

AIEEE-2003 Mathematics

1. A function f from the set of natural numbers to integers defined by

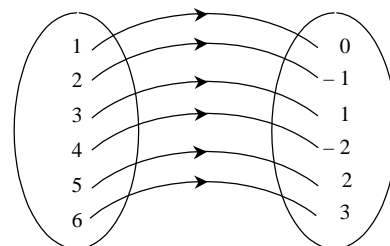
$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases}$$

- (1) One – one but not onto
(2) Onto but not one – one
(3) One – one and onto both
(4) Neither one – one nor onto

Sol. (3) $f : N \rightarrow I$

$$f(1) = 0, f(2) = -1, f(3) = -1, f(4) = -2, f(5) = 2 \\ f(6) = -3 \text{ so on.}$$

and



In this type of function every element of set A has unique image in set B and there is no element left in set B .

Hence f is one-one and onto function.

2. Let z_1 and z_2 be two roots of the equation. $z^2 + az + b = 0$, z being complex.

Further, assume that the origin, z_1 and z_2 form an equilateral triangle. Then

- (1) $a^2 = b$ (2) $a^2 = 2b$ (3) $a^2 = 3b$ (4) $a^2 = 4b$

Sol. (3) $z^2 + az + b = 0$

$$z_1 + z_2 = -a \text{ \& } z_1 z_2 = b$$

0, z_1 , z_2 form an equilateral Δ

$$\therefore 0^2 + z_1^2 + z_2^2 = 0 \cdot z_1 + z_1 \cdot z_2 + z_2 \cdot 0 \text{ (for eq. } \Delta; z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1 \text{)}$$

$$z_1^2 + z_2^2 = z_1 z_2$$

$$(z_1 + z_2)^2 = 3z_1 z_2$$

$$\therefore a^2 = 3b.$$

3. If z and ω are two non – zero complex numbers such that $|z\omega| = 1$, and $\text{Arg}(\omega) = \frac{\pi}{2}$, then $\bar{z}\omega$ is equal to

- (1) 1 (2) -1 (3) i (4) -i

Sol. (4) $|z||\omega| = 1$ (1)

$$\text{As } \text{Arg}\left(\frac{z}{\omega}\right) = \frac{\pi}{2} \therefore \frac{z}{\omega} = i$$

$$\therefore \left|\frac{z}{\omega}\right| = 1 \text{ (2) from (1) and (2) } |z|=|\omega|=1 \text{ and } \frac{z}{\omega} + \frac{\bar{z}}{\bar{\omega}} = 0; z\bar{\omega} + \bar{z}\omega = 0$$

$$\bar{z}\omega = -z\bar{\omega} = \frac{-z}{\omega} \cdot \bar{\omega} \cdot \omega$$

$$\bar{z}\omega = -i|\omega|^2 = -i.$$

4. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then

(1) $x = 4n$, where n is any positive integer

(2) $x = 2n$, where n is any positive integer

(3) $x = 4n + 1$, where n is any positive integer

(4) $x = 2n + 1$, where n is any positive integer

Sol. (1) $\left(\frac{1+i}{1-i}\right)^x = 1 \Rightarrow \left[\frac{(1+i)^2}{1-i^2}\right]^x = 1$

$$\left(\frac{1+i^2+2i}{1+1}\right)^x = 1 \Rightarrow (i)^x = 1$$

$$\therefore x = 4n; \quad n \in I^+.$$

5. If $\begin{vmatrix} a & a^2 & 1+a^2 \\ b & b^2 & 1+b^2 \\ c & c^2 & 1+c^2 \end{vmatrix} = 0$ and vectors $(1, a, a^2)$, $(1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar, then the product $a b c$ equals.

(1) 2

(2) -1

(3) 1

(4) 0

Sol. (2) $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$(a-b)(b-c)(c-a) + abc(a-b)(b-c)(c-a) = 0$$

$$(abc+1)[(a-b)(b-c)(c-a)] = 0$$

$$\text{As } \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \text{ (given condition)}$$

$$\therefore abc = -1.$$

6. If the system of linear equations

$$x + 2ay + az = 0$$

$$x + 3by + bz = 0$$

$$x + 4cy + cz = 0$$

has a non-zero solution, then a, b, c

(1) Are in A.P.

(2) Are in G.P.

(3) Are in H.P.

(4) Satisfy $a + 2b + 3c = 0$

Sol. (3) $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0 \quad c_2 \rightarrow c_2 - 2c_3$

$$\Rightarrow \begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix} \quad R_3 \rightarrow R_3 - R_2, R_2 \rightarrow R_2 - R_1$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & a \\ 0 & b & b-a \\ 0 & 2c-b & c-b \end{vmatrix} = 0$$

$$b(c-b) - (b-a)(2c-b) = 0$$

On simplification

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

$\therefore a, b, c$ are in Harmonic Progression.

7. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in

- (1) Arithmetic Progression
(3) Harmonic Progression

- (2) Geometric Progression
(4) Arithmetic – Geometric Progression

Sol. (3) $ax^2 + bx + c = 0$

$$\alpha + \beta = \frac{-b}{a}; \quad \alpha\beta = \frac{c}{a}$$

As for given condition

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2}$$

$$-\frac{b}{a} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}}$$

On simplification $2a^2c = ab^2 + bc^2$

$$\Rightarrow \frac{2a}{b} = \frac{c}{a} + \frac{b}{c}$$

$$\therefore \frac{a}{c}, \frac{b}{a} \& \frac{c}{b} \text{ are in H.P.}$$

8. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is

(1) 2

(2) 4

(3) 1

(4) 3

Sol. (2) $x^2 - 3|x| + 2 = 0$

$$(|x| - 2)(|x| - 1) = 0$$

$$|x| = 1; 2$$

$$x = \pm 1; \pm 2$$

\therefore No. of Solution = 4.

9. The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other, is

(1) $\frac{2}{3}$

(2) $-\frac{2}{3}$

(3) $\frac{1}{3}$

(4) $-\frac{1}{3}$

Sol. (1) $3\alpha = \frac{1-3a}{a^2-5a+3} \& 2\alpha^2 = \frac{2}{a^2-5a+3}$

$$2 \left[\frac{1}{9} \frac{(1-3a)^2}{(a^2-5a+3)^2} \right] = \frac{2}{a^2-5a+3}$$

$$\frac{(1-3a)^2}{(a^2-5a+3)} = 9$$

$$9a^2 - 6a + 1 = 9a^2 - 45a + 27$$

$$39a = 26$$

$$a = \frac{2}{3}$$

10. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(1) $\alpha = a^2 + b^2, \beta = ab$

(2) $\alpha = a^2 + b^2, \beta = 2ab$

(3) $\alpha = a^2 + b^2, \beta = a^2 - b^2$

(4) $\alpha = 2ab, \beta = a^2 + b^2$

Sol. (2) $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$$\alpha = a^2 + b^2; \beta = 2ab$$

11. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five question. The number of choices available to him is

(1) 140

(2) 196

(3) 280

(4) 346

Sol. (2) As for given question two cases are possible.

(i) Selecting 4 out of first five question & 6 out of remaining 8 Question = ${}^5C_4 \times {}^8C_6 = 140$ choices.

(ii) Selecting 5 out of first five Question & 5 out of remaining 8 Qs. = ${}^5C_5 \times {}^8C_5 = 56$ choices.

\therefore Total No. of choices = $140 + 56 = 196$.

12. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by

(1) $6! \times 5!$

(2) 30

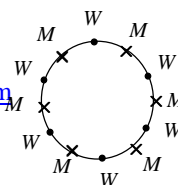
(3) $5! \times 4!$

(4) $7! \times 5!$

Sol. (1) No. of ways in which 6 men can be arranged at a round table = $(6-1)!$

Now women can be arranged in $6!$ Ways.

<http://www.123iitjee.com> OR <http://www.indianetgroup.com>



Total Number of ways = $6! \times 5!$

13. If $1, \omega, \omega^2$ are the cube roots of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to

- (1) 0 (2) 1 (3) ω (4) ω^2

Sol. (1) Applying $R_1 \rightarrow R_1 + R_2 + R_3$

As, $1 + \omega^n + \omega^{2n} = 0$ (if you don't remember this, put $n=1$ and you will get the answer)

$\therefore \Delta = 0$.

14. If nC_r denotes the number of combinations of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals

- (1) ${}^{n+2}C_r$ (2) ${}^{n+2}C_{r+1}$ (3) ${}^{n+1}C_r$ (4) ${}^{n+1}C_{r+1}$

Sol. (2) Using ${}^nC_r + {}^nC_r - 1 = {}^{n+1}C_r$

$${}^nC_{r+1} + \underbrace{{}^nC_{r-1} + {}^nC_r + {}^nC_r}_{{}^{n+1}C_r}$$

$${}^nC_{r+1} + {}^{n+1}C_r + {}^nC_r$$

$${}^{n+1}C_{r+1} + {}^{n+1}C_r$$

$$\Rightarrow {}^{n+2}C_{r+1}.$$

15. The number of integral terms in the expansion of $(\sqrt{3} + \sqrt[8]{5})^{256}$ is

- (1) 32 (2) 33 (3) 34 (4) 35

Sol. (2) $(\sqrt{3} + \sqrt[8]{5})^{256}$

$$T_{r+1} = {}^{256}C_r (\sqrt{3})^{256-r} (\sqrt[8]{5})^r$$

$$T_{r+1} = {}^{256}C_r (3)^{\frac{256-r}{2}} (5)^{r/8}$$

Terms would be integral if $\frac{256-r}{2}$ & $\frac{r}{8}$ both are +ve Integers.

As $0 \leq r \leq 256$

$\therefore r = 0, 8, 16, 24, \dots, 256$

For above values of r , $\left(\frac{256-r}{2}\right)$ is also an integer.

\therefore Total no. of Values of $r = 33$.

16. If x is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is

- (1) 7th term (2) 5th term (3) 8th term (4) 6th term

Sol. (3) $(1+x)^{\frac{27}{5}}$

$$T_{r+1} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!} (x)^r$$

For first negative term $n-r+1 < 0$

$$r > \frac{32}{5} \quad \therefore r = 7$$

\therefore First negative term is T_8 .

17. The sum of the series $\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4} - \dots$ upto ∞ is equal to

- (1) $2 \log_e 2$ (2) $\log_e 2 - 1$ (3) $\log_e 2$ (4) $\log_e \left(\frac{4}{e} \right)$

Sol. (4) $\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4} - \dots \infty$

$$\text{Let } T_n = \frac{1}{n(n+1)} = \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

$$S = T_1 - T_2 + T_3 - T_4 + T_5 - \dots \infty$$

$$\left(\frac{1}{1} - \frac{1}{2} \right) - \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) - \left(\frac{1}{4} - \frac{1}{5} \right) + \dots$$

$$\Rightarrow 1 - 2 \left[\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \dots \infty \right]$$

$$1 - 2[-\log(1+1) + 1]$$

$$2 \log 2 - 1 = \log \left(\frac{4}{e} \right).$$

18. Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in A.P. then $f'(1), f'(2)$ and $f'(3)$ are in

- (1) A.P. (2) G.P.
(3) G.P. (4) Arithmetic – geometric Progression

Sol. (1) $f(x) = ax^2 + bx + c$

$$f(1) = f(-1) \Rightarrow a + b + c = a - b + c$$

$$b = 0$$

$$\therefore f(x) = ax^2 + c$$

$$f'(x) = 2ax$$

$$\text{Now } f'(a); f'(b) \text{ \& } f'(c) \text{ are } 2a(a); 2a(b); 2a(c)$$

If a, b, c are in A.P. then $f'(a), f'(b) \text{ \& } f'(c)$ are also in A.P.

19. If x_1, x_2, x_3 , and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points (x_1, y_1) , (x_2, y_2) and (x_3, y_3)

- (1) Lie on a straight line (2) Lie on an ellipse (3) Lie on a circle (4) Are vertices of a triangle

Sol. (1) Taking co-ordinates as $\left(\frac{x}{r}, \frac{y}{r}\right)$; (x, y) & (xr, yr)

Above co-ordinates satisfy the relation $y = mx$

\therefore Lies on the straight line.

20. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a , is

- (1) $a \cot\left(\frac{\pi}{n}\right)$ (2) $\frac{a}{2} \cot\left(\frac{\pi}{2n}\right)$ (3) $a \cot\left(\frac{\pi}{2n}\right)$ (4) $\frac{a}{4} \cot\left(\frac{\pi}{2n}\right)$

Sol. (2) $\tan\left(\frac{\pi}{n}\right) = \frac{a}{2r}$

$$\sin\left(\frac{\pi}{n}\right) = \frac{a}{2R}$$

$$r + R = \frac{a}{2} \left[\cot\frac{\pi}{n} + \operatorname{cosec}\frac{\pi}{n} \right]$$

$$r + R = \frac{a}{2} \cdot \cot\left(\frac{\pi}{2n}\right).$$

21. If in a triangle ABC $a \cos^2\left(\frac{C}{2}\right) + c \cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides a, b , and c

- (1) Are in A.P. (2) Are in G.P. (3) Are in H.P. (4) Satisfy $a + b = c$

Sol. (1) If $a \cos^2\left(\frac{c}{2}\right) + c \cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$

$$a[\cos c + 1] + c[\cos A + 1] = 3b$$

$$(a + c) + (a \cos c + c \cos b) = 3b$$

$$a + c + b = 3b$$

$$a + c = 2b$$

a, b, c are in A.P.

22. In a triangle ABC , medians AD and BE are drawn. If $AD = 4$, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of the ΔABC is

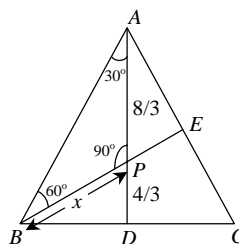
- (1) $\frac{8}{3}$ (2) $\frac{16}{3}$ (3) $\frac{32}{3}$ (4) $\frac{64}{3}$

Sol. (1) No given option is correct

$$\tan 60^\circ = \frac{8/3}{x}$$

$$x = \frac{8}{3\sqrt{3}}$$

$$\text{Area of } \Delta ABD = \frac{1}{2} \times 4 \times \frac{8}{3\sqrt{3}} = \frac{16}{3\sqrt{3}}$$



$$\therefore \text{Area of } \triangle ABC = 2 \times \frac{16}{3\sqrt{3}} = \frac{32}{3\sqrt{3}}$$

23. The trigonometric equation $\sin^{-1} x = 2 \sin^{-1} a$, has a solution for

- (1) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$ (2) all real values of a (3) $|a| < \frac{1}{2}$ (4) $|a| \geq \frac{1}{\sqrt{2}}$

Sol. (1) $\sin^{-1} x = 2 \sin^{-1} a$

$$\therefore \frac{-\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$$

$$\therefore -\frac{\pi}{2} \leq 2 \sin^{-1} a \leq \frac{\pi}{2}$$

$$-\frac{\pi}{4} \leq \sin^{-1} a \leq \frac{\pi}{4}$$

$$\therefore \frac{-1}{\sqrt{2}} \leq a \leq \frac{1}{\sqrt{2}}$$

$$\therefore |a| \leq \frac{1}{\sqrt{2}} \quad (\text{As } \frac{1}{\sqrt{2}} > \frac{1}{2})$$

Out of given four options no one is absolutely correct but 3. Could be taken into consideration. $\rightarrow |a| \leq \frac{1}{\sqrt{2}}$ is correct, if

$|a| < \frac{1}{2}$ is taken as correct then it domain satisfy for $a = \frac{1}{\sqrt{3}}$ but equation is satisfied. $\frac{1}{\sqrt{2}} > \frac{1}{\sqrt{3}} > \frac{1}{2}$.

