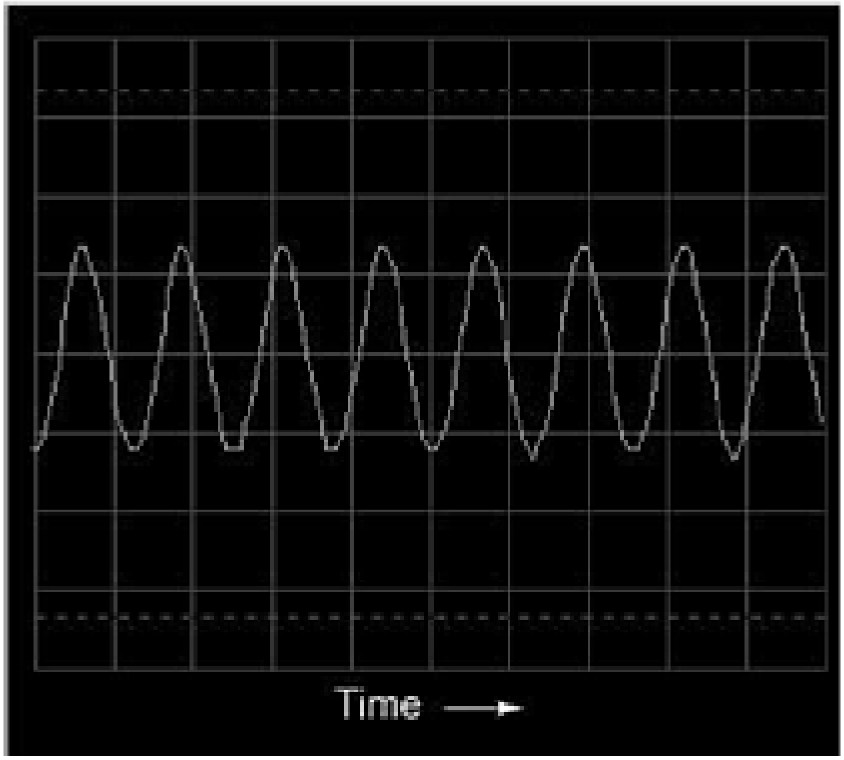
More on Spectrum Analysis

Chapter 7 - Mixed-Frequency AC Signals

Computerized Fourier analysis, particularly in the form of the *FFT* algorithm, is a powerful tool for furthering our understanding of waveforms (https://www.allaboutcircuits.com/textbook/alternating-current/chpt-1/ac-waveforms/) and their related spectral components. This same mathematical routine programmed into the SPICE simulator as the `.fourier` option is also programmed into a variety of electronic test instruments to perform real-time Fourier analysis on measured signals. This section is devoted to the use of such tools and the analysis of several different waveforms.

First we have a simple sine wave at a frequency of 523.25 Hz. This particular frequency value is a “C” pitch on a piano keyboard, one octave above “middle C”. Actually, the signal measured for this demonstration was created by an electronic keyboard set to produce the tone of a panflute, the closest instrument “voice” I could find resembling a perfect sine wave. The plot below was taken from an oscilloscope display, showing signal amplitude (voltage) over time: (Figure below)

