# Real Numbers

*"Numbers rule the universe."*

*- Pythagoras*

## Learning Objectives

**At the end of the lesson, you should be able to**

- **identify the subsets of integers**
- **recognize the various forms of rational numbers**
- **distinguish rational numbers from irrational numbers**
- **locate numbers on the real number line**

## Solve the puzzle, get an interview

**The Web search engine company Google made a very public display of its attempt to attract math enthusiasts.**

**It launched a billboard advertising campaign featuring a puzzle in front of the domain root ".com" – making the solution a Web address.**

{ first 10-digit prime found in consecutive digits of e }.com

**Candidates who made it to that page were asked to solve a harder second problem, which in turn guided them to yet another Web page that asked for their resume.**

The prime number they were asking for is

7427466391

## Sense of Touch

**The body has 450 touch cells per square inch of skin. We can detect a smooth plane of glass from one etched with lines only 1/2500 th of an inch deep. We can "feel" pressure on our fingertips or face that depresses the skin a microscopic 0.0004 inch.**

## Sense of Touch

We can tell the difference between a letter weighing **1¼** ounces and one weighing **1½** ounces, but not between **10** pounds and **10¼** pounds – the difference needs to be at least **2 percent**.

## The Real Number System

- evolved over time by expanding the notion of what we mean by the word "***number***."
- at first, "***number***" meant something you could count, like
  - how many children a father sired
  - how many legs an insect has

  These are called …

## NATURAL NUMBERS

***All natural numbers are truly natural. We find them in nature.***

The set of

**NATURAL NUMBERS**,

(also called

**COUNTING NUMBERS**)

is denoted by

N = {1, 2, 3, 4, 5, 6, 7, 8, 9,…}

## Subsets of Natural Numbers

- **Even Numbers** (2k: *multiples of* 2)
- **Odd Numbers (2k + 1, not even)**
- **Prime Numbers (divisible only by 1 and itself)**

  **{ 2, 3, 5, 7, 11, 13, 17, 19,..}**
- **Composite Numbers**

  **(not prime, not 1)**

  **{4, 6, 8, 9, 10,12,14,15,…}**

## The Number Zero

- **Is zero a number?**
- *How can the number of nothing be a number?*
- 0 is a rather ***special number*** because it does not quite obey the same laws as other numbers

  *( We can't divide by zero, for example.)*

## WHOLE NUMBERS

**W = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9…}**

***What is the only difference between this set and the set of natural (or counting) numbers?***

## NEGATIVE NUMBERS

- **Something less than nothing?**
- **Having less than 0 means that you have to add some to get it up to zero.**
- **Negative numbers – additive opposites of their positive versions**

## INTEGERS

**Z = {…,-4,-3,-2,-1,0,1,2,3,4,…}**

**This set adds on the negative counterparts to the already existing whole numbers.**

**Subsets of the Set of Integers**

**1. Negative integers {…-4, -3, -2, -1}**
**2. The set consisting of 0 alone : {0}**
**3. Positive integers : N= {1, 2, 3, 4, …}**

**Are there other subsets of Z?**

## RATIONAL NUMBERS

A **rational number** is a number that can be expressed as the **ratio** or **quotient** of two integers p and q where q ≠ 0. The set of rational numbers is denoted as

$$Q = \{\frac{p}{q} \mid p, q \in Z, q \neq 0\}$$

## Examples of Rational Numbers

a) $-\frac{1}{4} = -0.25$

b) $\frac{1}{2} = 0.5$

c) $\frac{11}{2} = 5.5$

d) $\frac{20}{5} = 4$

e) $\frac{2}{3} = 0.666...$

## Forms of Rational Numbers

- **Integers**
- **Fractions** (same root word as ***"fragment"*** and **"fracture"** suggesting breaking something up)
  - proper fraction
  - improper fraction
- **Decimals**
  - terminating
  - non-terminating but repeating decimals

## POSITIVE, NEGATIVE NUMBERS

- **Positive rational numbers**
- **Negative rational numbers**
- **Zero is neither positive nor negative**

## Positive Rational Numbers

Harry Guffee

http://www.songsforteaching.com/guffee/positiverationalnumbers.htm

Ladies and Gentlemen

The numbers that fall in between consecutive whole numbers include positive rational numbers.

Positive rational numbers can be written in any of three forms, any of three notations

as fractions

as decimals

as percents

Each form has its own special characteristics

The forms are interchangeable.

But the ways of doing standard operations with the three notations are very different.

FRACTIONS are used to name part of a whole object or part of a whole collection of objects or to compare two quantities.

A fraction is a number in the form a/b where a and b are whole numbers and b is not zero.

The fraction $\frac{2}{3}$

can be thought of as

**2 divided by 3**

so that when you divide a numerator by a denominator you can express that fraction as a **DECIMAL** equivalent.

*A* **PERCENT** is a fraction with 100 in the denominator

The word percent comes from the Latin word *"per centum"*

*"per"* meaning FOR

and *"centum"* meaning ONE HUNDRED

Ergo, 60% means 60 out of 100

A percent always represents a percent of something

And that something is the whole thing, or 100 %.

## Multiple Representations

- Fractions and mixed numbers
  – appear in recipes
- Decimals – occur in scientific measurements
- Percentages – used in commerce

# IRRATIONAL NUMBERS

– are those real numbers that can **not** be expressed as the ratio of two integers

–denote the set of irrational numbers as **Q**