



Special Products and Factoring

Upon completion, you should be able to

- Find special products
- Factor a polynomial completely



Special Products

- rules for finding products in a faster way
- long multiplication is the last resort



Product of two binomials

$$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$$

Examples:

1. $(x + 3)(x + 5)$
2. $(2x + 3)(x - 5)$
3. $(3w - 5v)(2w + 3v)$



Square of a binomial

$$(A \pm B)^2 = A^2 \pm 2AB + B^2$$

Examples:

1. $(3x + 5y)^2$
2. $(2x - 3y^2)^2$
3. $(a^3w^2 - bu^3)^2$



Product of Sum and Difference

$$(A + B)(A - B) = A^2 - B^2$$

Examples:

1. $(3x + 5y)(3x - 5y)$
2. $(2x - 3y^2)(2x + 3y^2)$



Cube of a binomial

$$(A \pm B)^3 = A^3 \pm 3A^2B + 3AB^2 \pm B^3$$

Examples:

1. $(x + y^2)^3$
2. $(2x - 3y)^3$



Product of a binomial and a trinomial

$$(A \pm B)(A^2 \mp AB + B^2) = A^3 \pm B^3$$

Examples:

$$1. (x + y^2)(x^2 - xy^2 + y^4)$$

$$2. (2x - 3y)(4x^2 + 6xy + 9y^2)$$



Square of a trinomial

$$(A + B + C)^2 = A^2 + B^2 + C^2 + 2AB + 2AC + 2BC$$

Examples:

$$1. (x + y^2 + w)^2$$

$$2. (2x - 3y + z)^2$$



Factoring

-process of writing an expression as product of its factors

-reverse process of finding products



Factoring

Example: Factor 124.

Example: Factor 124 completely.

For numbers, **factoring completely** is the same as finding the prime decomposition of a number.



Factoring

A polynomial with integer coefficients is said to be **prime** if it has no monomial or polynomial factors with integer coefficients other than itself and one.

A polynomial with integer coefficients is said to be **factored completely** when each of its polynomial factor is prime.



Factoring a Common Monomial Factor

$$ab + ac = a(b + c)$$

Examples:

$$1) 8x^2 + 4x$$

$$2) x^5 - 3x^4 + x^3$$



Difference of Two Squares

$$A^2 - B^2 = (A - B)(A + B)$$

Examples:

- 1) $16 - y^2$
- 2) $4u^2 - 9v^2$
- 3) $(a + 2b)^2 - (3b + c)^2$



Factoring Trinomials

$$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$$

Examples:

- 1) $x^2 + 7x + 10$
- 2) $a^2 - 10a + 24$
- 3) $a^2 + 4ab - 21b^2$
- 4) $20x^2 + 43xy + 14y^2$



Perfect Square Trinomials

$$A^2 \pm 2AB + B^2 = (A \pm B)^2$$

Examples:

- 1) $y^2 - 10y + 25$
- 2) $16x^2 + 8x + 1$
- 3) $9x^2 - 30xy + 25y^2$
- 4) $24a^2 + 72a^2b + 54ab^2$



Sum and Difference of Cubes

$$A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$$

Examples:

- 1) $27 - x^3$
- 2) $t^3 + 8$
- 3) $y^6 - 2y^3 + 1$
- 4) $x^6 - 2x^3y^3 + y^6$



Factoring by Grouping

Examples:

- 1) $10a^3 + 25a - 4a^2 - 10$
- 2) $3xy - yz + 3xw - zw$
- 3) $a^3 + 27b^3 + a + 3b$



Special Products and Factoring Summary

Factors	Products
$(ax+b)(cx+d)$	$acx^2 + (ad + bc)x + bd$
$(A \pm B)^2$	$A^2 \pm 2AB + B^2$
$(A+B)(A-B)$	$A^2 - B^2$
$(A \pm B)(A^2 \mp AB + B^2)$	$A^3 \pm B^3$
$(A \pm B)^3$	$A^3 \pm 3A^2B + 3AB^2 \pm B^3$
$(A + B + C)^2$	$A^2 + B^2 + C^2 + 2AB + 2AC + 2BC$



Special Products and Factoring Summary

- Always look for a common monomial factor, FIRST.
- Factoring can also be done by grouping some terms to yield a common polynomial factor.