

GEOMETRY: THE LANGUAGE OF SIZE AND SHAPE

A Short History

- The word "geometry" comes from two Greek words "geo" and "metria" meaning "earth measuring."
- In c. 2000 - 500 B.C. Ancient Egyptians demonstrated a practical knowledge of geometry through surveying and construction projects. The Nile River overflowed its banks every year, and the river banks would have to be re-surveyed.

- Ancient clay tablets reveal that the Babylonians knew the Pythagorean relationships.
- In c. 750-250 B.C., Ancient Greeks practiced centuries of experimental geometry like Egypt and Babylonia had, and they absorbed the experimental geometry of both of those cultures.
- Then they created the first formal mathematics of any kind by organizing geometry with rules of logic.
- Around 300 BC, the geometry of Euclid was presented in its complete form in the book *The Elements* which formed the basis for most of the geometry studied in schools ever since.

- We could say that mathematics itself became an independent science with *The Elements*.
- (c. 400 B.C. - 1800 A. D.) When Euclid organized his thoughts on geometry, he chose four undefined terms (*point, line, plane, space*) assumed relations, and five axioms or postulates. From here he logically deduced theorems or properties of lines triangles, circles, etc.
- Postulates (also called axioms) are basic assumptions - rules that seem to be obvious and are therefore accepted without proof.

The Five Postulates of Euclid

- * The shortest distance between any two points is a straight line.
- * Any line segment can be extended infinitely in either direction.
- * A circle can be made with any center and any radius.
- * All right angles are equal to each other.
- ⊕ Given a point and a line not on the point, only one line can be drawn parallel to the given line through the given point

- Euclid was not satisfied with accepting the Fifth Postulate (also known as the Parallel Postulate) without proof. Many mathematicians throughout the next centuries unsuccessfully attempted to prove Euclid's Fifth.
- In c. 1600 A.D., Descartes made one of the greatest advances in geometry by connecting algebra and geometry. A myth is that he was watching a fly on the ceiling when he conceived of locating points on a plane with a pair of numbers. Maybe this has something to do with the fact that he stayed in bed everyday until 11:00 A.M.
- Fermat also discovered coordinate geometry, but it is Descartes' version that we use today.

- While mathematicians tried proving the Fifth Postulates from the first four, they instead derived equivalent statements such as:
 - Two parallel lines are equidistant. (Posidonius, 1st century BC)
 - A line that intersects one of two parallel lines must also intersect the other line. (Proclus, 410-485 AD)
 - For each triangle, one can form construct a similar triangle of any size. (Wallis, 1616-1703)
 - The sum of the angles in a triangle is equal to two right angles.
 - Three non-collinear points determine a circle.

- The Fifth Postulate could not be deduced from the first four postulates. Thus it is possible to replace P5 with another postulate, and together with the first four, another type of geometry may be devised.
- Johann Bolyai (1802-1860) and Nikolai Lobachevsky (1792-1856) are credited with devising the first non-euclidean geometries, which is called hyperbolic geometry.
- A model of hyperbolic geometry is due to Jules Henri Poincare (1854-1912). He introduced the upper half plane model and open unit disc model.

Some results from hyperbolic geometry

- If L is a line and P a point not on L , then there are infinitely many lines through P that do not meet L .
- The sum of the interior angles of a triangle is less than 180 degrees.

In the late 1800's - 1900's

- **Differential geometry** - combines geometry with the techniques of calculus to provide a method for studying geometry on curved surfaces. Gauss and Riemann (his student) laid the foundation of this field. Einstein credits Gauss with formulating the mathematical fundamentals of the theory of relativity.
- **Fractal Geometry** - Fractals are geometric figures that model many natural structures like ferns or clouds. They are complex and detailed in structure at any level of their magnification. Often fractals are *self-similar*—that is, they have the property that each small portion of the fractal can be viewed as a reduced-scale replica of the whole.

- One example of a fractal is the “snowflake” curve constructed by taking an equilateral triangle and repeatedly erecting smaller equilateral triangles on the middle third of the progressively smaller sides. Theoretically, the result would be a figure of finite area but with a perimeter of infinite length, consisting of an infinite number of vertices. In mathematical terms, such a curve cannot be differentiated.
- A turning point in the study of fractals came with the discovery of fractal geometry by the Polish-born French mathematician Benoit B. Mandelbrot in the 1970s. Mandelbrot adopted a much more abstract definition of dimension than that used in Euclidean geometry, stating that the dimension of a fractal must be used as an exponent when measuring its size.

- The result is that a fractal cannot be treated as existing strictly in one, two, or any other whole-number dimensions. Instead, it must be handled mathematically as though it has some fractional dimension. The “snowflake” curve of fractals has a dimension of 1.2618.
- Fractal geometry is not simply an abstract development. A coastline, if measured down to its least irregularity, would tend toward infinite length just as does the “snowflake” curve. Mandelbrot has suggested that mountains, clouds, galaxy clusters, and other natural phenomena are similarly fractal in nature, and fractal geometry's application in the sciences has become a rapidly expanding field. In addition, the beauty of fractals has made them a key element in computer graphics.

- Fractals have also been used to compress still and video images on computers. In 1987, English-born mathematician Dr. Michael F. Barnsley discovered the Fractal Transform™ which automatically detects fractal codes in real-world images (digitized photographs). The discovery spawned fractal image compression, used in a variety of multimedia and other image-based computer applications.

- One of the most intricate and beautiful images in all of mathematics is the Mandelbrot set, discovered by Benoit Mandelbrot in 1980.