24. The upper $\frac{3}{4}th$ portion of a vertical pole subtends an angle $\tan^{-1} \frac{3}{5}$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is

- (1) 20m (2) 40m (3) 60m (4) 80m

Sol. (2) $\theta = \alpha + \beta$

$$\beta = \theta - \alpha$$

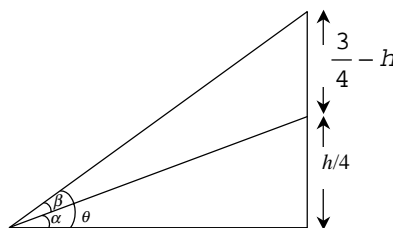
$$\tan \beta = \frac{\tan \theta - \tan \alpha}{1 + \tan \theta \tan \alpha}$$

$$\frac{3}{5} = \frac{\frac{h}{40} - \frac{h}{160}}{1 + \frac{h}{40} \cdot \frac{h}{160}}$$

$$h^2 - 200h + 6400 = 0$$

$$h = 40 \text{ or } 160 \text{ meter}$$

$$\therefore \text{possible height.} = 40 \text{ metre}$$



25. The real number x when added to its inverse gives the minimum value of the sum at x equal to

- (1) 2 (2) 1 (3) -1 (4) -2

Sol. (2) $y = x + \frac{1}{x}$

$$\frac{dy}{dx} = 1 - \frac{1}{x^2}$$

For max. or min.

$$1 - \frac{1}{x^2} = 0 \Rightarrow x = \pm 1$$

$$\frac{d^2y}{dx^2} = \frac{2}{x^3} \Rightarrow \left(\frac{d^2y}{dx^2} \right)_{x=2} = 2 \text{ (+ve minima)}$$

$$\therefore x = 1$$

26. If $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(x+y) = f(x) + f(y)$, for all $x, y \in \mathbb{R}$ and $f(1) = 7$, then $\sum_{r=1}^n f(r)$ is

- (1) $\frac{7n}{2}$ (2) $\frac{7(n+1)}{2}$ (3) $7n(n+1)$ (4) $\frac{7n(n+1)}{2}$

Sol. (4) $f(x+y) = f(x) + f(y)$

$$\text{Let } f(x) = mx$$

$$f(1) = 7; \therefore m = 7$$

$$f(x) = 7x$$

$$\sum_{r=1}^n f(r) = 7 \sum_{r=1}^n r$$

$$= \frac{7n(n+1)}{2}$$

27. If $f(x) = x^n$, then the value of $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \dots + \frac{(-1)^n f^{(n)}(1)}{n!}$ is

- (1) 2^n (2) 2^{n-1} (3) 0 (4) 1

Sol. (3) $f(x) = x^n \Rightarrow f(1) = 1$

$$f'(x) = nx^{n-1} \Rightarrow f'(1) = n$$

$$f''(x) = n(n-1)x^{n-2} \Rightarrow f''(1) = n(n-1)$$

.....

.....

$$f^{(n)}(x) = n! \Rightarrow f^{(n)}(1) = n!$$

$$= 1 - \frac{n}{1!} + \frac{n(n-1)}{2!} - \frac{n(n-1)(n-2)}{3!} + \dots + (-1)^n \frac{n!}{n!}$$

$$= {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots + (-1)^n {}^nC_n = 0$$

Alternatively, put $n=1$ and $n=2$ and in both the cases you will get the answer as 0.

28. Domain of definition of the function $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is

- (1) $(1,2)$ (2) $(-1,0) \cup (1,2)$ (3) $(1,2) \cup (2, \infty)$ (4) $(-1,0) \cup (1,2) \cup (2, \infty)$

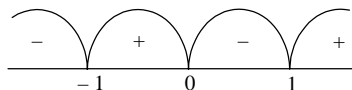
Sol. (4) $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$

$$4 - x^2 \neq 0 \quad x^3 - x > 0$$

$$x \neq \pm\sqrt{4}$$

$$\therefore D = (-1,0) \cup (1,\infty) - \{\sqrt{4}\}$$

$$D = (-1, 0) \cup (1, 2) \cup (2, \infty).$$



29. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\left[1 - \tan\left(\frac{x}{2}\right)\right] [1 - \sin x]}{\left[1 + \tan\left(\frac{x}{2}\right)\right] [\pi - 2x]^3}$ is

- (1) $\frac{1}{8}$ (2) 0 (3) $\frac{1}{32}$ (4) ∞

Sol. (3) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right) \cdot (1 - \sin x)}{(\pi - 2x)^3}$

Let $x = \frac{\pi}{2} + y$; $y \rightarrow 0$

$$\lim_{y \rightarrow 0} \frac{-\tan\left(\frac{-y}{2}\right) \cdot (1 - \cos y)}{(-2y)^3}$$

$$\lim_{y \rightarrow 0} \frac{-\tan \frac{y}{2} \cdot 2 \sin^2 \frac{y}{2}}{(-8) \cdot \frac{y^3}{8} \cdot 8}$$

$$\lim_{y \rightarrow 0} \frac{1}{32} \frac{\tan \frac{y}{2}}{\left(\frac{y}{2}\right)} \cdot \left[\frac{\sin y / 2}{y / 2}\right]^2$$

$$\Rightarrow \frac{1}{32}.$$

30. If $\lim_{x \rightarrow 0} \frac{\log(3+x) - \log(3-x)}{x} = k$, the value of k is

- (1) 0 (2) $-\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $-\frac{2}{3}$

Sol. (3) $\lim_{x \rightarrow 0} \frac{\log(3+x) - \log(3-x)}{x} = K$ (By L Hospital rule)

$$\lim_{x \rightarrow 0} \frac{\frac{1}{3+x} - \frac{-1}{3-x}}{1} = K$$

$$\therefore \frac{2}{3} = K.$$

31. Let $f(1) = g(1) = k$ and their n^{th} derivatives $f^n(1)$, $g^n(1)$ exist and are not equal for some n . Further if

$$\lim_{x \rightarrow a} \frac{f(a)g(x) - f(x)g(a)}{g(x) - f(x)} = 4$$

(1) 4

(2) 2

(3) 1

(4) 0

Sol. (1) $\lim_{x \rightarrow a} \frac{k(9(x) - kf(x))}{9(k) - f(x)} = 4$

(By L Hospital' rule)

$$\lim_{x \rightarrow a} k \left[\frac{9'(x) - f'(x)}{9'(x) - f'(x)} \right] = 4$$

$$\therefore k = 4.$$

32. The function $f(x) = \log(x + \sqrt{x^2 + 1})$, is

(1) An even function

(2) An odd function

(3) A periodic function

(4) Neither an even nor odd function

Sol. (2) $f(x) = \log(x + \sqrt{x^2 + 1})$

$$f(-x) = -\log(x + \sqrt{x^2 + 1})$$

$$f(-x) = -f(x)$$

$f(x)$ is odd function.

33. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$ then $f(x)$ is

(1) Continuous as well as differentiable for all x

(2) Continuous for all x but not differentiable at $x = 0$

(3) Neither differentiable nor continuous at $x = 0$

(4) Discontinuous every where

Sol. (2) $f(0) = 0$

$$f(x) = xe^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}$$

$$\text{R.H.L.} \quad \lim_{h \rightarrow 0} (0+h)e^{-2/h} = \lim_{h \rightarrow 0} \frac{h}{e^{2/h}} = 0$$

$$\text{L.H.L.} \quad \lim_{h \rightarrow 0} (0-h)e^{-\left(\frac{1}{h} - \frac{1}{h}\right)} = 0$$

$\therefore f(x)$ is continuous.

$$\text{R.H.D.} \quad \lim_{h \rightarrow 0} \frac{(0+h)e^{-\left(\frac{1}{h}+\frac{1}{h}\right)} - h e^{-\left(\frac{1}{h}+\frac{1}{h}\right)}}{h} = 0$$

$$\text{L.H.D.} \quad \lim_{h \rightarrow 0} \frac{(0-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)} - h e^{-\left(\frac{1}{h}+\frac{1}{h}\right)}}{-h} = 1$$

\therefore L.H.D. \neq R.H.D.

$f(x)$ is not differentiable at $x = 0$.

34. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals

- (1) 3 (2) 1 (3) 2 (4) $\frac{1}{2}$

Sol. (3) $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$$f'(x) = 6x^2 - 18ax + 12a^2$$

$$f''(x) = 12x - 18a$$

for max. or min.

$$6x^2 - 18ax + 12a^2 = 0 \Rightarrow x^2 - 3ax + 2a^2 = 0$$

$x = a$ or $x = 2a$, at $x = a$ max. and at $x = 2a$ min

$$\therefore p^2 = q$$

$$a^2 = 2a \Rightarrow a = 2 \text{ or } a = 0$$

but $a > 0$, therefore $a = 2$.

35. If $f(y) = e^y$, $g(y) = y$; $y = 0$ and $f(t) = \int_0^t f(t-y) g(y) dy$, then

- (1) $F(t) = 1 - e^{-t}(1+t)$ (2) $F(t) = e^t - (1+t)$ (3) $f(t) = t e^t$ (4) $F(t) = t e^{-t}$

Sol. (2) $F(t) = \int_0^t f(t-y) g(y) dy$

$$= \int_0^t e^{t-y} y dy = e^t \int_0^t e^{-y} \cdot y dy$$

$$= e^t \left[-ye^{-y} - e^{-y} \right]_0^t = -e^t \left[ye^{-y} + e^{-y} \right]_0^t$$

$$= -e^t \left[te^{-t} + e^{-t} - 0 - 1 \right]$$

$$= e^t \left[\frac{t+1-e^t}{e^t} \right] = e^t - (1+t)$$

36. If $f(a+b-x) = f(x)$, then $\int_a^b x f(x) dx$ is equal to

- (1) $\frac{a+b}{2} \int_a^b f(b-x) dx$ (2) $\frac{a+b}{2} \int_a^b f(x) dx$ (3) $\frac{b+a}{2} \int_a^b f(x) dx$ (4)

Sol. (2 & 4)

$$I = \int_a^b x f(x) dx$$

$$I = \int_a^b (a+b-x)f(a+b-x) dx$$

$$= (a+b) \int_a^b f(a+b-x) dx - \int_a^b x f(a+b-x) dx = (a+b) \int_a^b f(a+b-x) dx - \int_a^b x f(x) dx$$

$$2I = (a+b) \int_a^b f(x) dx$$

$$I = \frac{(a+b)}{2} \int_a^b f(x) dx$$

$$I = \frac{a+b}{2} \int_a^b f(a+b-x) dx$$

37. The value of $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sec^2 t dt}{x \sin x}$ is

(1) 3

(2) 2

(3) 1

(4) 0

Sol. (3) $\lim_{x \rightarrow 0} \frac{\frac{d}{dx} \int_0^{x^2} \sec^2 t dt}{\frac{d}{dx} (x \sin x)} = \lim_{x \rightarrow 0} \frac{\sec^2 x^2 \cdot 2x}{\sin x + x \cos x}$ (by L hospital's rule)

$$\lim_{x \rightarrow 0} \frac{2 \sec^2 x^2}{\left(\frac{\sin x}{x} + \cos x \right)} = \frac{2 \times 1}{1+1} = 1$$

38. The value of the integral $I = \int_0^1 x(1-x)^n dx$ is

(1) $\frac{1}{n+1}$

(2) $\frac{1}{n+2}$

(3) $\frac{1}{n+1} - \frac{1}{n+2}$

(4) $\frac{1}{n+1} + \frac{1}{n+2}$

Sol. (3) $I = \int_0^1 x(1-x)^n dx$

$$-I = \int_0^1 -x(1-x)^n dx = \int_0^1 (1-x-1)(1-x)^n dx$$

$$= \int_0^1 (1-x)^{n+1} dx - \int_0^1 (1-x)^n dx = \left[\frac{(1-x)^{n+2}}{-(n+2)} \right]_0^1 - \left[\frac{(1-x)^{n+1}}{-(n+1)} \right]_0^1 = \frac{1}{n+2} - \frac{1}{n+1}$$

$$I = \frac{1}{n+1} - \frac{1}{n+2}$$

39. $\lim_{n \rightarrow \infty} \frac{1+2^4+3^3+\dots+n^4}{n^5} - \lim_{n \rightarrow \infty} \frac{1+2^3+3^3+\dots+n^3}{n^5}$

(1) $\frac{1}{30}$

(2) Zero

(3) $\frac{1}{4}$

(4) $\frac{1}{5}$

Sol. (4) $\lim_{n \rightarrow \infty} \frac{1}{n} \left\{ \left(\frac{1}{n} \right)^4 + \left(\frac{2}{n} \right)^4 + \left(\frac{3}{n} \right)^4 + \dots + \left(\frac{n}{n} \right)^4 \right\} - \lim_{n \rightarrow \infty} \frac{1}{n} \left\{ \frac{1}{n^4} + \frac{2^3}{n^4} + \dots + \frac{n^3}{n^4} \right\}$

$$\int_0^1 (x)^4 dx - 0 = \frac{x^5}{5} \Big|_0^1 = \frac{1}{5}$$

40. Let $\frac{d}{dx} F(x) = \left(\frac{e^{\sin x}}{x} \right)$, $x > 0$. If $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(k) - F(1)$, then one of the possible values of k, is

(1) 15

(2) 16

(3) 63

(4) 64

Sol. (4) $\frac{d}{dx} F(x) = \frac{e^{\sin x}}{x}$

$$\int_1^4 \frac{3}{x} e^{\sin x^3} dx = \int_1^4 \frac{3x^2}{x^3} e^{\sin x^3} dx$$

Put $x^3 = t$, $3x^2 dx = dt$

When $x = 1$, $t = 1$ & $x = 4$, $t = 64$

$$F(t) = \int_1^{64} \frac{e^{\sin t}}{t} dt = \int_1^{64} F(t) dt = F(64) - F(1)$$

