Electron energy distribution function and electron characteristics of conventional and micro hollow cathode discharges

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The conventional hollow cathode discharge, micro hollow cathode discharge, and the transition between them have been analyzed. The time independent and spatially averaged electron energy distribution function, electron density, mean electron energy, excitation, and ionization rates have been calculated and compared. The direct comparison showed substantial differences between the conventional and micro hollow cathode discharges, particularly in absorbed power per unit volume, degree of ionization, and excitation and ionization rates. © 2002 American Institute of Physics.  [DOI: 10.1063/1.1466819]

I. INTRODUCTION

The conventional hollow cathode discharge (HCD) has been widely investigated both theoretically and experimentally. The HCD possesses unique properties that have led to many technological and scientific applications. The requirement for sustaining a HCD at low gas pressure p is that the HCD diameter D should be in the centimeter range according to the relation \( pD > \ln(f^{-1})(\sigma_{\text{tot}} n_0) = (pD)_{\text{crit}} \). In this expression, \( n_0 \) is the gas density at a pressure of 1 Torr, \( f \) is a loss factor (\( f = 0.6 \) for Ar for a cathode diameter in the cm range), and \( \langle \sigma_{\text{tot}} \rangle \) is the average total electron–atom interaction cross section, which is the sum of elastic and inelastic cross sections (\( \langle \sigma_{\text{tot}} \rangle = 5.5 \times 10^{-16} \text{ cm}^2 \) for Ar).\(^1\)\(^2\) If the gas pressure times the cathode diameter is less than the critical value, \( (pD)_{\text{crit}} \), the “Pendel” electrons reaching the opposite cathode fall are removed from the discharge, which results in an insufficient number to ionize the discharge. An extension of this rule to high-pressure discharges is desirable due to enhanced excitation and ionization rates.

The high-pressure range of HCD (10\(^2\) – 10\(^3\) Torr) has been less investigated. In the early work of White\(^3\) the HCD operates in the so-called micro HCD mode with the D value in the submillimeter range. Generally, at these pressures and small geometric sizes these micro HCDs operate at voltages of 150–500 V.\(^4\) Research on high-pressure glow discharges has discovered some HCD applications such as surface treatment, thin-film deposition, detoxification, VUV spectroscopy, and light sources. Some essential electrical and light emission properties of micro HCD have been systematically pointed out by Schoenbach and co-workers in a series of experiments: the current–voltage characteristics,\(^5\) Paschen’s Law,\(^6\) direct current glow discharges in atmospheric air,\(^6,7\) generation of intense excimer radiation,\(^8\) micro HCD in mixtures of hydrochloric acid, hydrogen, xenon, and neon,\(^9\) electron density measurements in atmospheric pressure, air plasma,\(^10\) micro HCD excimer lamps,\(^11\) resonant energy transfer from argon dimers to atomic oxygen in micro HCD,\(^12\) electron heating in atmospheric pressure glow discharges,\(^13\) qualitative information on the electron energy distribution function (EEDF),\(^14\) etc. Finally, experiments on these microdischarges with plane cathodes and their operation in single or end-on coupled devices are summarized in Ref. 4.

The EEDF is one of the main characteristics of the discharge. Kagan and co-authors found an analytical expression for the EEDF.\(^15,16\) Their model is supported by results from potential probe and electron energy electrostatic measurements, as well as on the spectral emissions observed in HCDs. Other models have also dealt with the EEDF in conventional (low pressure) HCDs.\(^17–21\) The EEDF in micro HCDs has only been investigated using indirect methods. It has been partially analyzed in atmospheric pressure glow discharges that are sustained with nanosecond voltage pulses.\(^14\) The objectives of this work are to calculate the EEDF for a micro HCD and to compare it and other related properties to those found in conventional HCDs. A self-consistent model has been developed for calculating the EEDF, the mean electron energy, and excitation and ionization rates in both conventional and micro HCD and these calculations are compared and analyzed.