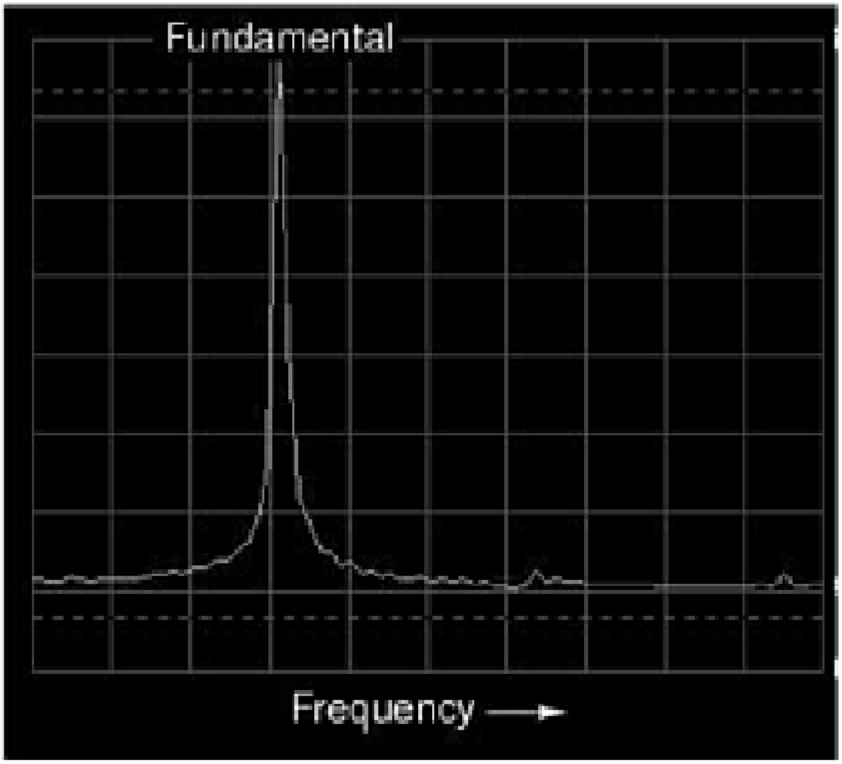


Oscilloscope display: voltage vs time.

Viewed with an oscilloscope (https://www.allaboutcircuits.com/textbook/experiments/chpt-4/pc-oscilloscope/), a sine wave looks like a wavy curve traced horizontally on the screen. The horizontal axis of this oscilloscope display is marked with the word “Time” and an arrow pointing in the direction of time’s progression. The curve itself, of course, represents the cyclic increase and decrease of voltage over time.

Close observation reveals imperfections in the sine-wave shape. This, unfortunately, is a result of the specific equipment used to analyze the waveform. Characteristics like these due to quirks of the test equipment are technically known as *artifacts*: phenomena existing solely because of a peculiarity in the equipment used to perform the experiment.

If we view this same AC voltage on a spectrum analyzer, the result is quite different: (Figure below)



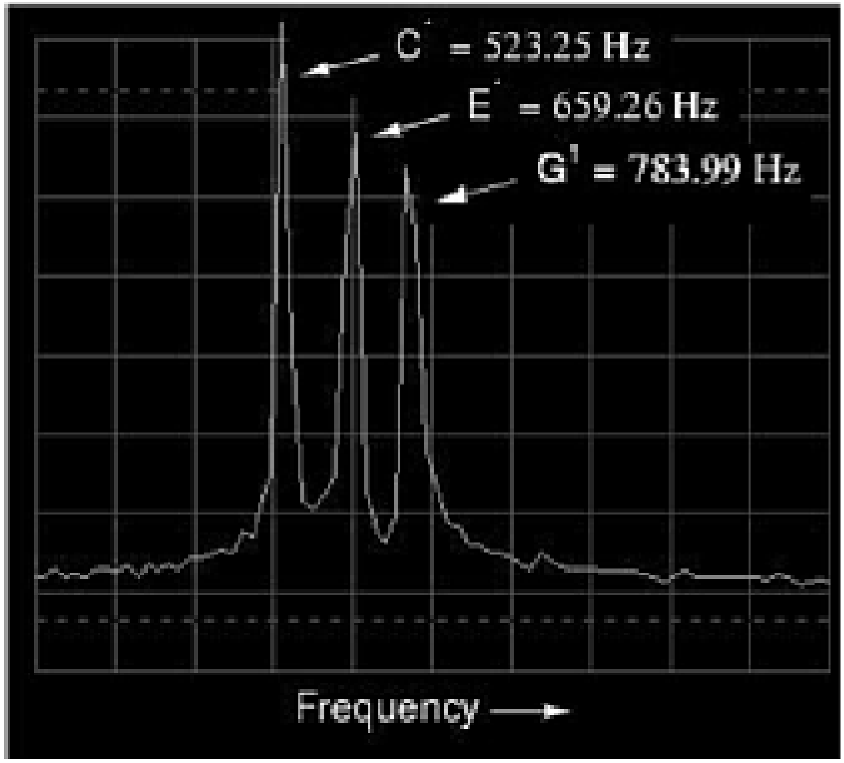
Spectrum analyzer display: voltage vs frequency.