$K = 64$

41. The area of the region bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is

(1) 2 sq. units

(2) 3 sq. units

(3) 4 sq. units

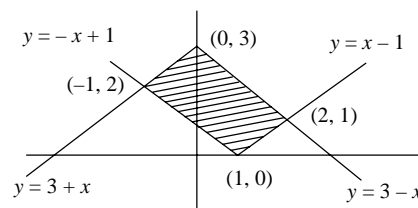
(4) 6 sq. units

Sol. (3) $A = \int_{-1}^0 \{(3+x) - (-x+1)\} dx + \int_0^1 \{(3-x) - (-x+1)\} dx + \int_1^2 \{(3-x) - (x-1)\} dx$

$$= \int_{-1}^0 (2+2x) dx + \int_0^1 2 dx + \int_1^2 (4-2x) dx$$

$$= [2x - x^2]_{-1}^0 + [2x]_0^1 + [4x - x^2]_1^2$$

$$= 0 - (-2+1) + (2-0) + (8-4) - (4-1) = 1 + 2 + 4 - 3 = 4 \text{ sq. units}$$



42. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $f(x)$ be a function that satisfies $f(x) + f(x) = x^2$. Then the value of the integral $\int_0^1 f(x) g(x) dx$, is

(1) $e - \frac{e^2}{2} - \frac{5}{2}$

(2) $e + \frac{e^2}{2} - \frac{3}{2}$

(3) $e - \frac{e^2}{2} - \frac{3}{2}$

(4) $e + \frac{e^2}{2} + \frac{5}{2}$

Sol. (3) Let $f(x) = e^x$

$$\therefore \int_0^1 f(x) g(x) dx = \int_0^1 e^x (x^2 - e^x) dx$$

$$= \int_0^1 x^2 e^x dx - \int_0^1 e^{2x} dx$$

$$= [x^2 e^x]_0^1 - 2[xe^x - e^x]_0^1 - \frac{1}{2}[e^{2x}]_0^1$$

$$= e - \left[\frac{e^2}{2} - \frac{1}{2} \right] - 2[e - e + 1]$$

$$= e - \frac{e^2}{2} - \frac{3}{2}$$

43. The degree and order of the differential equation of the family of all parabolas whose axis is x - axis, are respectively
 (1) 2, 1 (2) 1, 2 (3) 3, 2 (4) 2, 3

Sol. (2) $y^2 = 4a(x - h)$

$$2yy_1 = 4a \Rightarrow yy_1 = 2a \Rightarrow y_1^2 + yy_2 = 0$$

Degree = 1, order = 2

44. The solution of the differential equation $(a + y^2) + (x - e^{\tan^{-1} y}) \frac{dy}{dx} = 0$, is

(1) $(x - 2) = k e^{\tan^{-1} y}$ (2) $2x e^{\tan^{-1} y} = e^{2 \tan^{-1} y} + k$ (3) $s e^{\tan^{-1} y} = \tan^{-1} y + k$ (4)

Sol. (2) $(1 + y^2) + (x - e^{\tan^{-1} y}) \frac{dy}{dx} = 0$

$$(1 + y^2) \frac{dx}{dy} + x = e^{\tan^{-1} y}$$

$$\frac{dx}{dy} + \frac{x}{(1 + y^2)} = \frac{e^{\tan^{-1} y}}{(1 + y^2)}$$

$$\text{I.F.} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$$

$$x(e^{\tan^{-1} y}) = \int \frac{e^{\tan^{-1} y}}{1 + y} e^{\tan^{-1} y} dy$$

$$x(e^{\tan^{-1} y}) = \frac{e^{2 \tan^{-1} y}}{2} + C$$

$$\therefore 2xe^{\tan^{-1} y} = e^{2 \tan^{-1} y} + k$$

45. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then the value of 'c' is

(1) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$ (2) $a_1^2 - a_2^2 + b_1^2 - b_2^2$ (3) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$ (4) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

Sol. (1) $(h - a_1)^2 + (k - b_1)^2 = (h - a_2)^2 + (k - b_2)^2$

$$(a_1 - a_2)x + (b_1 - b_2)y + \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2) = 0$$

$$C = \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$$

46. Locus of centroid of the triangle whose vertices are $(a \cos t, a \sin t)$, $(b \sin t, -b \cos t)$ and $(1, 0)$ where t is a parameter, is

(1) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$ (2) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

$$(3) (3x+1)^2 + (3y)^2 = a^2 + b^2$$

$$(4) (3x+1)^2 + (3y)^2 = a^2 + b^2$$

Sol. (2) $x = \frac{\cos t + b \sin t + 1}{3} \Rightarrow a \cos t + b \sin t = 3x - 1$

$$y = \frac{a \sin t - b \cos t}{3} \Rightarrow a \sin t - b \cos t = 3y$$

Squaring & adding

$$(3x-1)^2 + (3y)^2 = a^2 + b^2$$

47. If the pair of straight line $x^2 - 2pxy - x^2 = 0$ and $x^2 - 2pxy - y^2 = 0$ be such that each pair, then

(1) $p = q$

(2) $p = -q$

(3) $pq = 1$

(4) $pq = -1$

Sol. (4) Equation of bisectors of both pair of st. lines

$$px^2 + 2xy - py^2 = 0 \quad \dots\dots (1)$$

$$qx^2 + 2xy - qy^2 = 0 \quad \dots\dots (2)$$

From (1) & (2)

$$\frac{q}{1} = \frac{2}{-2p} = \frac{-q}{-1} \Rightarrow pq = -1$$

48. A square of side a lies above the x -axis and has one vertex at the origin. The side passing through the origin makes an angle α ($0 < \alpha < \frac{\pi}{4}$) with the positive direction of x -axis. The equation of its diagonal not passing through the origin is

(1) $y(\cos \alpha - \sin \alpha) - x(\sin \alpha - \cos \alpha) = a$ (2)

$y(\cos \alpha + \sin \alpha) + x(\sin \alpha - \cos \alpha) = a$

(3) $y(\cos \alpha + \sin \alpha) + x(\sin \alpha + \cos \alpha) = a$ (4)

$y(\cos \alpha + \sin \alpha) + x(\sin \alpha - \cos \alpha) = a$

Sol. (4) co-ordinates of $A = (a \cos \alpha, a \sin \alpha)$

Equation of OB

$$y = \tan\left(\frac{\pi}{4} + \alpha\right) x$$

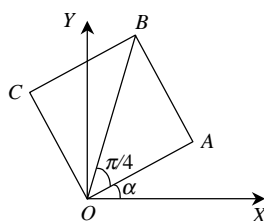
$\therefore CA \perp r$ to OB

$$\therefore \text{slope of } CA = -\cot\left(\frac{\pi}{4} + 2\right)$$

Equation of CA

$$y - a \sin \alpha = -\cot\left(\frac{\pi}{4} + 2\right)(x - a \cos \alpha)$$

$$y(\sin \alpha + \cos \alpha) + x(\cos \alpha - \sin \alpha) = a$$



49. If the two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

(1) $2 < r < 8$

(2) $r < 2$

(3) $r = 2$

(4) $r > 2$

Sol. (1) $|r_1 - r_2| < C_1 C_2$ for intersection

$$\Rightarrow r - 3 < 5 \Rightarrow r < 8 \quad \dots\dots (1)$$

$$\text{and } r_1 + r_2 > C_1 C_2$$

$$r + 3 > 5 \Rightarrow r = 2 \quad \dots\dots (2)$$

From (1) & (2)

$$2 < r < 8$$

50. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq. units. Then the equation of the circle is

- (1) $x^2 + y^2 + 2x - 2y = 62$ (2) $x^2 + y^2 + 2x - 2y = 47$ (3) $x^2 + y^2 - 2x + 2y = 47$ (4) $x^2 + y^2 - 2x + 2y = 62$

