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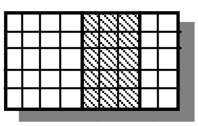
Teaching the Distributive Property

This excerpts reflections I journaled during my student teaching in 2004.

… Essentially all my math in junior high was definition and symbol manipulation. We moved letters and symbols around according to rules. Later we saw these symbols had something to do with graphs.

…In my middle school internship, I observed teachers presenting the distributive property via stacks of blocks, pictures of stacks of blocks, and stories about stacks of blocks. The students loved stacks of blocks. The students followed procedures to find



areas in a rectangle made of blocks. The students also agreed:

$$(5\times4) + (5\times3) + (5\times2) = 5 \times (4+3+2)$$

Two Troubles

However, students did not connect the manipulative to the symbolic. They did not regard either representation as supporting the other. There were two games: one you played with blocks and one you played with + and ×.

How do we know the distributive property is true? We mimic the teacher in drawing arrows from the multiplier to the terms to be distributed:

$$5\ (4 + 3) = 5\ (4) + 5\ (3)$$

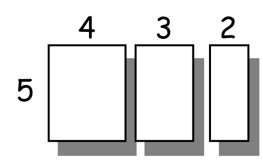
I was happy that arrows allowed the middle school students to pass their tests. Unfortunately, magic arrows started showing up everywhere:

$$5 + (4 + 3) = 5 + 4 + 5 + 3$$

At a different school I found whole rooms of college-bound high school seniors drawing those same magic arrows! Sometimes the arrows were correct, sometimes not. A student told me, "I draw arrows to show my work." For another student distributing was drawing "an arrow to each term".

A Successful Experiment

It seemed to me that regarding the manipulatives, there was a perceptual problem in partitioning the 5x9 array of blocks. Partitioning seemed arbitrary and extra work. Later, when I taught the distributive property to middle school students, I tried reversing the sequence of presentation, to focus first on "putting together" rather than "taking apart": Instead of students manipulating blocks, I sketched:



I said, "… How do we find the total area of these three rectangles? Multiply 5×4=what? 20. And 5×3=15, and 5×2=10. Then Jim adds: 20+15+10 =what? 45. Or Brittany can just add 4+3+2=9 and multiply once 5×9=45. Do we get the same result? Which is easier?"

The students paused for a beat and exclaimed, "Ooooh. Less work! This is good!"

We discussed that this worked in reverse. Instead of teaching the distributive property, in effect I taught them "the aggregative property" or what teachers called "reverse distribution".

Questions

Is there anything wrong with presenting the distributive property in the “backwards” way that was productive for me? Is the traditional presentation only that, a tradition? For pre-calculus, I’ve seen commentary that introducing trig with the unit circle was an artifact of the period 1920-1980. Trig was originally introduced with simple right triangles.

What mental catalyst leads a kid to generalize from manipulatives? If kids do not learn from manipulatives, ought I just stick to sketches and symbols? Kids would like to have manipulatives every day. But what they remember is not always what the teacher wants remembered.

How can I help students go beyond scaffolding such as drawing arrows? For example, I might simply show the symbolic property and examples, and use arrows later only if students have trouble with the plain symbols.

I wonder if math lessons that emphasize making life easier would help motivate students.

Can we predict particular topics most likely to merit re-teaching--or careful effort to get them right the first time?