Greg Johnson - Sample Lesson Plan

Updated April 30, 2005

This document contains my lesson plans and pages from the teacher's guide for the video-recorded lessons of Thursday, October 28, 2004. Students were eighth graders in the "CMP 8", Connected Mathematics Program. This lesson occurred at the midpoint of a 30-day geometry unit on the Pythagorean Theorem. Students had discovered the Pythagorean relationship for right triangles. They were transitioning from constructing squares on the sides of triangles to symbolic application of the formula.

In this class, students apply the Pythagorean relationship to find the shortest distance between any dots on a square dot grid.

The unit frequently references a fictional town named "Euclid" where streets follow a square grid. See page 2 below. Thus the lessons speak of "driving distance" versus "helicopter distance".

These plans contain enhancement material to be ready to insert if appropriate, which was not necessarily used.

Lesson Plans for Week of October 25, 2004 CMP 8
Greg Johnson

Text: Looking For Pythagoras, Investigation 3: The Pythagorean Theorem.

Content Objective for week: Students will discover (geometrically derive) the Pythagorean Theorem and apply it.

[Especially as a new teacher, I find I profit from a checklist of skills I must practice!]

My Classroom-Management Improvement Objectives for Week:

- Prepare: Greet students at door. Provide engaging content.
- Rehearse a procedure: Respond quietly to a call for attention, "Heads up." (Tell students: "If nearby, hand signal others. Calling another student to be quiet is like blurting out the answer to a math question. People need to learn for themselves.")
- Start and end tasks clearly, waiting for full class attention.
- Detect off-task signs at all times and from every student.
- Consistently respond to off-task behavior with appropriate escalation.

[Monday & Tuesday lessons omitted]

[Wednesday's schedule gives a bit of the context for the focal Thursday lesson.]

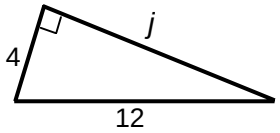
Wednesday 10/27 Investigation 3.2 "Puzzling Through a Proof"

Objective: Students investigate a puzzle that verifies that the sum or the areas of the squares on the legs of a right triangle equals the are of the square on the hypotenuse. They also encounter symbolic expression of this idea: $a^2 + b^2 = c^2$.
Classwork: Problem 3.2 & FU
Homework: **ACE p 34 #7, p 35 #8, #9.** (Book suggests also 10,11, 17, 20 & any skipped previously.)
Prepare: Labsheet 3.2 (3 versions. Make 24 copies of each version using 3 colors. Each team member gets a different color.) Make sure each group box has scissors. Oldest student should go first.

. . .

[Following are Wednesday's three Homework problems discussed at start of Thursday's class.]

p 34 #7: Find the missing length:



p 35:

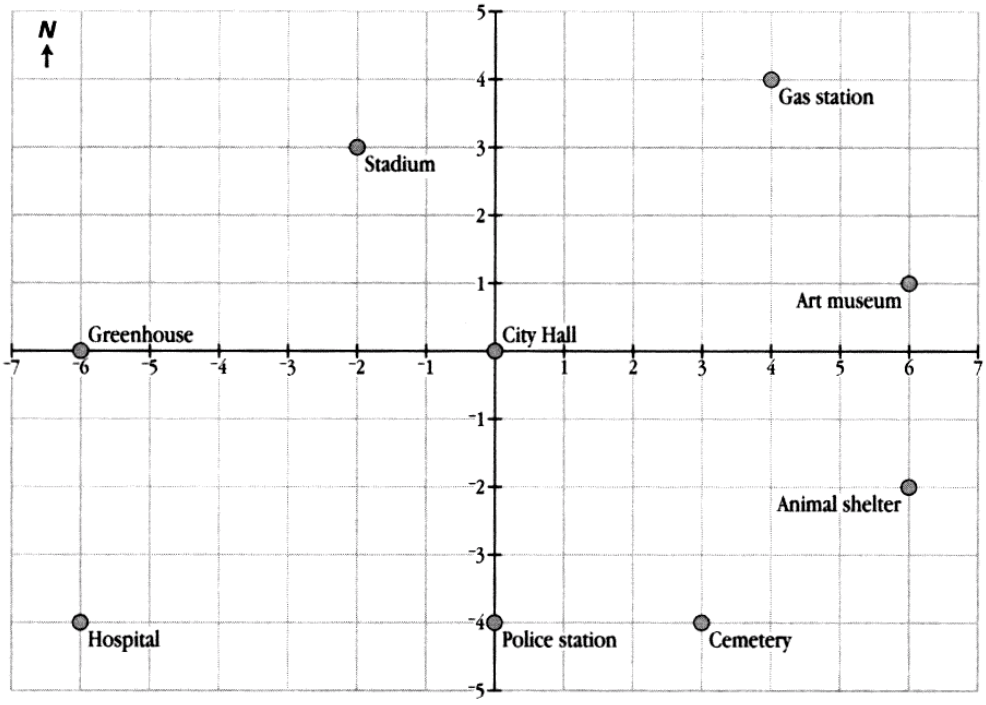
In 8–11, find the helicopter distance between the two locations in blocks without using a ruler, and explain how you found your answer.