Sol. (3) $\pi r^2 = 154 \Rightarrow r = 7$

For centre on solving equation $2x - 3y = 5$ & $3x - 4y = 7$

$$x = 1, y = 1$$

centre = (1, -1)

$$\text{Equation of circle } (x-1)^2 + (y+1)^2 = 7^2$$

$$x^2 + y^2 - 2x + 2y = 47$$

51. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then

- (1) $t_2 = -t_1 - \frac{2}{t_1}$ (2) $t_2 = -t_1 + \frac{2}{t_1}$ (3) $t_2 = t_1 - \frac{2}{t_1}$ (4) $t_2 = t_1 + \frac{2}{t_1}$

Sol. (1) Fundamental theorem (fact)

$$t_2 = -t_1 - \frac{2}{t_1}$$

52. The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then the value of the b^2 is

- (1) 1 (2) 5 (3) 7 (4) 9

Sol. (3) $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$$a = \sqrt{\frac{144}{25}}, b = \sqrt{\frac{81}{25}}, e_1 = \sqrt{1 + \frac{81}{144}} = \frac{15}{12} = \frac{5}{4}$$

$$\text{Foci} = (3, 0), \text{ focus of ellipse} = (3, 0) \Rightarrow e = \frac{3}{4}$$

$$b^2 = 16 \left(1 - \frac{9}{16} \right) = 7$$

53. A tetrahedron has vertices at $O(0, 0, 0)$, $A(1, 2, 1)$, $B(2, 1, 3)$ and $C(-1, 1, 2)$. Then the angle between the faces OAB and ABC will be

- (1) $\cos^{-1}\left(\frac{19}{35}\right)$ (2) $\cos^{-1}\left(\frac{17}{31}\right)$ (3) 30° (4) 90°

Sol. (1) Vector perpendicular to the face OAB

$$= \overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 5i - j - 3k$$

Vector perpendicular to the face ABC

$$= \vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix} = i - 5j - 3k$$

Angle between the faces = angle between their normals

$$\cos \theta = \frac{5+5+9}{\sqrt{35}\sqrt{35}} = \frac{19}{35}$$

$$\theta = \cos^{-1}\left(\frac{19}{35}\right)$$

54. The radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$ is

- (1) 1 (2) 2 (3) 3 (4) 4

Sol. (3) centre of sphere = (-1, 1, 2)

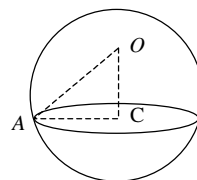
$$\text{Radius of sphere} = \sqrt{1+1+4+19} = 5$$

Perpendicular distance from centre to the plane

$$OC = d = \left| \frac{-1+2+4+7}{\sqrt{1+4+4}} \right| = \frac{12}{3} = 4$$

$$AC^2 = AO^2 - OC^2 = 5^2 - 4^2 = 9$$

$$AC = 3$$



55. The line $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if

- (1) $k = 0$ or -1 (2) $k = 0$ or -1 (3) $k = 0$ or -3 (4) $k = 3$ or -3

Sol. (3)
$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 0 & 0 & -1 \\ 2 & 1+k & -k \\ k+2 & 1 & 1 \end{vmatrix} = 0$$

$$k^2 + 3k^2 = 0 \Rightarrow k(k+3) = 0$$

$$k = 0 \text{ or } -3$$

56. The two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ will be perpendicular, if and only if

- (1) $aa' + bb' + cc' + 1 = 0$ (2) $aa' + bb' + cc' = 0$
(3) $(a+a')(b+b') + (c+c') = 0$ (4) $aa' + cc' + 1 = 0$

Sol. (4)
$$\frac{x-b}{a} = \frac{y}{1} = \frac{3-d}{c}$$

$$\frac{x-b'}{a'} = \frac{y}{1} = \frac{3-d'}{c'}$$

For perpendicular $aa' + 1 + cc' = 0$

57. The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is

- (1) 26 (2) $11\frac{4}{13}$ (3) 13 (4) 39

Sol. (3) \because shortest distance = perpendicular distance

$$= \left| \frac{-2 \times 12 + 4 \times 1 + 3 \times 3 - 327}{\sqrt{144 + 9 + 16}} \right| = 26$$

$$\therefore \text{shortest distance} = 26 - \sqrt{4 + 1 + 15 + 9} \quad [\because 26 - r] \\ = 26 - 13 = 13$$

58. Two systems of rectangular axes have the same origin. If a plane cuts them at distance a, b, c and a', b', c' from the origin, then

- (1) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2} = 0$ (2) $\frac{1}{a^2} + \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} - \frac{1}{c'^2} = 0$
 (3) $\frac{1}{a^2} - \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$ (4) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

Sol. (4) Equation of planes be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and $\frac{x}{a_1} + \frac{y}{b_1} + \frac{z}{c_1} = 1$

(Perpendicular distance on plane from origin is same)

$$\therefore \left| \frac{-1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} \right| = \left| \frac{-1}{\sqrt{\frac{1}{a_1^2} + \frac{1}{b_1^2} + \frac{1}{c_1^2}}} \right|$$

$$\therefore \Sigma \frac{1}{a^2} - \Sigma \frac{1}{a_1^2} = 0$$

59. $\vec{a}, \vec{b}, \vec{c}$ are 3 vectors, such that $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 1$, $|\vec{b}| = 2$, $|\vec{c}| = 3$, then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is equal to

- (1) 0 (2) -7 (3) 7 (4) 1

Sol. (2) $\vec{a} + \vec{b} + \vec{c} = 0$

$$(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \frac{-1 - 4 - 9}{2} = -7$$

60. If \vec{u}, \vec{v} and \vec{w} are three non-coplanar vectors, then $(\vec{u} + \vec{v} - \vec{w}) \cdot (\vec{u} - \vec{v}) \times (\vec{v} - \vec{w})$ equals

- (1) 0 (2) $\vec{u} \cdot \vec{v} \times \vec{w}$ (3) $\vec{u} \cdot \vec{w} \times \vec{v}$ (4) $3 \vec{u} \cdot \vec{v} \times \vec{w}$

Sol. (b) $(\vec{u} + \vec{v} - \vec{w}) \cdot (\vec{u} \times \vec{v} - \vec{u} \times \vec{w} - \vec{v} \times \vec{v} + \vec{v} \times \vec{w})$

$$= (\vec{u} + \vec{v} - \vec{w}) \cdot (\vec{u} \times \vec{v} - \vec{u} \times \vec{w} + \vec{v} \times \vec{w})$$

$$\begin{aligned}
 &= \frac{\vec{u} \cdot (\vec{u} \times \vec{v})}{0} - \frac{\vec{u} \cdot (\vec{u} \times \vec{w})}{0} + \vec{u} \cdot (\vec{v} \times \vec{w}) + \frac{\vec{v} \cdot (\vec{u} \times \vec{v})}{0} - \vec{v} \cdot (\vec{u} \times \vec{w}) + \frac{\vec{v} \cdot (\vec{v} \times \vec{w})}{0} - \vec{w} \cdot (\vec{u} \times \vec{v}) + \frac{\vec{w} \cdot (\vec{u} \times \vec{w})}{0} - \frac{\vec{w} \cdot (\vec{v} \times \vec{w})}{0} \\
 &= \vec{u} \cdot (\vec{v} \times \vec{w}) - \vec{v} \cdot (\vec{u} \times \vec{w}) - \vec{w} \cdot (\vec{u} \times \vec{v}) \\
 &= [\vec{u} \ \vec{v} \ \vec{w}] + [\vec{v} \ \vec{w} \ \vec{u}] - [\vec{w} \ \vec{u} \ \vec{v}] \\
 &= \vec{u} \cdot (\vec{v} \times \vec{w})
 \end{aligned}$$

