

## Operations on Polynomials

Learning Objectives: Upon completion you should be able to perform the following operations on polynomials:

- Addition
- Subtraction
- Multiplication
- Division

## Operations on Polynomials

### Polynomial

-an algebraic expression involving only **nonnegative-integral powers** of one or more variables and containing **no variable in the denominator**

## Operations on Polynomials

### Polynomial

Which of the following is a polynomial?

1.  $x^2 - 3y^2$
2.  $\sqrt{x} - 3y^3$
3.  $\frac{x-y}{x+y}$
4.  $x^{-2} + y^{-3}$
5.  $x^3 + 2x^2$
6.  $\frac{x^2}{3} + \frac{y^4}{7}$

## Operations on Polynomials

### Term of a polynomial

-a constant or a constant multiplied by nonnegative-integral powers of variables

Example: Name the terms of these polynomials:

1.  $x^2 + 2x + 1$
2.  $x^3 - y^3$

## Operations on Polynomials

Types of polynomials according to the number of terms:

- Monomial** –one term
- Binomial** – two terms
- Trinomial** – three terms
- Multinomial** – more than three terms

## Operations on Polynomials

Identify each polynomial according to the number of terms:

1.  $x^2y^3z^4$
2.  $x^8 - y^4$
3.  $y^3 - 3y^2 + 7$
4.  $x^4 + 4x^3y - 6x^2y^2 + 4xy^3 - y^4$

## Operations on Polynomials

### Degree of a term

-sum of the powers of all variables in the term

Ex. Find the degree of each term in the polynomial:

$$x^5 + x^4y + x^3y^3 - 3x^2 + 12y^7$$

## Operations on Polynomials

### Degree of a polynomial

-degree of the term with the highest degree in the polynomial

Ex. Find the degree of the polynomial:

$$x^5 + x^4y + x^3y^3 - 3x^2 + 12y^7$$

## Operations on Polynomials

Identify the degree of each polynomial:

1.  $x^2y^3z^4$
2.  $x^8 - y^4$
3.  $y^3 - 3y^2 + 7$
4.  $x^4 + 4x^3y - 6x^2y^2 + 4xy^3 - y^4$

## Operations on Polynomials

Polynomials are also classified according to degrees:

- Constant – degree 0  $5$
- Linear – degree 1  $x + y$
- Quadratic – degree 2  $xy + x$
- Cubic – degree 3  $3 + x + 3x^2 + x^3$

ETC.

## Operations on Polynomials

A term is composed of

- Numerical coefficient
- Literal coefficient

Example: Identify the numerical and literal coefficients.

$$3xy^2 + 7xy^3 - 12x^2y^2 + x^2y^2$$

## Operations on Polynomials

Similar terms

-Terms that have the same literal coefficient

Example: Identify similar terms.

$$3xy^2 + 7xy^3 - 12x^2y^2 + x^2y^2$$

### Operations on Polynomials

#### Addition of polynomials

RULE: Add only similar terms

Example: Simplify by adding similar terms.

$$3x^2y^2 + 7xy^3 - 12x^2y^2 + xy^3$$

### Operations on Polynomials

#### Addition of polynomials :Column Method

RULE: Arrange similar terms in column, then add.

Example: Add the polynomials below.

$$4x^3 - 7x^2 + 2x - 4$$

$$3x^3 + 8x^2 + 3x - 7$$

### Operations on Polynomials

#### Subtraction of Polynomials

RULE:  $a - b = a + (-b)$

Example: Subtract the polynomials below.

$$4x^3 - 7x^2 + 2x - 4$$

$$3x^3 + 8x^2 + 3x - 7$$

### Operations on Polynomials

#### Multiplication of Polynomials

RULE:  $a(b+c) = ab + ac$

Example: Multiply the polynomials below.

$$4x^3 - 7x^2 + 2x - 4$$

$$3x^3 + 8x^2 + 3x - 7$$

### Operations on Polynomials

#### Division of Polynomials

RULE:  $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

Example: Perform the indicated division.

$$\frac{a^3 - 3a^2 - a + 3}{a^2 - 2}$$

### Operations on Polynomials

#### Division of Polynomials : Synthetic Division

Example: Perform the indicated division using synthetic division.

$$\frac{a^3 - 3a^2 - a + 3}{a - 2}$$

## Operations on Polynomials

In this section you learned to perform the following operations on polynomials:

- Addition
- Subtraction
- Multiplication
- Division