ally occur in **k** (for instance, such a square follows the initial factor of **k** of length 324), as well as squares of words of length 1, 2, 3, and 9. On the contrary, one can check that, for all $x \in S$ of length 18, either $\varphi(xx) = 111$ or $\varphi(xx) = 1111$, and therefore xx does not occur in **k.** Thus we can state our first result.

Proposition 1. *The lengths of the non-empty words x such that xx occurs in the KOLAKOVSKI sequence are:* 1, 2, 3, 9, 27.

As a consequence, one derives the following

Proposition 2. *The KOLAKOVSKI sequence is cube-free.*

To prove Proposition 2, it would be sufficient to check that, for each one of the finitely many squares xx occurring in **k**, xxx does not occur. We can avoid long and tedious verifications remarking that, by an argument similar to that used in the proof of Lemma 2, if xxx is a non-empty factor of **k**, then **k** contains a square zz, with $|z|$ even and $|x| = |\sigma(z)|$. By Proposition 1, one has $|z| = 2$ and, consequently, $|x| \le 4$: but we know [2] that **k** does not contain cubes xxx with $|x| \le 4$.

It is also possible to compute the *maximal rational exponent* of repetitions contained in the KOLAKOVSKI sequence, that is the maximal value of $|(xy)^n x| / |xy|$, with $(xy)^n x$ a non-empty factor of **k**: indeed, one can prove that in **k** no factor of the form $xyxyx$ with $|x| \ge 2$ occurs, except 12112112, 12212212, 21121121, 21221221, which are exactly repetitions of exponent 8/3: hence, this is the searched value.

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A CODIFICATION OF COLUMN-CONVEX POLYOMINOES WHICH GENERATES A REGULAR LANGUAGE

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R O M A N I A

Abstract: The set of column-convex polyominoes $\mathcal{P}$ is considered. A codification which associates a word over a certain finite alphabet to each column-convex polyomino is defined. The resulting language M is determined and, finally, a proof of the fact that M is a regular language is given.

1. Introduction

We call *cell* a unit square in the plane whose vertices have integer coordinates. A *polyomino* is a connected finite union of cells which does not have a finite cut set. Two polyominoes are identified if a translation transforming one of them into the other exists. A polyomino P is *column-convex* if every one of its columns represents a noninterrupted line of cells (see Fig.1). We shall denote the set of column-convex polyominoes by $\mathcal{P}$.

The combinatorial object of polyomino has been studied mostly from an enumerative point of view (Klarner [3,4], Delest [1,2] , Viennot [2]). In the present paper we mean to study this concept from the point of view of formal language theory. Thus, we shall define a codification which associates a word over a certain finite alphabet to each column-convex polyomino. Using a technique similar to that of the M. LOTHAIRE group with