**Gilles Saint-Paul**
**Macro III**
**Problem #1**
We consider the following matching model.
The matching function is given by

$$m(u,v) = \frac{uv}{u+v}.$$

There is free entry in vacancy posting and the cost of posting a vacancy per unit of time is equal to $c$. Wages are set by intertemporal Nash bargaining so that

$$V_e = V_u + \varphi W,$$

and

$$J = (1-\varphi)W$$

The job destruction rate is $s$ and the real interest rate is $r$.

1. What is the function $q(\theta)$? Check that $m(u,v)$ has constant returns to scale and is concave in each of its arguments. What is the elasticity of $m$ with respect to $u$ as a function of $\theta$?
2. Show that the Beveridge curve has the following equation:

$$\theta = \frac{s(1-u)}{u - s(1-u)}$$

3. Show that the equilibrium value of $\theta$ must satisfy:

$$y/c = \frac{r+s}{1-\varphi}(1+\theta) + \frac{\varphi}{1-\varphi}\theta \tag{1}$$

4. What is the steady-state unemployment rate $u$? How does it depend on $r, s, c, y$, and $\varphi$? Why?
5. Show that the following relationship exists between the wage $w$ and $\theta$ :

$$w = \frac{\varphi c}{1-\varphi}[(r+s)(1+\theta)+\theta]$$

6. How do wages depend on $r, s, c, \varphi, \theta$ according to this formula? Explain?
7. What would be the observed correlation between wages and unemployment if the driving force for changes in unemployment was:
-changes in productivity ?
-changes in the discount rate?
-changes in the cost of posting vacancies?
-changes in workers' bargaining power?

8. Write down the central planner's problem, the Hamiltonian, and the corresponding first-order conditions.

9. Show that along the optimal path the marginal value of an extra job $\lambda$ is related to $\theta$ according to:

$$\lambda = c(1+\theta)^2.$$

Explain the economic intuition for why it increases with $c$ and $\theta$.

10. Show that in steady state the optimal value of $\theta$ must satisfy

$$y/c = (r+s)(1+\theta)^2 + c\theta^2 \tag{2}$$

11. What relationship must hold among the model's parameters for (2) to have the same solution as (1) ?

12. Check that this is equivalent to

$$\varphi = \frac{\theta}{1+\theta},$$

with $\theta$ being the common solution to these two equations.

13. How does this last condition relate to the Hosios efficiency condition?