8. the greenhouse and the stadium

9. the police station and the animal shelter

10. the greenhouse and the hospital

11. City Hall and the gas station



Thursday 10/28 Investigation 3.3 "Finding Distances" [This is the lesson on the video!]

Objective: Students discover how the Pythagorean Theorem can be used to find the distance between two dots on a grid.
Classwork: p 31, 3.3 & Follow-Up [One page lesson! Attached below.]
Homework: **ACE p 34 #1, p 36 #13, p 37 #17.**
(2,19,21,22,23,24,25,26 also suggested by text.) [Labsheet 3.3 attached below]
Prepare: Labsheet 3.3. Check supply of dot paper on shelf.
* SSR today, shorter morning classes. [Not applicable to classes in video]
* During guided practice, temporarily return quizzes from last Thursday 10/21. [Skipped]

[This info stays on board for kids to copy.]

10/28 CMP 8
P 31, Inv 3.3 Finding Distances
Classwork: Problem 3.3 & FU
Homework:
ACE p 34 #1, p 36 #13, p 37 #17.

[As usual I projected homework solutions but not methods. The 3 problems from Wednesday are on previous page. Students correct their answers, and we discuss. Subsequent assignment quiz requires students to give HW answers & show work.]

Open (4 minutes): Grade & discuss HW.
Launch (4 minutes):
Ask: "How can you find the shortest distance between two points?" A ruler may be suggested. "What if there is a mountain in between the points?" "The Pythagorean Theorem is a fast way to find the distance without drawing."
Model: "Let's pick two points on a dot grid, that are not on the same line. (overhead) How can we use the Pythagorean Theorem to find the length of this slanted line?"
Sketch solutions with target distance as hypotenuse. "What are the lengths of the legs?" "Can we find the length of the hypotenuse?"

Guided Practice (8 minutes): Students proceed with Labsheet 3.3. [Attached below]
Discussion (4 minutes): Call for attention. Ask for answers. "How did you get that?"

Exploration (4+ minutes): 3.3 Follow-Up is short, but less procedural: "On a sheet of dot grid paper, find two dots exactly $\sqrt{13}$ units apart. You can't use a ruler to measure."
[Centimeter dot grid paper is always available in this classroom.]
Some students may guess and check, connecting dots and finding distance. If no progress, after 3 minutes, hint: "If a square has area 13 square units, what's the length of a side?" If students come up with a correct triangle (legs that are 2 & 3 units long), have them find a line segment on the dot grid with length $\sqrt{40}$.
If they get past that... "Is it possible to find a line segment with length $\sqrt{7}$ units?" (No.)

Closure (about 4 minutes before end of hour!): How do you find the shortest distance between two points on a coordinate grid? Do you think this would work even if the points are at any locations, not just integer locations, say (0,0.5) & (2.7, 5)?
For fast students: Does the Pythagorean Theorem work for triangles that are not right triangles? Triangle on dot grid with base 5, & vertex 3 from left at height 1.

[Teacher's Guide for this lesson. This includes the single page from student text.]

3.3 Finding Distances

In Problem 2.3, you used squares to help you find the lengths of line segments connecting dots on a grid. The Pythagorean Theorem can also help you find these lengths.