As you can see, the horizontal axis of the display is marked with the word “Frequency,” denoting the domain of this measurement. The single peak on the curve represents the predominance of a single frequency within the range of frequencies covered by the width of the display. If the scale of this analyzer instrument were marked with numbers, you would see that this peak occurs at 523.25 Hz. The height of the peak represents the signal amplitude (voltage).

If we mix three different sine-wave tones together on the electronic keyboard (C-E-G, a C-major chord) and measure the result, both the oscilloscope display and the spectrum analyzer display reflect this increased complexity: (Figure below)

Oscilloscape display: three tones.

The oscilloscope display (time-domain) shows a waveform with many more peaks and valleys than before, a direct result of the mixing of these three frequencies. As you will notice, some of these peaks are higher than the peaks of the original single-pitch waveform, while others are lower. This is a result of the three different waveforms alternately reinforcing and canceling each other as their respective phase shifts change in time.



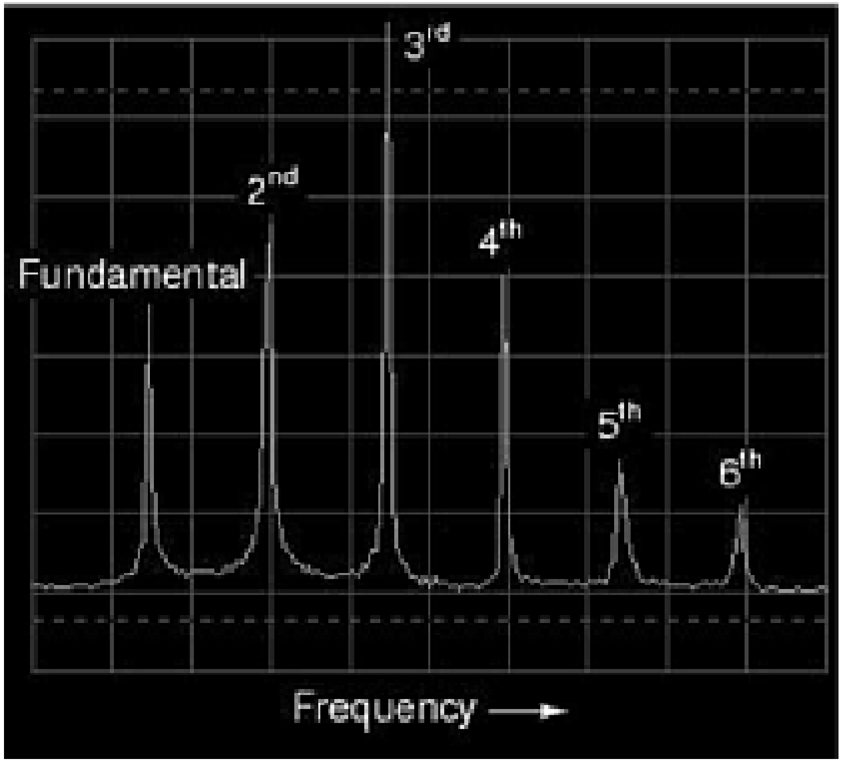
Spectrum analyzer display: three tones.

The spectrum display (frequency-domain) is much easier to interpret: each pitch is represented by its own peak on the curve. (Figure above) The difference in height between these three peaks is another artifact of the test equipment: a consequence of limitations within the equipment used to generate and analyze these waveforms, and not a necessary characteristic of the musical chord itself.