61. Consider point A, B, C and D with position vectors $7\hat{i} - 4\hat{j} + 7\hat{k}$, $\hat{i} - 6\hat{j} + 10\hat{k}$, $-\hat{i} - 3\hat{j} + 4\hat{k}$ and $5\hat{i} - \hat{j} + 5\hat{k}$ respectively. Then ABCD is a

- (1) Square (2) Rhombus
(3) Rectangle (4) Parallelogram but not a rhombus

Sol. (0) No option satisfied wrong.

$$A = (7, -4, 7), B = (1, -6, 10), C = (-1, -3, 4) \text{ and } D = (5, -1, 5)$$

$$AB = \sqrt{(7-1)^2 + (-4+6)^2 + (7-10)^2} = \sqrt{36+4+9} = 7$$

$$BC = \sqrt{(1+1)^2 + (-6+3)^2 + (10-4)^2} = \sqrt{4+9+36} = 7$$

$$CD = \sqrt{(-1-5)^2 + (-3+1)^2 + (4-5)^2} = \sqrt{36+4+1} = \sqrt{41}$$

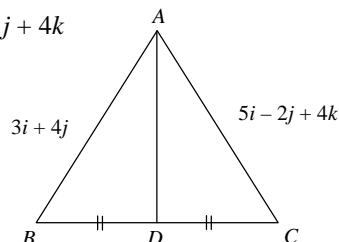
$$DA = \sqrt{(5-7)^2 + (-1+4)^2 + (5-7)^2} = \sqrt{4+9+4} = \sqrt{17}$$

62. The vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$, and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is

- (1) $\sqrt{18}$ (2) $\sqrt{72}$ (3) $\sqrt{33}$ (4) $\sqrt{288}$

Sol. (3) Position Vector of $\overrightarrow{AD} = \frac{(3+5)\hat{i} + (0-2)\hat{j} + (4+4)\hat{k}}{2} = 4\hat{i} - \hat{j} + 4\hat{k}$

$$|\overrightarrow{AD}| = \sqrt{16+16+1} = \sqrt{33}$$



63. A particle acted on by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ is displaced from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The total work done by the force is

- (1) 20 units (2) 30 units (3) 40 units (4) 50 units

Sol. (3) $\vec{F} + \vec{F}_1 + \vec{F}_2 = 7\hat{i} + 2\hat{j} - 4\hat{k}$

$$\vec{d} = P.V. \text{ of } \vec{B} - P.V. \text{ of } \vec{A} = 4\hat{i} + 2\hat{j} - 2\hat{k}$$

$$W = \vec{F} \cdot \vec{d} = 28 + 4 + 8 = 40 \text{ unit}$$

64. Let $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If \hat{n} is a unit vector such that $\vec{u} \cdot \hat{n} = 0$ and $\vec{v} \cdot \hat{n} = 0$, then $|\vec{w} \cdot \hat{n}|$ is equal to

- (1) 0 (2) 1 (3) 2 (4) 3

Sol. (4) $\because \vec{n}$ is perpendicular \vec{u} and \vec{v}

$$\vec{n} = \vec{u} \times \vec{v}$$

$$\hat{n} = \frac{\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}}{\sqrt{2} \times \sqrt{2}} = \frac{-2\hat{k}}{2} = -\hat{k}$$

$$|\vec{w} \cdot \hat{n}| = |(i + 2j + 3k) \cdot (-k)| = |-3| = 3$$

65. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observation of the set is increased by 2, then the median of the new set

- (1) Is increased by 2 (2) Is decreased by 2
(3) Is two times the original median (4) Remains the same as that of the original set

Sol. (4) $n = 9$ then median term = $\left(\frac{9+1}{2}\right)^{th} = 5^{th}$ term last four observations are increased by 2

\therefore the median is 5^{th} observation which is remaining unchanged.

\therefore There will be no change in median.

66. In an experiment with 15 observations on x , the following results were available

$$\Sigma x^2 = 2830, \Sigma x = 170$$

One observation that was 20 was found to be wrong and was replaced by the correct value 30. Then the corrected variance is

- (1) 78.00 (2) 188.66 (3) 177.33 (4) 8.33

Sol. (1) $\Sigma x = 170, \Sigma x^2 = 2830$ increase in $\Sigma x = 10$, then $\Sigma x' = 170 + 10 = 180$

$$\text{Increase in } \Sigma x^2 = 900 - 400 = 500 \text{ then } \Sigma x'^2 = 2830 + 500 = 3330$$

$$\text{Variance} = \frac{1}{n} \Sigma x'^2 - \left(\frac{1}{n} \Sigma x' \right)^2 = \frac{1}{15} \times 3330 - \left(\frac{1}{15} \times 180 \right)^2 = 222 - 144 = 78$$

67. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is

- (1) $\frac{4}{5}$ (2) $\frac{3}{5}$ (3) $\frac{1}{5}$ (4) $\frac{2}{5}$

Sol. (4) $n(S) = {}^5C_2$

$$n(E) = {}^2C_1 + {}^2C_1$$

$$p(E) = \frac{n(E)}{n(S)} = \frac{{}^2C_1 + {}^2C_1}{{}^5C_2} = \frac{2}{5}$$

68. Events A, B, C are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$. The set of possible values of x are in the interval

- (1) $\left[\frac{1}{3}, \frac{1}{2} \right]$ (2) $\left[\frac{1}{3}, \frac{2}{3} \right]$ (3) $\left[\frac{1}{3}, \frac{13}{3} \right]$ (4) $[0, 1]$

Sol. (1) $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$, $P(C) = \frac{1-2x}{2}$

\therefore These are mutually exclusive

$$0 \leq \frac{3x+1}{3} \leq 1, \quad 0 \leq \frac{1-x}{4} \leq 1 \text{ and } 0 \leq \frac{1-2x}{2} \leq 1$$

$$-1 \leq 3x \leq 2, \quad -3 \leq x \leq 1 \text{ and } -1 \leq 2x \leq 1$$

$$-\frac{1}{3} \leq x \leq \frac{2}{3} \leq -3 \leq x \leq 1, \text{ and } -\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$\text{Also } 0 \leq \frac{1+3x}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \leq 1$$

$$0 \leq 13-3x \leq 12 \Rightarrow 1 \leq 3x \leq 13$$

$$\Rightarrow \frac{1}{3} \leq x \leq \frac{13}{3}$$

$$\max. \left\{ -\frac{1}{3}, -3, -\frac{1}{2}, \frac{1}{3} \right\} \leq x \leq \min \left\{ \frac{2}{3}, 1, \frac{1}{2}, \frac{13}{3} \right\}$$

$$\frac{1}{3} \leq x \leq \frac{1}{2} \Rightarrow x \in \left[\frac{1}{3}, \frac{1}{2} \right]$$

69. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively, then $P(X=1)$ is

(1) $\frac{1}{32}$

(2) $\frac{1}{16}$

(3) $\frac{1}{8}$

(4) $\frac{1}{4}$

Sol. (1) $\left. \begin{matrix} np=4 \\ npq=2 \end{matrix} \right\} \Rightarrow q=\frac{1}{2}, p=\frac{1}{2}, n=8$

$$P(X=1) = {}^8C_1 \left(\frac{1}{2} \right) \left(\frac{1}{2} \right)^7$$

$$= 8 \cdot \frac{1}{2^8} = \frac{1}{2^5} = \frac{1}{32}$$

70. The resultant of force \vec{P} and \vec{Q} is \vec{R} . If \vec{Q} is double then \vec{R} is doubled. If the direction of \vec{Q} is reversed, then \vec{R} is again doubled. Then $P^2 : Q^2 : R^2$ is