II. THE ELECTRON BOLTZMANN EQUATION

The time independent spatially averaged electron Boltzmann equation is solved for a cylindrical hollow cathode with radius R and length L. The collisional processes have been limited to elastic scattering, Coulomb collisions between electrons, excitation to electronically excited states and ionization. Without an external electric field, the electron Boltzmann equation can be written in form,
The EEDF is normalized according to \( n_e = \int_0^\infty f(u) du \), where \( n_e \) is the electron density. The gas temperature \( T_g \) is in energy units, \( m \) and \( e \) are the mass and magnitude of electron charge, respectively, and \( v(u) \) is the absolute value of the electron velocity with kinetic energy \( u \). Furthermore, \( M, N, \) and \( \sigma_{\text{mon}}(u) \) are the mass, density, and the momentum transfer cross section of the heavy particle, and \( \sigma^{\text{exc}}(u) \) is a lumped electron impact excitation cross section from the ground state, having energy threshold \( u^{\text{exc}} \). The Coulomb collisions are taken into account using the conventional Fokker–Planck expression, having two integral terms \( P(u) = 2u^{-1/2} \int_0^u \sigma^{\text{ion}}(e) v(e) d\epsilon + 2u^{-1/2} \int_u^\infty \sigma^{\text{ion}}(e) v(e) d\epsilon \) and \( Q(u) = 3u^{-1/2} \int_0^u \sigma^{\text{ion}}(e) v(e) d\epsilon \). \( \ln(L) \) is the Coulomb logarithm. The electron impact ionization is characterized by an energy threshold \( u^{\text{ion}} \) and differential cross section \( \sigma^{\text{ion}}(e,u) \) in which a primary electron with energy \( e \) creates a secondary electron with energy \( u \). The total ionization cross section \( \sigma^{\text{ion}}(e) = \int_0^{e_{\text{max}}} \sigma^{\text{ion}}(e,u) du \) is obtained by integration of the differential ionization cross section over the available energy \( u \leq e_{\text{max}} = (e-u^{\text{ion}})/2 \) of the secondary electron. The next term describes ionization by the primary (beam) electrons, which enter the plasma with energy \( u^{\text{beam}} \) equal to the energy the electrons gain in the cathode fall. The distribution of these electrons is formed in the cathode dark space and in most practical cases it does not play significant role. We assume that it is a monoenergetic electron beam described with a profile \( \psi^{\text{beam}}(u) = (\pi u_w)^{-1} e^{-\left(u-u^{\text{beam}}\right)^2/2u_w^2} \) with normalization \( \int_0^\infty \psi^{\text{beam}}(u) du = 1 \). The width of the energy distribution, \( u_w \) has been taken few eV. The ionization rate \( R^{\text{beam}} \) by the primary (beam) electrons is calculated from the electron particle balance equation.

Another distinguished feature of the HCD is the electron diffusion to the anode. The electron loss in HCD occurs in axial direction. The electrons cannot escape in radial direction because they are repelled by the cathode fall potential. The majority of the electrons are also trapped by an axial confining potential and only a small fraction of the total number of electrons is capable of overcoming the confining potential and escaping to the anode. Diffusion is possible only for those electrons having kinetic energy \( u > U_{\text{conf}} \) (\( U_{\text{conf}} \) is the axial confining potential) and the diffusion rate can be written in form,

\[
\frac{\partial f}{\partial t} = 0 - \frac{\partial}{\partial u} \left[ \frac{2m}{M} \sigma^{\text{mon}}(u) v(u) \right] \left[ \frac{1}{2} T_g - u \right] f(u) - T_g \frac{\partial f}{\partial u} \]

\[-\frac{\partial}{\partial u} \left[ \frac{2 \pi e^4}{3} \frac{u^{1/2}}{m} \ln(L) \right] \left[ P(u) \left( \frac{f(u)}{2u} - \frac{\partial f}{\partial u} \right) - Q(u) f(u) \right] + \sigma^{\text{exc}}(u + u^{\text{exc}}) v(u + u^{\text{exc}}) N f(u + u^{\text{exc}})
\]

\[-\sigma^{\text{exc}}(u) v(u) N f(u) + \int_{u + u^{\text{ion}}}^{u + u^{\text{ion}}} \sigma^{\text{ion}}(u) v(u) N f(u) d\epsilon + R^{\text{beam}} \psi^{\text{beam}}(u) - \nu^{\text{diff}}(u) f(u).
\]

These electrons form the anode current and their number is uniquely determined. The diffusion rate \( \nu_0 \) is inversely proportional to the time of free diffusion \( \tau_0 = L^2 / \nu_0 \) of an untrapped electron. \( \Lambda = L / \sqrt{8} \) is the characteristic length and \( D_e = \lambda \nu / 3 \) is the electron diffusion coefficient, where \( \nu = \sqrt{2e u/m} \) is the electron velocity and \( \lambda = \left( \sigma^{\text{mon}}(u) N \right)^{-1} \) is the mean free path of the electron. The confining potential \( U_{\text{conf}} \) is calculated from the anode current, as explained in Sec. III.

