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Macroeconomics III
Problem 2

There exists a single final good, produced with a continuum of intermediate goods, indexed by $i \in [0,1]$. The production function is given by

$$Y_t = \left[\int_0^1 x_{it}^{\alpha} di \right]^{1/\alpha},$$

where Y_t is the output of the final good at date t, and x_{it} is the input of the intermediate good, and $\alpha < 1$.

The final good is produced by a competitive industry. The nominal price of input i at date t is p_{it}.

1. Show that the demand for input i at date t is given by:

$$x_{it} = Y_t \left(\frac{p_{it}}{p_t} \right)^{-\sigma},$$

where p_t is the price of the final good, which in equilibrium must satisfy:

$$p_t = \left(\int_0^1 p_{it}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \tag{1}$$

How is σ related to α?

The production function for any intermediate good is

$$x_{it} = l_{it},$$

where l_{it} is the amount of labor employed. The nominal wage at date Each intermediate good is produced by a monopoly. Each monopoly sets its price p_{it}, which is fixed in nominal terms. With probability λ per unit of time, however, an idiosyncratic shock occurs which allows the monopoly to revise its price. This shock is independent across monopolies. The nominal interest rate at any given date t is i_t. Firms maximize the expected present discounted value of their profits.

2. Show that at a date t when it is allowed to change its price, a monopoly sets the new price at a level $\bar{p}_i(t)$ so as to maximize

$$\int_{\bar{t}}^{+\infty} (\bar{p}_i(t) - w_s) Y_s \left(\frac{\bar{p}_i(t)}{p_s} \right)^{-\sigma} e^{-\int_t^s i_v dv - \lambda(s-t)} ds$$

3. Show that when adjusting its price at t, a monopoly sets it equal to Z_t, where

$$Z_t = \Omega_t / Q_t$$

$$\Omega_t = \frac{\sigma}{\sigma - 1} \int_t^{+\infty} w_s Y_s p_s^{\sigma} e^{-\int_t^s i_v dv - \lambda(s-t)} ds$$

$$Q_t = \int_t^{+\infty} Y_s p_s^{\sigma} e^{-\int_t^s i_v dv - \lambda(s-t)} ds$$

We assume that at each instant t, the flow of firms which adjust their price is randomly selected, and that the distribution of their initial price levels is the same as the aggregate one–that is, firms which are allowed to adjust at t are a representative sample of the distribution of price levels among firms.

4. Show that the aggregate price level p_t evolves according to

$$(1 - \sigma) p_t^{-\sigma} \frac{dp_t}{dt} = \lambda (Z_t^{1-\sigma} - p_t^{1-\sigma})$$

There exists a representative consumer who maximizes his intertemporal utility given by

$$\max \int_0^{+\infty} \left[\ln C_t + \ln \frac{M_t}{p_t} + \ln(1 - L_t) \right] e^{-\rho t} dt$$

, where C_t is consumption of the final good, M_t money holdings, L_t total labor supply.

5. Show that

$$L_t = q_t Y_t,$$

where

$$q_t = \int_0^1 \left(\frac{p_{it}}{p_t} \right)^{-\sigma} di.$$

Using (1) along with Jensen's inequality, show that $q_t > 1$ and that a mean preserving spread in the cross-sectional distribution of prices reduces measured labor productivity. Why?

6. Show that q_t evolves according to

$$\frac{dq_t}{dt} = \sigma \dot{p}_t \frac{q_t}{p_t} + \lambda (\left(\frac{Z_t}{p_t} \right)^{-\sigma} - q_t)$$

Let A_t be the consumer's net financial wealth at date t.

7. Show that the consumer's instantaneous budget constraint is

$$\dot{A}_t = i_t A_t + w_t L_t - p_t C_t - T_t - \dot{M}_t + \Pi_t,$$

where $T_t =$ (lump-sum) taxes paid by the consumer and $\Pi_t = \int_0^1 \pi_{it} di$ are the monopolies' profits rebated to the consumer. Taxes are used to finance public expenditures, which are equal to G_t units of the final good.