As was stated before, the device used to generate these waveforms is an electronic keyboard: a musical instrument designed to mimic the tones of many different instruments. The panflute "voice" was chosen for the first demonstrations because it most closely resembled a pure sine wave (a single frequency on the spectrum analyzer display). Other musical instrument "voices" are not as simple as this one, though. In fact, the unique tone produced by *any* instrument is a function of its waveshape (or spectrum of frequencies). For example, let’s view the signal for a trumpet tone: (Figure below)

Oscilloscope display: waveshape of a trumpet tone.

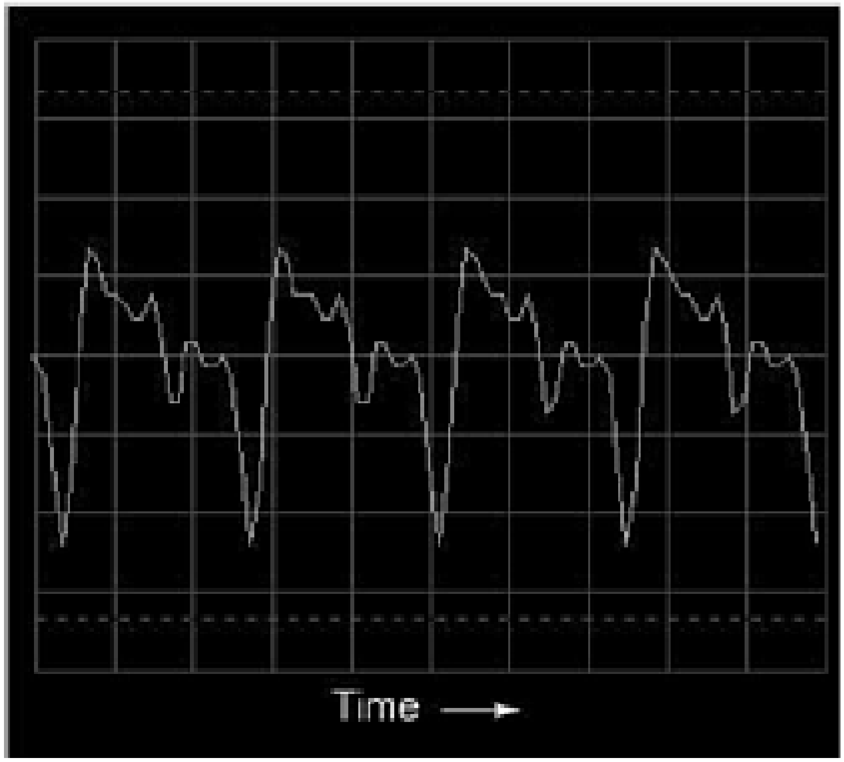
The fundamental frequency of this tone is the same as in the first panflute example: 523.25 Hz, one octave above “middle C.” The waveform itself is far from a pure and simple sine-wave form. Knowing that any repeating, non-sinusoidal waveform is equivalent to a series of sinusoidal waveforms at different amplitudes and frequencies, we should expect to see multiple peaks on the spectrum analyzer display: (Figure below)



