Set Relations

Learning Objectives:

Upon completion you should be able to determine set relations such as

- equality
- subset and superset
- equivalence

Set Relations

Equal Sets

Two sets A and B are *equal* if and only if they have the same elements.

Example:

Let A = {a, e, i, o, u} and B = {e, i, o, a, u}.

Are all the elements in A also in B?

Are all the elements in B also in A?

YES! Therefore A and B have the same elements.

Sets A and B are ________.

Set Relations

Equal Sets

If sets A and B are equal, we write $A = B$.

Otherwise, we write $A \neq B$.

Example: Let C = {a, b, c, d} and D = {a, b, b, c, c, d}.

Are all the elements in C also in D?

Are all the elements in D also in A?

YES! Therefore C = D.

Set Relations

Equal Sets

REMEMBER:

1. IN A SET, AN ELEMENT MAY NOT BE LISTED MORE THAN ONCE.

2. IN A SET, THE ORDER OF LISTING THE ELEMENTS DOES NOT MATTER.

3. TWO SETS ARE EQUAL IF AND ONLY IF THEY HAVE THE SAME ELEMENTS.

Set Relations

Subset

If all elements of set A are also elements of set B, we say, A is a subset of B or B is a superset of A.

Example: Let A be the set of all students in this room.

Let B be the set of of all boys in this room.

Let C be the set of all girls in this room.

Is B a subset of A?

Is C a subset of A?

Set Relations

Subset

If A is a subset of B, we write $A \subseteq B$.

In our previous example,

A is the set of all students in this room.

B is the set of of all boys in this room.

C is the set of all girls in this room.

$B \subseteq A$ and $C \subseteq A$.

However, B is not a subset of C. Why?

In this case, we write $B \not\subseteq C$.

Set Relations

Subset

If A $\subseteq$ B and A $\neq$ B, then we call A a proper subset of B.

If A = {♣, ♠} and B = {♣,♦,♥,♠} then A is a proper subset of B and we write A $\subset$ B.

Set Relations

Subset

True or False: Let A, B, and C be sets.

1. A $\subseteq$ A.
2. If A $\subseteq$ B then B $\subseteq$ A.
3. If A $\subseteq$ B and B $\subseteq$ C then A $\subseteq$ C
4. $\phi \subseteq$ A
5. $\phi \subseteq \phi$
6. A $\subseteq$ U

Set Relations

Set Equivalence

What can you observe about the following pairs of sets?

A = {1, 2, 3, 4, 5} B = {a, e, i, o,u}

C= {guava, melon, avocado} D = {do, re, mi}

Two sets are in 1-1 correspondence if they have the same number of elements.

When two sets A and B are in 1-1 correspondence, we say they are equivalent and we write A ~ B.

Thus, in our example, A ~ B and C ~ D.

Is A ~ C? Why?

Set Relations

Set Equivalence

Time to think:

1. Are all equal sets equivalent?
2. Are all equivalent sets equal?
3. Can a set be equivalent to any of its subsets?
4. Can a set be equal to any of its subsets?

Set Relations

Summary

In this section, we learned

- When two sets are equal or not
- When a set is a subset or superset of another
- When two sets are equivalent or not