(1) $3 : 1 : 1$

(2) $2 : 3 : 2$

(3) $1 : 2 : 3$

(4) $2 : 3 : 1$

Sol. (2) $R^2 = P^2 + Q^2 + 2PQ \cos \theta$ (1)

$$4R^2 = P^2 + 4Q^2 + 4PQ \cos \theta$$
 (2)

$$4R^2 = P^2 + Q^2 - 2PQ \cos \theta$$
 (3)

On (i) + (ii), $5R^2 = 2P^2 + 2Q^2$ (4)

On (iii) $\times 2$ + (ii), $12R^2 = 3P^2 + 6Q^2$ (5)

$$2P^2 + 2Q^2 - 5R^2$$
 (6)

$$3P^2 + 6Q^2 - 12R^2$$
 (7)

by cross multiplication

$$\frac{P^2}{-24+30} = \frac{Q^2}{24-15} = \frac{R^2}{12-6}$$

$$\frac{P^2}{6} = \frac{Q^2}{9} = \frac{R^2}{6}$$

$$P^2 : Q^2 : R^2 = 2 : 3 : 2$$

71. Let R_1 and R_2 respectively be the maximum ranges up and down an inclined plane and R be the maximum range on the horizontal plane. Then R_1, R, R_2 are in

- (1) Arithmetic-Geometric Progression (A.G.P.) (2) A.P.
(3) G.P. (4) H.P.

Sol. (4) Let β be the inclination of the plane to the horizontal and u be the velocity of projection of the projectile

$$R_1 = \frac{u^2}{g(1 + \sin \beta)} \text{ and } R_2 = \frac{u^2}{g(1 - \sin \beta)}$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2g}{u^2}$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{R} \quad \left[\because R = \frac{u^2}{g} \right]$$

$\therefore R_1, R, R_2$ are in H.P.

72. A couple is of moment \vec{G} and the force forming the couple is \vec{P} . If \vec{P} is turned through a right angle, the moment of the couple thus formed is \vec{H} . If instead, the force \vec{P} are turned through an angle α , then the moment of couple becomes

- (1) $\vec{G} \sin \alpha - \vec{H} \cos \alpha$ (2) $\vec{H} \cos \alpha + \vec{G} \sin \alpha$ (3) $\vec{G} \cos \alpha + \vec{H} \sin \alpha$ (4) $\vec{H} \sin \alpha - \vec{G} \cos \alpha$

Sol. (3) $\vec{a} = \vec{r} \times \vec{p}$

$$|\vec{a}| = rp \sin \theta$$

$$|\vec{H}| = rp \cos \theta \quad [\because \sin(90^\circ + \nu) = \cos \theta]$$

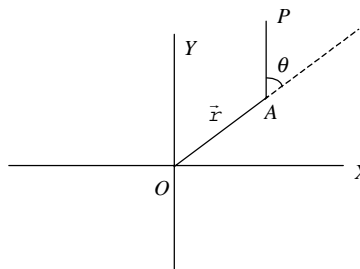
$$G = rp \sin \theta \quad \dots\dots (1)$$

$$H = rp \cos \theta \quad \dots\dots (2)$$

$$x = rp \sin(\theta + \alpha) \quad \dots\dots (3)$$

from (1), (2) and (3)

$$x = \vec{a} \cos \alpha + \vec{H} \sin \alpha$$



73. The particles start simultaneously from the same point and move along two straight lines, one with uniform velocity \vec{u} and the other from rest with uniform acceleration \vec{f} . Let α be the angle between their directions of motion. The relative velocity of the second particle w.r.t. the first is least after a time

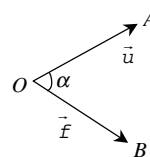
- (1) $\frac{u \sin \alpha}{f}$ (2) $\frac{f \cos \alpha}{u}$ (3) $u \sin \alpha$ (4) $\frac{u \cos \alpha}{f}$

Sol. (4) After t ; velocity = $f \times t$

$$\vec{V}_{BA} = \vec{f} t + (-\vec{u})$$

$$V_{BA} = \sqrt{f^2 t^2 + u^2 - 2fut \cos \alpha}$$

$$\text{For max. and min. } \frac{d}{dt}(V_{BA}^2) = 2f^2 t - 2fu \cos \alpha = 0$$



$$t = \frac{u \cos \alpha}{f}$$

74. Two stones are projected from the top of a cliff h metres high, with the same speed u so as to hit the ground at the same spot. If one of the stones is projected horizontally and the other is projected at an angle θ to the horizontal then $\tan \theta$ equals

- (1) $\sqrt{\frac{2u}{gh}}$ (2) $2g\sqrt{\frac{u}{h}}$ (3) $2h\sqrt{\frac{u}{g}}$ (4) $u\sqrt{\frac{2}{gh}}$

Sol. (4) $R = u\sqrt{\frac{2h}{g}} = (u \cos \theta) \times t$

$$t = \frac{1}{\cos \theta} \sqrt{\frac{2h}{g}} \quad \dots\dots (1)$$

$$\text{Now } h = (-u \sin \theta)t + \frac{1}{2}gt^2$$

' t ' from (1)

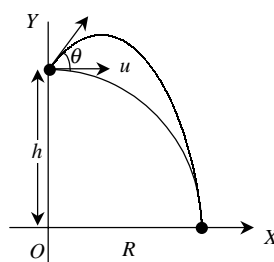
$$h = -\frac{u \sin \theta}{\cos \theta} \sqrt{\frac{2h}{g}} + \frac{1}{2}g \left[\frac{2h}{g \cos^2 \theta} \right]$$

$$h = -u \sqrt{\frac{2h}{g}} \tan \theta + h \sec^2 \theta$$

$$h = -u \sqrt{\frac{2h}{g}} \tan \theta + h \tan^2 \theta + h$$

$$\tan^2 \theta - u \sqrt{\frac{2}{hg}} \tan \theta = 0$$

$$\therefore \tan \theta = u \sqrt{\frac{2}{hg}}$$



75. A body travels a distance s in t seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r . The value of t is given by

- (1) $2s \left(\frac{1}{f} + \frac{1}{r} \right)$ (2) $\frac{2s}{\frac{1}{f} + \frac{1}{r}}$ (3) $\sqrt{2s(f+r)}$ (4) $\sqrt{2s \left(\frac{1}{f} + \frac{1}{r} \right)}$

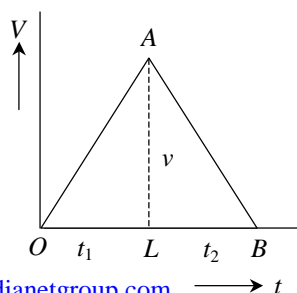
- Sol. (4) Portion OA , OB corresponds to motion with acceleration ' f ' and retardation ' r ' respectively.

Area of $\Delta OAB = S$ and $OB = t$. Let $OL = t_1$, $LB = t_2$, and $AL = v$

$$S = \frac{1}{2}OB.AL$$

$$= \frac{1}{2}t.v$$

$$v = \frac{2S}{t}$$



$$\text{Also } f = \frac{v}{t_1}, t_1 = \frac{v}{f} = \frac{2s}{tf} \text{ and } r = \frac{v}{t_2}, t_2 = \frac{v}{r} = \frac{2s}{tr}$$

$$t = t_1 + t_2 = \frac{2S}{tf} + \frac{2S}{tr}$$

$$t = \left(\frac{1}{f} + \frac{1}{r} \right) \frac{2S}{t} \Rightarrow t = \sqrt{2s \left(\frac{1}{f} + \frac{1}{r} \right)}$$