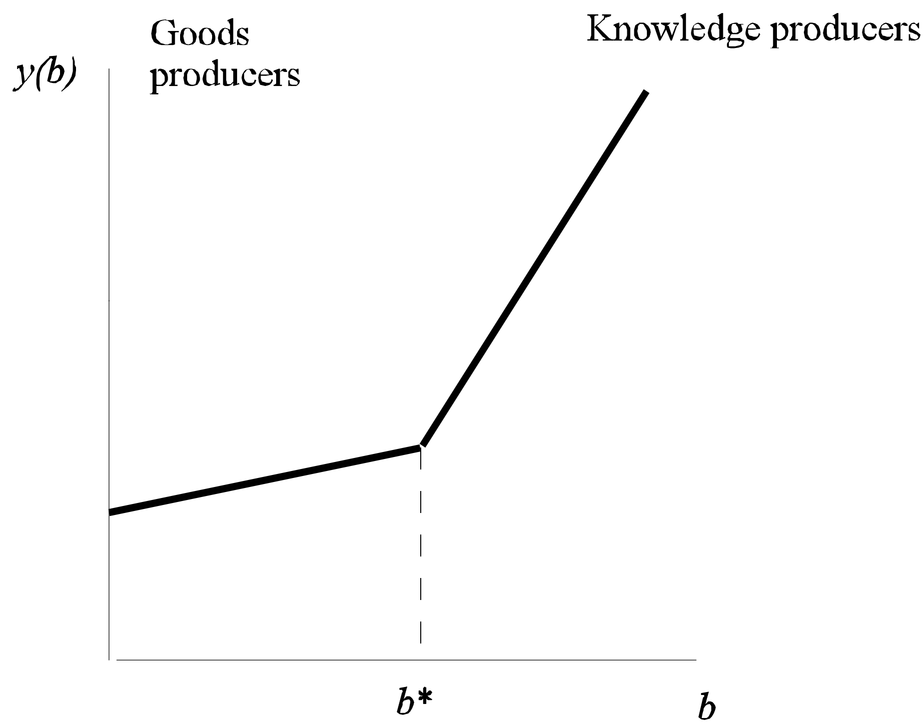


Figure 7: The distribution of income

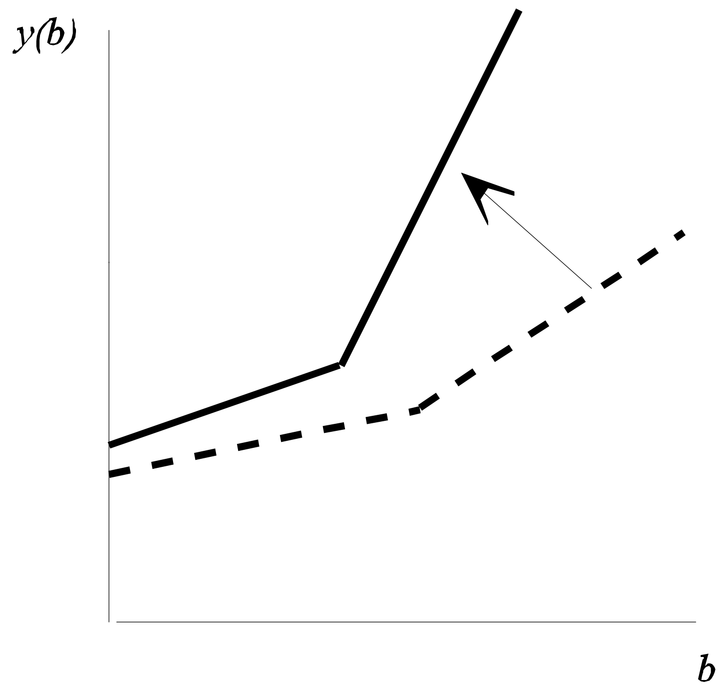


Figure 8a: Effect of productivity growth on the distribution of income (Solovian zone).

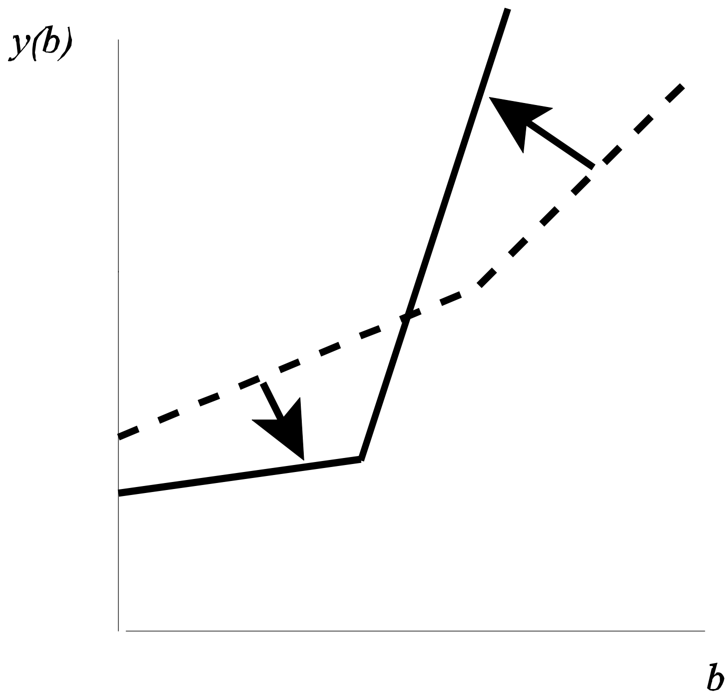


Figure 8b: Effect of a productivity increase on the distribution of income (Marxian zone).

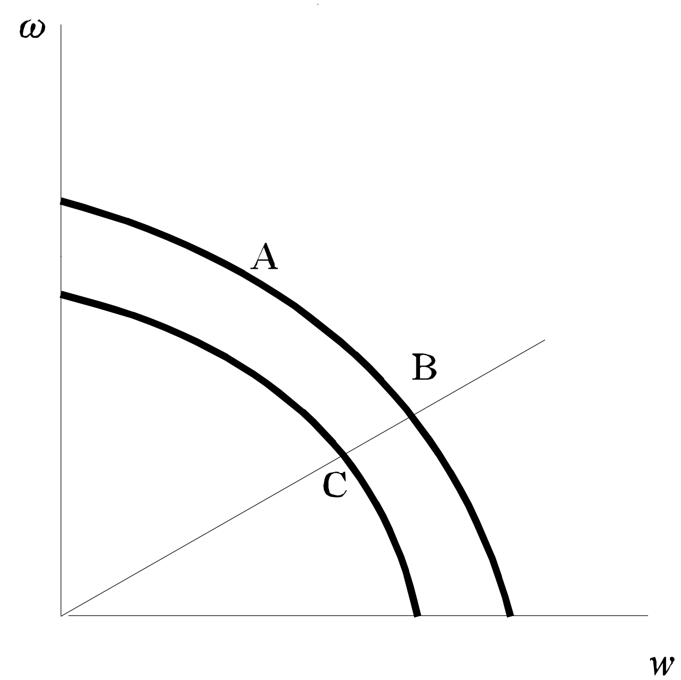


Figure 9: The dominance of redistribution over work rules