Algebraic Expressions 1

Learning Objective:

Upon completion you should be able to perform operations involving integer exponents.

a^n a^{-n}

Algebraic Expressions 1

ALGEBRA – the language of mathematics

- uses symbols and numbers to represent quantities

- has definite rules for simplifying these symbols and numbers

Algebraic Expressions 1

Algebra is concerned with the following operations on numbers:

- Addition
- Subtraction
- Multiplication
- Division

Algebraic Expressions 1

Algebra is concerned with the following operations on numbers:

- Raising to a power (Exponentiation)
- Getting roots (Involution)

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Symbols can be a

- Variable- symbol representing a quantity but whose value is not known at the moment; usually latter letters of the English alphabet, e.g., u, v, w, x, y, z are used for variables

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Symbols can be a

- Constant-symbol that represents a specific value, e.g. 2, ½, 0.43, π

-may use also starting letters of the English alphabet such as a, b, c

in which case the value of each letter is fixed

Algebraic Expressions 1

Algebraic expression

-Combination of variables and constants which use any of the operations on numbers

Examples are:

$2xy$ $\quad 3x + y$ $\quad \frac{x+y}{z-2}$ $\quad \sqrt{x-y}$

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Integer Exponents

Shortcut for writing repeated products

$a^n = a \times a \times a \ldots \times a$

(**n** copies of **a**, provided n is a positive integer, **a** is the base, **n** is the exponent)

Ex: $3^4 = 3\times3\times3\times3 = 81$

$(-2)^3 = (-2)(-2)(-2) = -8$

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Integer Exponents

$$a^{-n} = \frac{1}{a^n}$$

Ex: $5^{-3} = \frac{1}{5^3} = \frac{1}{5\times5\times5} = \frac{1}{125}$

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Integer Exponents

$a^0 = 1, \quad$ provided $a \neq 0$

Ex:

$(\frac{3}{4})^0 = 1, \qquad (\sqrt[3]{54})^0 = 1$

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Rules on Integer Exponents

$$a^p a^q = a^{p+q}$$

Examples:

$2^5 2^3 = 2^{5+3} = 2^8$

$(-3)^{-2}(-3)^5 = (-3)^{-2+5} = (-3)^3$

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Rules on Integer Exponents

$$\frac{a^p}{a^q} = a^{p-q}$$

Example:

$\frac{2^5}{2^3} = 2^{5-3} = 2^2 = 4$

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Rules on Integer Exponents

$$\frac{a^p}{a^q} = a^{p-q}$$

Example:

$$\frac{7^3}{7^5} = 7^{3-5} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$$

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Rules on Integer Exponents

$$(ab)^p = a^p b^p$$

Example:

$$(2c)^5 = 2^5 c^5 = 32c^5$$

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Rules on Integer Exponents

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

Example:

$$\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} = \frac{32}{243}$$

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Rules on Integer Exponents

$$\left(a^p\right)^q = a^{pq}$$

Example:

$$(2s^3)^4 = 2^4(s^3)^4 = 16s^{3\bullet 4} = 16s^{12}$$

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Rules on Integer Exponents

More example: Simplify

$$\left(\frac{2a^{-2}b^2c^{-4}}{3a^{-3}b^{-1}c^2}\right)^{-2}$$

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In this section you learned the rules on integer exponents such as:

$a^p a^q = a^{p+q}$ $\quad (ab)^p = a^p b^p$

$\frac{a^p}{a^q} = a^{p-q}$ $\quad (\frac{a}{b})^p = \frac{a^p}{b^p}$

$(a^p)^q = a^{pq}$ $\quad a^{-n} = \frac{1}{a^n}$ $\quad a^0 = 1$