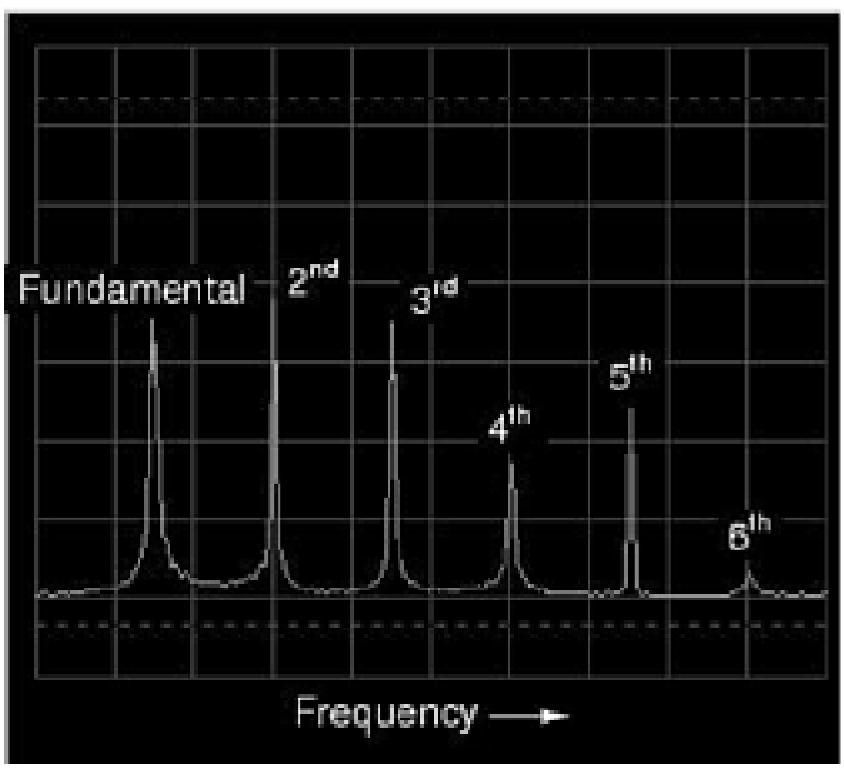
Spectrum of a trumpet tone.

Indeed we do! The fundamental frequency component of 523.25 Hz is represented by the left-most peak, with each successive harmonic represented as its own peak along the width of the analyzer screen. The second harmonic is twice the frequency of the fundamental (1046.5 Hz), the third harmonic three times the fundamental (1569.75 Hz), and so on. This display only shows the first six harmonics, but there are many more comprising this complex tone.

Trying a different instrument voice (the accordion) on the keyboard, we obtain a similarly complex oscilloscope (time-domain) plot (Figure below) and spectrum analyzer (frequency-domain) display: (Figure below)



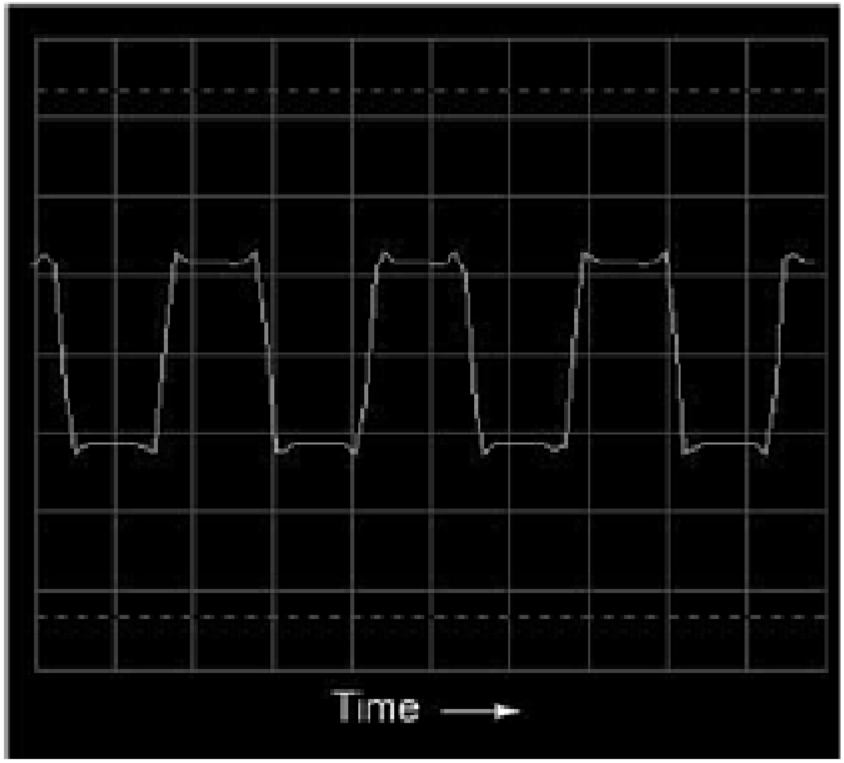
Oscilloscope display: waveshape of accordion tone.