Argon has been used as a model gas. The lumped cross section for electron impact excitation (sum of all cross sections) compiled recently, is displayed in Fig. 1 along with the momentum transfer and valence shell ionization cross sections. The momentum transfer cross sections is taken from Ref. 23. The valence shell differential ionization cross section \( \sigma^{\text{ion}}(e,u) \) for energies of the primary electron up to 500 eV has been tabulated by Opal et al. Bretagne employed

\[
S^{\text{ion}}(e,u) = A(e) \left( \frac{\Gamma^2}{(u-u_0)^2 + \Gamma^2} \right)
\]

FIG. 1. The set of cross sections for Ar used in the model.
with \( A(\varepsilon) = 2.65 \times 10^{-15} \ln(\varepsilon/\mu_{\text{ion}}^m) \), \( \Gamma = 4.6 \) and \( \mu_0 = 1.2 - 250/(\varepsilon + 2u_{\text{ion}}^m) \).

### III. ELECTRON MACROSCOPIC PROPERTIES AND SELF-CONSISTENT TREATMENT OF THE HCD

The electron particle and power balances are derived from Eq. (1). The particle balance reads

\[
R_{\text{ion}} + R_{\text{beam}} = R_{\text{dif}},
\]

where \( R_{\text{ion}} = N \int_0^\infty \sigma_{\text{ion}}(u)v(u)f_0(u)\,du \) is the ionization rate with secondary electrons and \( R_{\text{dif}} = \int_0^\infty p(u)f(u)\,du \) is the diffusion rate. The electron power balance equation,

\[
p_{\text{beam}} = p_{\text{exc}} + p_{\text{ion}} + p_{\text{el}} + p_{\text{dif}}
\]

consists of power gain \( p_{\text{beam}} = R_{\text{beam}}u_{\text{beam}} \) due to ionization with primary (beam) electrons, power loss in excitation \( p_{\text{exc}} = R_{\text{exc}}u_{\text{exc}} \), ionization \( p_{\text{ion}} = R_{\text{ion}}u_{\text{ion}} \), and elastic scattering

\[
p_{\text{el}} = -\frac{2m}{M}N \int_0^\infty \sigma_{\text{mom}}(u)v(u)N \left( \frac{1}{2} T_g - u \right) f(u) - T_g \frac{df(u)}{du} \, du
\]

with secondary electrons and diffusion \( p_{\text{dif}} = \int_0^\infty p(u)f(u)\,du \). \( P \) denotes the power gain or loss per unit volume.

The self-consistent treatment of the HCD includes calculation of the discharge current at the cathode and at the anode. The ions are lost in radial direction through ambipolar diffusion in the negative glow, forming the cathode current \( i \). The discharge current \( i = eVR_{\text{amb}} \) is a product of the magnitude of the electron charge \( e \), discharge volume \( V = \pi R^2 L_{\text{conf}} \), and the ambipolar diffusion rate \( R_{\text{amb}} \). The ambipolar diffusion frequency \( \nu_{\text{amb}} = (\mu_0/R)^2 D_{\lambda^+} \). The ambipolar diffusion coefficient \( D_{\lambda^+} \) is given by \( D_{\lambda^+} = 2.405 \) the root of the Bessel function. Substituting \( R_{\text{amb}} \) and \( V \) in the discharge current expression, one gets

\[
i = e \mu_0^2 \pi D_{\lambda^+} L_{\text{conf}}.
\]

The electrons vanish in axial direction, forming the anode current \( i = eVR_{\text{dif}} \). Making use of (2) the anode current takes form

\[
i = \pi R^2 L_0 \int_{U_{\text{conf}}}^\infty f(u)\,du.
\]

The electron Boltzmann equation (1) is treated as a second order differential equation. It is solved on a nonequidistant grid \( u_i, i = 0, \ldots, n \) from zero kinetic energy to energy \( U_{\text{conf}} \) slightly exceeding the beam energy. The boundary conditions used are (i) \( f(U_{\text{conf}}) = 0 \) and (ii) \( \sum_{i=0}^n f(u_i)\Delta u_i = n_e \). The second boundary condition comes from the discretization of the electron density \( n_e = \int_0^\infty f(u)\,du \) by presenting the integral as a sum. The electron density is calculated from Eq. (5a).