8. Show that the consumer's intertemporal budget constraint can be written as

$$\int_0^{+\infty} e^{-\int_0^t i_v dv} \left[w_t L_t + \Pi_t - p_t C_t - T_t - i_t M_t \right] dt + M_0 + A_0 = 0.$$

What is the economic interpretation of this budget constraint ? (Hint: if in trouble, integrate by parts!)

9. What are the consumers' first order conditions with respect to $C, M,$and L? Interpret them.

10. Show that the model's solution in terms of aggregate variables is given by the following set of equations:

$$\begin{aligned}
\dot{C}_t / C_t &= i_t - \frac{\dot{p}}{p_t} - \rho \\
\frac{1}{M_t} &= \frac{i_t}{p_t C_t} \\
\frac{1}{1 - L_t} &= \frac{w_t}{p_t C_t} \\
L_t &= q_t Y_t \\
Y_t &= C_t + G_t \\
(1-\sigma) p_t^{-\sigma} \frac{dp_t}{dt} &= \lambda (Z_t^{1-\sigma} - p_t^{1-\sigma}) \\
\frac{dq_t}{dt} &= \sigma \dot{p}_t \frac{q_t}{p_t} + \lambda (\left(\frac{Z_t}{p_t} \right)^{-\sigma} - q_t) \\
Z_t &= \Omega_t / Q_t \\
\dot{\Omega}_t &= -\frac{\sigma}{\sigma - 1} w_t Y_t p_t^{\sigma} + (i_t + \lambda) \Omega_t \\
\dot{Q}_t &= -Y_t P_t^{\sigma} + (i_t + \lambda) Q_t
\end{aligned}$$

What are the 10 endogenous variables? Which are the exogenous variables? Tell which endogenous variables are state-variables, forward-looking variables, or determined intratemporally.

We now look for a balanced growth path of the economy. We assume the money stock grows at rate μ and that $G_t = G$ is constant.

11. Show that along such a balanced growth path $L, Y, C, q,$ and i must be constant, while $p, Z,$ and w must grow at rate μ. At what rate must Ω and Q grow?

12. Show that one must have

$$i = \mu + \rho$$

Explain.

13. Show that in a balanced growth path:

$$q = \frac{\lambda}{\lambda - \sigma\mu}\left(\frac{\lambda - (\sigma - 1)\mu}{\lambda}\right)^{\frac{\sigma}{\sigma - 1}}$$

How does nominal price inertia and inflation affect cross-sectional price dispersion and labor productivity? What is the rate of money growth which maximizes labor productivity? What are the levels of price dispersion and labor productivity when prices are fully flexible (i.e. $\lambda \to \infty$)?

14. Show that in a balanced growth path, the real wage is given by

$$\frac{w}{p} = \left(\frac{\lambda - (\sigma - 1)\mu}{\lambda}\right)^{-\frac{1}{\sigma - 1}}\left(\frac{\sigma - 1}{\sigma}\right)\frac{\rho + \lambda - \sigma\mu}{\rho + \mu(1 - \sigma) + \lambda} = \omega(\mu)$$

How does money growth affect real wages? What is the rate of money growth which maximizes real wages? Can you spell out the positive and negative effects that money growth has on real wages? How does nominal price inertia affect real wages? What is the value of the real wage when prices are fully flexible?

(Hint: can you compute the markup and express real wages as the product of a markup effect and a productivity effect due to price dispersion ?)

15. Show that consumption, output, and employment are given by

$$\begin{aligned} C &= \frac{\omega(\mu)(1 - qG)}{1 + q\omega(\mu)} \\ Y &= \frac{G + \omega(\mu)}{1 + q\omega(\mu)} \\ L &= \frac{q(G + \omega(\mu))}{1 + q\omega(\mu)} \end{aligned}$$

How do public expenditures affect these variables? Explain.

16. Assume $G = 0.$, and $\mu = 0$ initially. When does an increase in money growth increase steady state output? When does it increase steady state welfare?

17. Is there a long-run trade-off between output and inflation in this model? Why? What is it shape? Could you suggest alternative assumptions about price formation such that there would be no such trade-off?