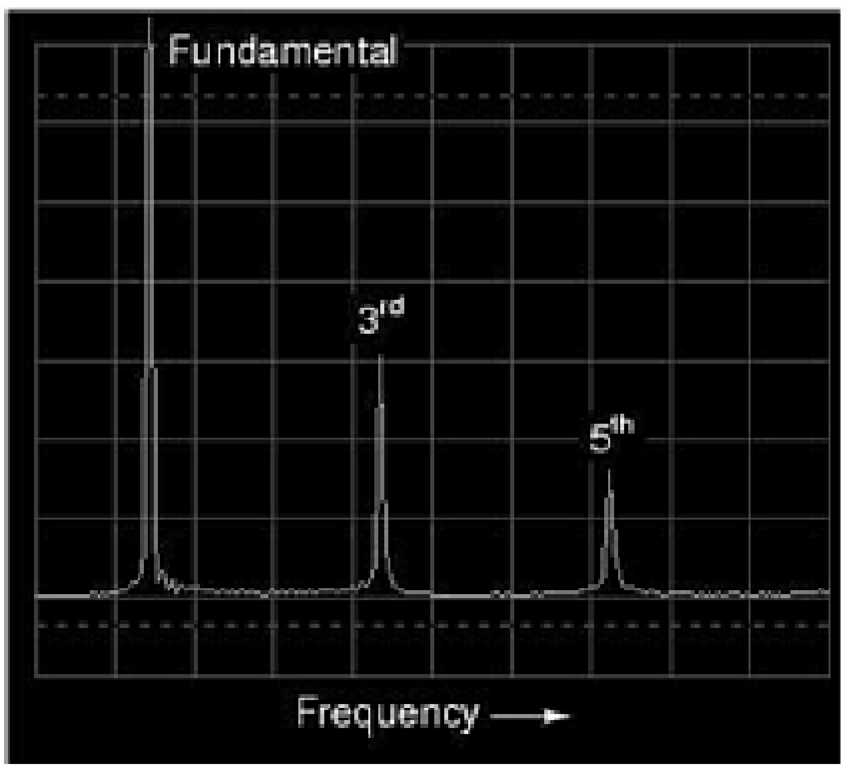
Spectrum of accordion tone.

Note the differences in relative harmonic amplitudes (peak heights) on the spectrum displays for trumpet and accordion. Both instrument tones contain harmonics all the way from 1st (fundamental) to 6th (and beyond!), but the proportions aren't the same. Each instrument has a unique harmonic "signature" to its tone. Bear in mind that all this complexity is in reference to *a single note* played with these two instrument "voices." Multiple notes played on an accordion, for example, would create a much more complex mixture of frequencies than what is seen here.

The analytical power of the oscilloscope and spectrum analyzer permit us to derive general rules about waveforms and their harmonic spectra from real waveform examples. We already know that any deviation from a pure sine-wave results in the equivalent of a mixture of multiple sine-wave waveforms at different amplitudes and frequencies. However, close observation allows us to be more specific than this. Note, for example, the time- (Figure below) and frequency-domain (Figure below) plots for a waveform approximating a square wave:

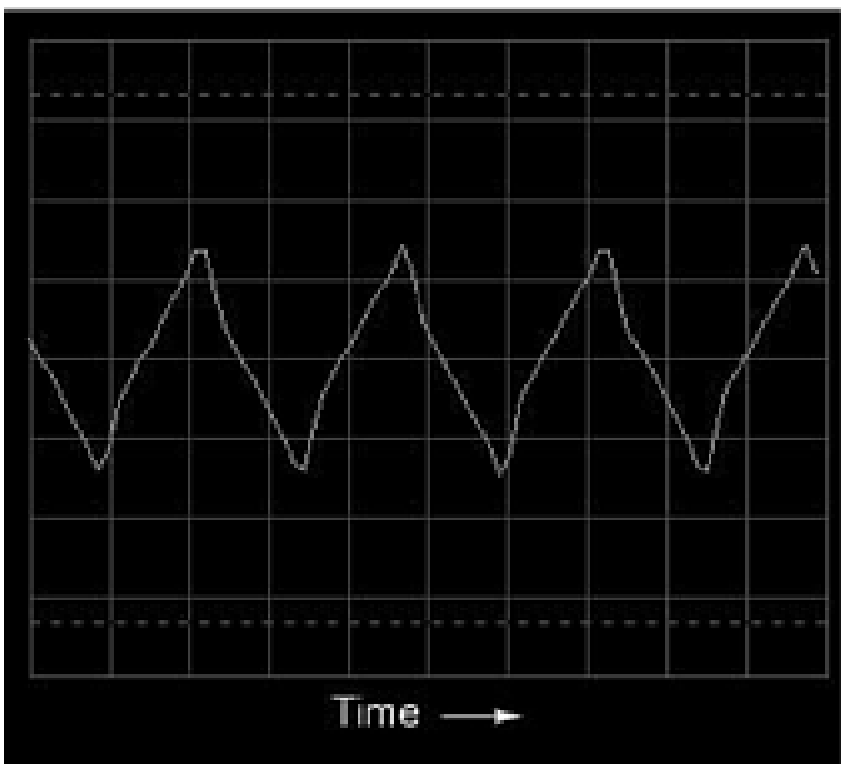


Oscilloscope time-domain display of a square wave

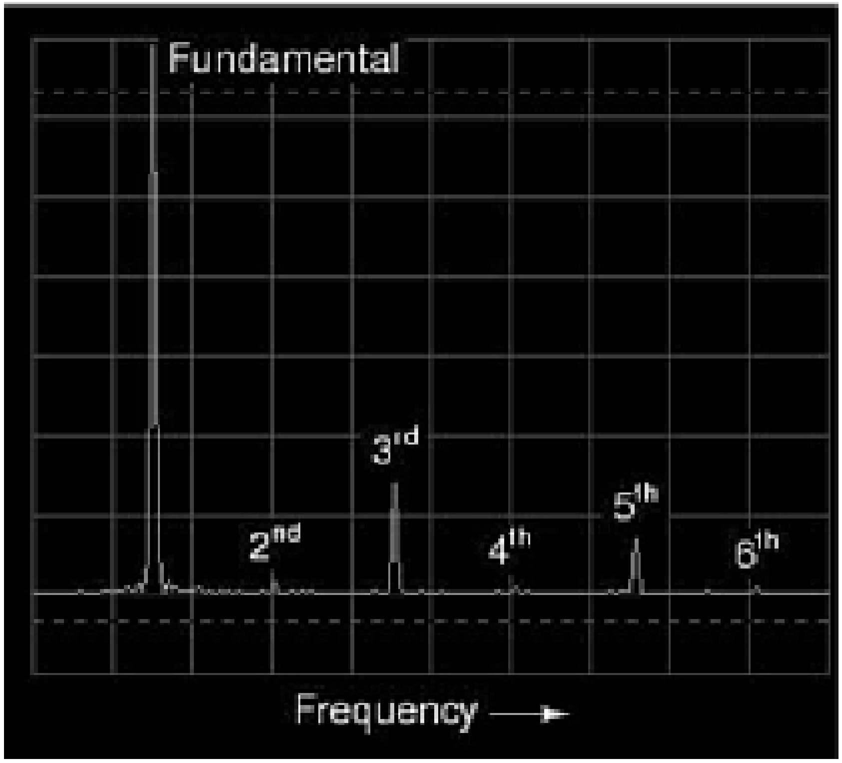


Spectrum (frequency-domain) of a square wave.