Equation (1) contains two unknown parameters: the ionization rate by beam electrons \( R_{\text{beam}} \) and the axial confining potential \( U_{\text{conf}} \). \( R_{\text{beam}} \) is promptly calculated from Eq. (3), keeping in mind that the right-hand side of (3), \( R_{\text{dif}} = i/eV \) is known. The confining potential plays an important role in the discharge by limiting the number of electrons, which go to the anode. That is why \( U_{\text{conf}} \) is uniquely determined by the anode current and it is calculated from Eq. (5b). Equation (1) also contains nonlinear terms, resulting from the Coulomb collisions, and must be treated iteratively. Another reason to treat Eq. (1) iteratively is the aforementioned parameters \( R_{\text{beam}} \) and \( U_{\text{conf}} \). On each iteration \( P(u), Q(u), \ln(L), R_{\text{beam}}, U_{\text{conf}} \) and \( n_e \) must be repeatedly updated until convergence is reached.

### IV. RESULTS AND DISCUSSIONS

The transition from conventional HCD to a micro HCD can be achieved by modeling the plasma parameters vs gas pressure. To keep the task simple, all other “external” parameters or combination of parameters have been kept constant. The investigation is performed for discharge current \( i = 10 \) mA and voltage \( 250 \) V \( (u_{\text{beam}} = 250 \) eV). The length-to-radius ratio has been kept \( L/R = 5:1 \). It is well known that the parameter \( pR \) in a HCD is of order of one; otherwise either the beam electrons cannot reach the discharge axis or a substantial fraction of them parish in the opposite cathode fall and cannot sustain the discharge. Throughout the calculations the parameter \( pR \) has been kept \( pR = 1 \) Torr cm. The modeling of the conventional HCD \( (p = 1 \) Torr), micro HCD \( (p = 10 \) Torr), and the transition between them has been done by varying the gas pressure from 1 Torr to 100 Torr. The tube radius has been correspondingly reduced from 1 cm (conventional HCD) to \( 10^{-2} \) cm (micro HCD).

The EEDF for \( p = 1, 10 \) and 100 Torr \( (R = 1, 0.1, \) and 0.01 cm, respectively) is shown in Fig. 2, where we plot \( \tilde{f}(u) = u^{-1/2}f(u)/n_e \) in a linear-log plot. In a conventional HCD, \( \tilde{f}(u) \) consists of a Maxwellian function for kinetic energies \( u \approx 5 \) eV, a plateau from \( u \approx 5 \) eV to \( u \approx 11 \) eV and a “tail,” which extends to kinetic energy \( u_{\text{beam}} \). The plateau is a result of secondary electrons created in ionization with...
fast electrons. For kinetic energies slightly exceeding the energy threshold for electron impact excitation of Ar (11.55 eV) the EEDF experiences a significant drop as a result of inelastic processes. For \( p = 10 \text{ Torr} \) the plateau is still appreciable, but the drop of the EEDF at 11.55 eV is moderate. For the highest pressure (micro HCD) neither the plateau, nor the drop of the EEDF near the first excitation threshold of Ar is evident; the Maxwellian part of the EEDF smoothly transforms into the tail of the EEDF.

The electron density, plotted in Fig. 3(a), increases as a square of the gas pressure. The reason for the electron density increase with gas pressure should be distinguished from that in a positive column. While in a positive column it is due to the change of the electron mobility, in a HCD it is due to the change of the electron density of the gas pressure can be derived, and for a constant discharge current the dependence of the electron density on the gas pressure has an important consequence: the degree of ionization in the conventional HCD is

\[
N_e \sim \frac{5}{3} \text{ Torr cm} \quad \text{and} \quad L = 5R \text{ constant, it follows that} \quad L \sim p^{-1}.
\]

From Eq. (5a) one gets \( n_e \sim (i D_{Ar}) L \sim (i p^{-2}) \) and for a constant discharge current the dependence of the electron density on the gas pressure can be derived, \( n_e \sim p^{-2} \). The quadratic increase of the electron density with gas pressure has an important consequence: the degree of ionization \( n_e/N \) increases linearly with the gas pressure. For the example presented the degree of ionization in the conventional HCD is \( n_e/N \sim 6 \times 10^{-5} \), while for the micro HCD it is \( n_e/N \sim 5 \times 10^{-3} \).

