RATIONAL EXPRESSIONS

Upon completion, you should be able to

- Simplify rational expressions
- Perform addition, subtraction, multiplication, and division of rational expressions
- Simplify complex fractions

RATIONAL EXPRESSIONS

A rational expression is the ratio of polynomials.

Thus if a and b are polynomials, a rational expression is of the form

$\frac{a}{b}$, where $b \neq 0$.

a is called the numerator, while

b is called the denominator.

RATIONAL EXPRESSIONS

The following are examples of rational expressions:

1) $\frac{1}{2}$ 2) $\frac{2}{x^2}$ 3) $\frac{x^2 - 2x + 3}{x^3 - x - 5}$ 4) $\frac{x^5 - 1}{x^2 - 2x}$

The following are NOT rational expressions:

1) $\frac{2}{x^{\frac{1}{2}}}$ 2) $\frac{\sqrt{x} - 1}{\sqrt{x} + 1}$

Simplifying Rational Expressions

Use the rule of cancellation

$$\frac{ac}{bc} = \frac{a}{b} \quad \text{provided } \mathbf{c} \neq 0$$

Examples: Simplify the following.

1) $\frac{7x^2y^5}{21x^3y^2}$ 2) $\frac{6x^2 + 17x + 7}{12x^2 + 13x - 35}$

Simplifying Rational Expressions

Examples: Simplify the following.

3) $\frac{x^3 - 27}{2x^2 - x - 15}$

4) $\frac{y^2 - 16}{8 + 2y - y^2}$

A rational expression is said to be in lowest terms if the numerator and denominator have no common factor except 1 and –1.

Multiplying Rational Expressions

Use the rule: $\frac{A}{B} \bullet \frac{C}{D} = \frac{AC}{BD}$

In using the rule, see to it that you cancel common factors first before multiplying the numerators and the denominators.

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Multiplying Rational Expressions

Example: Find these products.

1) $\dfrac{5x^3}{7y^4} \bullet \dfrac{21y^2}{25x^2}$

2) $\dfrac{x^2 - x - 12}{5x - 5} \bullet \dfrac{3x - 3}{x^2 - 9}$

3) $\dfrac{6c - 9}{c^2 - 25} \bullet \dfrac{c^2 - 3c - 10}{12 - 4c}$

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Multiplying Rational Expressions

Example: Find these products.

4) $\dfrac{9x^2 - 16y^2}{3x^2 - 5xy - 12y^2} \bullet \dfrac{xy - 2x^2}{3xy - 4y^2} \bullet \dfrac{x - 3y}{2x - y}$

Remember: Sometimes, you have to factor out a –1 to be able to cancel factors and simplify.

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Dividing Rational Expressions

Use the rule:

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \bullet \frac{D}{C} = \frac{AD}{BC}$$

Remember: To divide two rational expressions, multiply to the first the reciprocal of the second expression.

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Dividing Rational Expressions

Example: Find the quotients and simplify:

1) $\dfrac{81xz^3}{36y} \div \dfrac{27x^2z^2}{12xy}$

2) $\dfrac{x^2 + 9x + 14}{x^2 + 4x - 21} \div \dfrac{x^2 - 3x - 10}{x^2 + 2x - 35}$

3) $\dfrac{a}{b - a} \bullet \dfrac{a^2 + b^2}{a + b} \div \dfrac{a^2 - b^2}{a^2 - 2ab + b^2}$

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Adding and Subtracting Rational Expressions

Similar rational expressions are those that have the same denominators.

The following are similar rational expressions:

1) $\dfrac{1}{2}, \dfrac{3}{2}$ 2) $\dfrac{1}{x}, \dfrac{x - 1}{x}$

3) $\dfrac{x}{(x-1)^2}, \dfrac{x + 2}{(x-1)^2}$

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Adding and Subtracting Rational Expressions

Two or more rational expressions can be made similar by getting their least common denominator (LCD).

For example: Find the LCD of

1) $\dfrac{1}{2}, \dfrac{2}{3}$ 2) $\dfrac{1}{x}, \dfrac{2}{x^2}$

Remember: The LCD is the expression with the smallest power that can be divided by the denominators exactly.

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Adding and Subtracting Rational Expressions

Remember: To find the LCD,

1. Factor each denominator completely.
2. Get all unique factors of the denominators.
3. Get the highest power of each factor appearing in the denominators and multiply them.

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Adding and Subtracting Rational Expressions

To make two rational expressions similar, we multiply both numerator and denominator of each expression by the factor that will make the denominators equal to the LCD.

Example: Convert to similar rational expressions.

$$1)\ \frac{1}{2}, \frac{2}{3} \qquad 2)\ \frac{1}{x}, \frac{2}{x^2}$$

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Adding and Subtracting Rational Expressions

Example: Convert to similar rational expressions.

$$3)\ \frac{x-2y}{x+2y}, \frac{2x-y}{2x+y}$$

$$4)\ \frac{a-1}{2a^2-18}, \frac{a+2}{9a-3a^2}$$

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Adding and Subtracting Rational Expressions

To add(or subtract) two or more rational expressions, convert them to similar rational expressions then add(subtract) the numerators. The denominator of the sum(difference) is the LCD. In symbols,

$$\frac{a}{b} + \frac{c}{b} = \frac{a}{b}$$

or

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

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Adding and Subtracting Rational Expressions

Examples: Find the sum or difference, then simplify the result.

$$1)\ \frac{1}{2} + \frac{2}{3}$$

$$2)\ \frac{1}{x} - \frac{2}{x^2}$$

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Adding and Subtracting Rational Expressions

Examples: Find the sum or difference, then simplify the result.

$$3)\ \frac{x-2y}{x+2y} - \frac{2x-y}{2x+y}$$

$$4)\ \frac{a-1}{2a^2-18} + \frac{a+2}{9a-3a^2}$$

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Adding and Subtracting Rational Expressions

Examples: Find the sum or difference, then simplify the result.

$$5)\ \frac{4t^2}{s^2 - t^2} - \frac{t-s}{t+s} + \frac{s+t}{t-s}$$

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Complex Fractions

A complex fraction is the ratio of two or more rational expressions. To simplify a complex fraction, locate the main division bar and treat the problem as a division problem.

Example: Simplify.

$$1)\ \frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x} + \frac{1}{y}}$$

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Complex Fractions

Example: Simplify.

$$2)\ \frac{\frac{1+x}{1-x} - \frac{1-x}{1+x}}{\frac{1}{1+x} - \frac{1}{1-x}}$$

$$3)\ 1 + \frac{1}{1 - \frac{1}{1 - \frac{1}{x}}}$$

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Summary

In this section, we learned

- How to simplify a rational expression $\frac{ac}{bc} = \frac{a}{c}$
- How to multiply and divide rational expressions

$$\frac{A}{B} \bullet \frac{C}{D} = \frac{AC}{BD} \qquad \frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \bullet \frac{D}{C} = \frac{AD}{BC}$$

- How to add(subtract) rational expressions

$$\frac{a}{b} + \frac{c}{b} = \frac{a}{b} \qquad \frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

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Summary

In this section, we learned

- That we should not divide by zero
- That we should not cancel terms
- That factoring a −1 is sometimes needed to simplify a rational expression
- That in simplifying complex fractions, we should identify the main division bar and consider the problem as a division problem.