According to the spectrum analysis, this waveform contains *no* even harmonics, only odd. Although this display doesn't show frequencies past the sixth harmonic, the pattern of odd-only harmonics in descending amplitude continues indefinitely. This should come as no surprise, as we've already seen with SPICE that a square wave is comprised of an infinitude of odd harmonics. The trumpet and accordion tones, however, contained *both* even and odd harmonics. This difference in harmonic content is noteworthy. Let's continue our investigation with an analysis of a triangle wave: (Figure below)



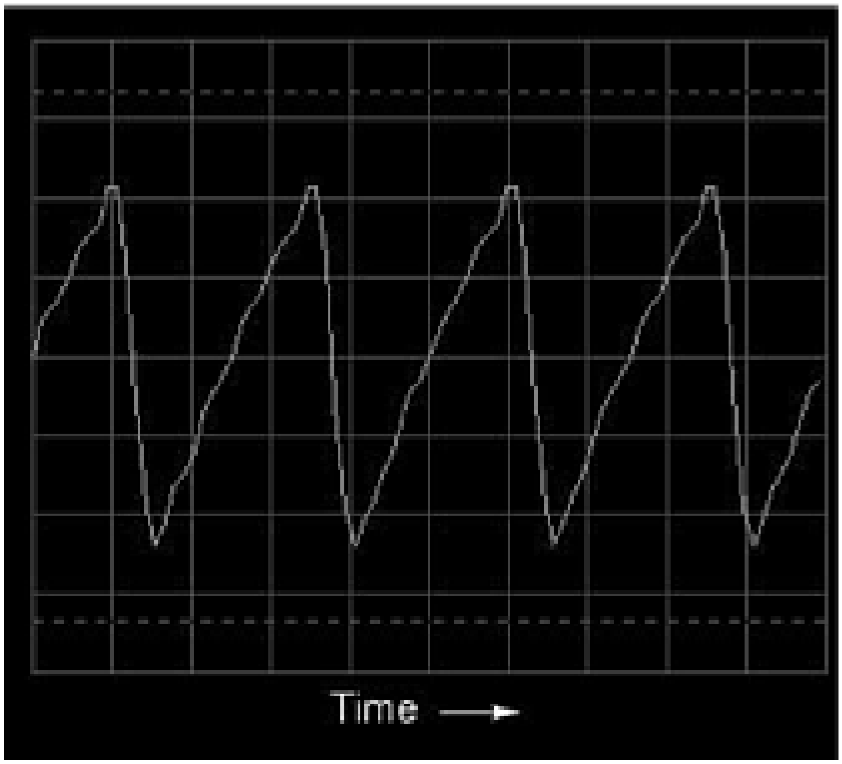
Oscilloscope time-domain display of a triangle wave.



Spectrum of a triangle wave.

In this waveform there are practically no even harmonics: (Figure above) the only significant frequency peaks on the spectrum analyzer display belong to odd-numbered multiples of the fundamental frequency. Tiny peaks can be seen for the second, fourth, and sixth harmonics, but this is due to imperfections in this particular triangle waveshape (once again, artifacts of the test equipment used in this analysis). A perfect triangle waveshape produces no even harmonics, just like a perfect square wave. It should be obvious from inspection that the harmonic spectrum of the triangle wave is not identical to the spectrum of the square wave: the respective harmonic peaks are of different heights. However, the two different waveforms are common in their lack of even harmonics.

Let’s examine another waveform, this one very similar to the triangle wave, except that its rise-time is not the same as its fall-time. Known as a *sawtooth wave*, its oscilloscope plot reveals it to be aptly named: (Figure below)



Time-domain display of a sawtooth wave.

When the spectrum analysis of this waveform is plotted, we see a result that is quite different from that of the regular triangle wave, for this analysis shows the strong presence of even-numbered harmonics (second and fourth): (Figure below)