Problem 3.3

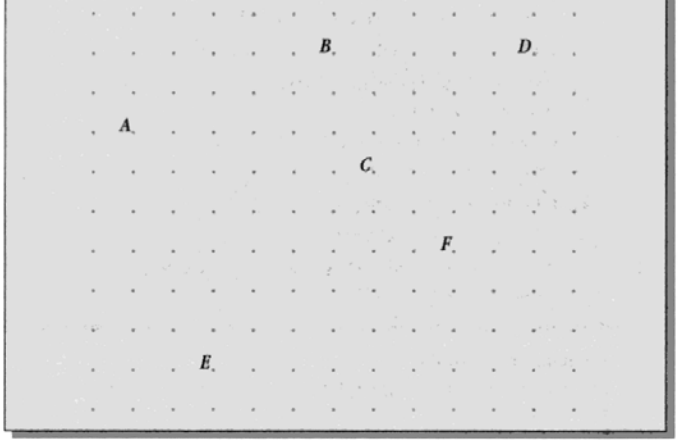
A. **1.** On the grid on Labsheet 3.3, draw a line segment connecting points *A* and *B*. Draw a right triangle with segment *AB* as its hypotenuse.

2. Find the lengths of the legs of the triangle.

3. Use the Pythagorean Theorem to find the length of the hypotenuse of the triangle.

B. Use the method described in part A to find the distance between points *C* and *D*.

C. Use the method described in part A to find the distance between points *E* and *F*.



Problem 3.3 Follow-Up

On a sheet of dot paper, find two points that are $\sqrt{13}$ units apart. Label the points *X* and *Y*. Explain how you know that the distance between the points is $\sqrt{13}$.

3.3 Finding Distances

At a Glance

Grouping: *pairs*

Launch

- Draw line segment *AB* as shown in the student edition, and ask how students could find its length.
- Do part A as a class.
- Have pairs work on the problem and follow-up.

Explore

- If students have trouble with the follow-up, ask questions to guide their thinking.

Summarize

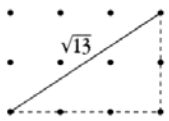
- Ask students to explain their solutions to the problem.
- Go over the follow-up question carefully.
- Help the class visualize how the Pythagorean relationship can be used to find the distance between two dots on a grid.

Answers to Problem 3.3

See page 40l.

Answer to Problem 3.3 Follow-Up

Since $(\sqrt{13})^2 = 2^2 + 3^2$, the hypotenuse of a right triangle with legs of lengths 2 and 3 will have a length of $\sqrt{13}$.



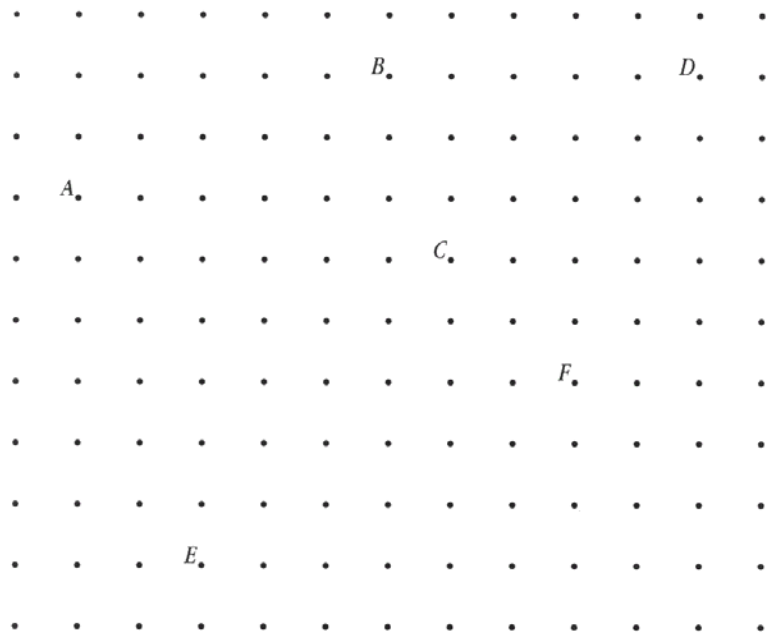
Assignment Choices

ACE questions 1, 2, 19, 21–26, and unassigned choices from earlier problems

[Handed out for class work.]

Labsheet 3.3

Points on a Grid



[Advice from Teacher's Guide for Investigation 3.3.]

3.3 • Finding Distances

In this problem, students discover how the Pythagorean Theorem can be used to find the distance between two dots on a grid.

Launch

Display Transparency 3.3 or a transparent grid, and indicate or label points *A* and *B* as shown in the student edition. Ask:

> How can you find the distance between these two points?

