2.3 Problem-Solving Strategies

Goals:

To apply critical thinking skills to the solutions of quantitative problems;

To enumerate and illustrate the processes involved in problem solving.

2.3.1 Problem Solving is an art.

- No fixed rules are prescribed
- No set of tools will always work
- May involve many different creative processes
- Many different ways may be available to address the same problem
- Requires ingenuity, experience and a bag of tools

Mathematics has a practical use.

- Misconception about Mathematics: too abstract, irrelevant , has nothing to do with real issues
- The dreaded *word problems* attempt to introduce relevance into mathematics courses.

2.3.2 Problem Solving through Unit Analysis

- Numbers in real problems almost always represent a quantity of *something*, such as length, distance, area, volume.
- Terms that describe a quantity are called *units* or *dimensions.*
- We learned that the meaning of a number depends on the units associated with it.

- Working with units along with numbers is essential in problem solving.
- Keeping track of units in calculations is one of the most powerful problem-solving tools.
- Units can help you find a solution to the problem.

2.3.3 The Process of Problem Solving

- Be *flexible* in solving problems because no particular strategy always works.
- Be *creative.*
- Be *organized.*
- Be *logical.*

Suggested Steps in Problem Solving

1. Understand the problem.
2. Devise a strategy for solving the problem.
3. Carry out your strategy, and revise if necessary.
4. Check, interpret and explain your solution.

1. Understand the problem.

- Think about the context of the problem.
- Make a list or table of the specific information given.
- Draw a picture or diagram to help you make sense of the problem.
- Restate the problem in different words to clarify its question.
- Make a mental or written model of the solution.

2. Devise a strategy for solving the problem.

Most difficult, requires creativity, organization and experience.

- Identify and obtain any information needed to solve the problem that was not provided in the problem statement, using recall, estimation or research.
- Make a list of possible strategies and hints that will help you select your overall strategy.
- Map out your strategy with a flow chart or diagram.
- Use your tools for constructing arguments to help formulate the strategy.

3. Carry out your strategy, and revise it if necessary.

Here you use analytical and computational tools as you work through the mathematical details of the problem.

- Keep an organized, neat, and written record of your work.
- Double-check each step you take to avoid errors.
- Constantly re-evaluate your strategy as you work.

4. Check, interpret and explain your solution.

- Be sure that your result makes sense. Does it have the expected units? Is the value reasonable?
- Check the specifics of your result.
- Identify and understand potential sources of uncertainty in your result.
- Consider and discuss any pertinent implications of your result.

Sample problem:

Apply the four-step process in finding how much sales tax revenue is generated by textbook sales at your school each year.

2.3.4 Strategic Hints for Problem Solving

To help you develop a "mind set" that is conducive to enjoyable and successful problem solving.

- There may be more than one answer. Non-unique solutions may occur because not enough information is available to distinguish among a variety of possibilities.

- There may be more than one method.
- Use appropriate tools.
- Consider simpler, similar problems.
- Consider equivalent problems with simpler solutions.
- Do not be reluctant to use approximations.
- Try alternative patterns of work.
- Do not spin your wheels.

Conclusion

- Working with the units of a problem, as well as with the numbers, is one of the best problem-solving techniques.
- Problem solving is more of an art than a science, and requires both creativity and organization.
- The only sure way to improve at problem solving is by doing it.