To derive the electron density, we assumed that \( \text{Ar}^+ \) is the only ion species. We neglected the molecular ion \( \text{Ar}_2^+ \) and we can justify this assumption by estimating the ratio of the molecular ion density to the atomic ion density. Consider the creation and loss of \( \text{Ar}_2^+ \) through the reactions \( \text{Ar}^+ + 2\text{Ar} \rightarrow \text{Ar}_2^+ + \text{Ar} \) and \( \text{Ar}_2^+ + e \rightarrow \text{Ar}^+ + \text{Ar} \), respectively. The balance equation for \( \text{Ar}_2^+ \) reads \( k_{\text{ass}} N_{\text{Ar}}^2 N_{\text{Ar}_2^+} = k_{\text{dis}} n_e N_{\text{Ar}_2^+} \) and the ratio of molecular to atomic ion density is \( N_{\text{Ar}_2^+}/N_{\text{Ar}^+} = k_{\text{ass}} N_{\text{Ar}}^2 / k_{\text{dis}} n_e \). \( k_{\text{ass}} = 2 \times 10^{-31} \text{ cm}^6 \text{ s}^{-1} \) is the rate coefficient for the formation of \( \text{Ar}_2^+ \), \( k_{\text{dis}} = 10^{-7} \text{ cm}^3 \text{ s}^{-1} \) is the dissociative recombination coefficient, and \( N = 3.2 \times 10^{16} \text{ cm}^{-3} \) is the atomic density. Taking the electron density \( n_e = 2 \times 10^{12} \text{ cm}^{-3} \) [Fig. 3(a)] one gets \( N_{\text{Ar}_2^+}/N_{\text{Ar}^+} \approx 10^{-3} \). Note that both the nominator and denominator are proportional to \( p^2 \) and this ratio holds regardless of the gas pressure.

The mean electron energy \( \bar{u} = \int_0^\infty u f(u) du / \int_0^\infty f(u) du \) is plotted in Fig. 3(b). At low gas pressure it is \( \bar{u} \approx 0.5 \text{ eV} \) and gradually increases to \( \bar{u} \approx 0.6 \text{ eV} \). The confining potential is \( U_{\text{conf}} \sim 2.3 \text{ eV} \) in a conventional HCD and \( \sim 20\% \) larger in a micro HCD. In both conventional and micro HCD the ratio of the confining potential to the electron temperature is about the same, \( U_{\text{conf}}/T_e \approx 6 \). Our result is in good agreement with other theoretical considerations for conventional HCD. 17 The rates of electron impact excitation, ionization, and free diffusion are given in the next figure (Fig. 4). All rates exhibit the same behavior; they increase as \( p^3 \) with the gas pressure.

As expected, the power per unit volume also increases as \( p^3 \) [Fig. 5(a)]. The fractional power losses \( (P_{\text{term}}/P_{\text{beam}})_{\alpha} = \text{el, exc, ion, dif from Eq. (4)} \) are plotted in the figure below.

\[
V \sim p^{-3}.
\]

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Most of the power loss occurs due to electron impact ionization with secondary electrons (~59%). The power loss due to electron impact excitation is 18%–21%, due to diffusion, 12%–15% and due to elastic scattering, 5%–8%. The fractional power losses depend on the gas pressure since the EEDF changes with the pressure. Hence the power loss in a micro HCD is distributed the same way as in a conventional HCD, but the power loss per unit volume is much larger. The energy per electron–ion pair $W_{ei}$ is a fundamental characteristic of any e-beam sustained plasma. We calculated value for $W_{ei}$ was 25.8 eV, regardless of the gas pressure.

The voltage–current ($V–I$) characteristic is another unique feature of the HCD. The voltage is low and depends weakly on the discharge current. In most cases it increases with the current, but negative differential conductivity is often observed. The $V–I$ characteristics are much more sensitive with respect to $pD$ ($D$, cathode diameter). For argon at low $pD$, of order of 0.2 Torr cm, the voltage is 400–600 V, for $pD$ ~ 1 Torr cm it is 300–450 V, and for $pD$ close to the upper limit (4 Torr cm), the voltage is rather low, 220–280 V. Experiments confirmed that the $V–I$ characteristics of conventional and micro HCD’s have a lot of similarities: low voltage and presence of negative differential conductivity. As an example, the $V–I$ characteristics are illustrated in Fig. 6. For both conventional and micro HCD the voltage is 180–250 V.

V. CONCLUSION

The conventional HCD, micro HCD, and the transition between them have been investigated by solving the time independent spatially averaged electron Boltzmann equation. The EEDF, electron density, mean electron energy, excitation, and ionization rates were calculated at different gas pressures, keeping the discharge current and the parameters $pR$ and $L/R$ constant. The direct comparison showed that the low energy part of the EEDF in micro HCD is somewhat different compared to conventional HCD. Drastic differences between conventional and micro HCD’s are observed for power deposition, excitation, and ionization rates. The power per unit volume, excitation and ionization rates increase as a cube of the gas pressure, the electron density—as a square and the degree of ionization—linearly with the gas pressure. The comparison showed that the conventional and micro HCDs have different properties, except for the $V–A$ characteristics, EEDF, and the energy per electron–ion pair.