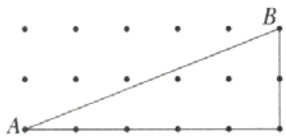
The class may suggest measuring the distance with a ruler. Explain that the Pythagorean Theorem can be used to find a more exact length. Use points *A* and *B* to illustrate the method of finding the distance. Draw line segment *AB* and ask:

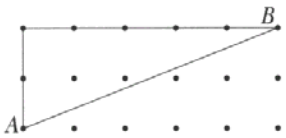
> How can we use the Pythagorean Theorem to find the length of this line segment?

Some students will probably suggest using the segment as the side of a square; others may suggest using it as the hypotenuse of a right triangle.

> What right triangle has this hypotenuse?

Sketch students' suggestions, which may be either of the triangles shown here:





What are the lengths of the legs? *(5 and 2)* How can you use this information to find the length of the hypotenuse? *(The square of the length of the hypotenuse = $5^2 + 2^2 = 29$, so the length is $\sqrt{29}$.)* So, what is the distance between points *A* and *B*? ($\sqrt{29}$)

Distribute Labsheet 3.3 to each student, and have the class work in pairs on the rest of the problem and the follow-up.

Explore

Students should find the problem a review of what they have learned so far. However, the follow-up is a bit difficult, so you may need to help guide their thinking.

Can the $\sqrt{13}$-unit line segment be a vertical or a horizontal segment? *(no)* Why not?

If it is a tilted line segment, can it be the hypotenuse of a right triangle? *(yes)*

Assume that this segment is the hypotenuse of a right triangle. What will the area of the square on the hypotenuse be? *[$(\sqrt{13})^2$, or 13, square units]*

What is the sum of the areas of the squares on the legs of this right triangle? *(13)*

What are two squares whose sum is 13? *(4 and 9)* So, what are the lengths of the legs? *($\sqrt{4}$ and $\sqrt{9}$, or 2 and 3)*

Now, draw a right triangle with legs of lengths 2 and 3. The hypotenuse has length $\sqrt{13}$.

Summarize

Ask students to demonstrate and explain how they found the answers to the problem.

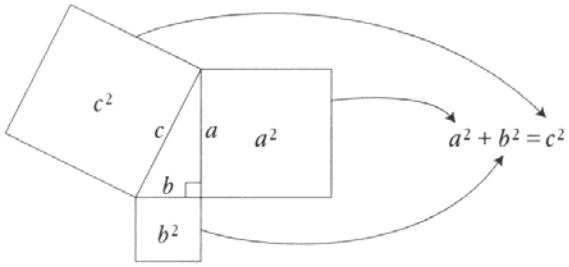
Then, go over the follow-up carefully. After someone has explained how he or she found two points that were $\sqrt{13}$ units apart, offer a similar problem.

How would you find a line segment with a length of $\sqrt{40}$?

Ask one or two students to describe their method. They will likely have used a guess-and-check procedure to find the two squares with a sum of 40, which are 36 and 4. From this they can determine that the lengths 6 and 2 will give a right triangle with a hypotenuse of length $\sqrt{40}$.

Students should be able to focus on the areas of the three squares on the sides of a right triangle and their relationship to the lengths of the sides. Typically, two lengths or two areas are known, and we must find the third length or area. Once we know the missing area, we can take its square root to find the length. Conversely, once we know the missing length, we can square it to find the area.

The following visual explanation will help some students to understand the essence of the Pythagorean Theorem:



The essential strategy for finding a tilted line with a certain length depends on finding two squares whose sum is equal to the square of that length. If students have done ACE question 20, the table of squares they created can help them to find the sum of the areas of upright squares on the legs of a right triangle to create a hypotenuse with the desired length. As a final check, ask this question:

> Can 7 be the length of a tilted line segment drawn between two dots on a dot grid? *(No, because 49 does not equal the sum of two squares of whole numbers.)*

3.4 • Measuring the Egyptian Way

In this problem, students investigate the converse of the Pythagorean Theorem: If a, b, and c are the lengths of the sides of a triangle and $a^2 + b^2 = c^2$, then the triangle is a right triangle.

Launch

Talk about the two questions in the introduction to Problem 3.4:

- Is any triangle whose side lengths satisfy the relationship $a^2 + b^2 = c^2$ a right triangle?
- If the side lengths of a triangle do not satisfy the relationship $a^2 + b^2 = c^2$, does this mean that the triangle is not a right triangle?

Distribute rulers and straws or string, and have the class work in pairs on the problem and follow-up.

[Lappan, Fey, Fitzgerald, Friel, Phillips. *Looking For Pythagoras, Teachers' Guide*. 1998. White Plains, NY: Dale Seymour Publications. (Connected Mathematics Series.)]