

Chapter 1.2

Arguments and Reasoning

1

⌘ There is a need to understand the general principles of reasoning before one can learn to reason with quantitative information.

⌘ Possessing skills in reasoning can be advantageous.

2

⌘ It gives confidence in one's ability to learn, ask questions and figure out things.

⌘ It helps us understand the many forces of persuasion and fallacy shown in media.

⌘ It is an important ingredient in sound decision making.

3

Chapter Outline

⌘ What is Logic?

⌘ Logical Arguments

⌘ Propositions

⌘ Deductive Arguments

⌘ Inductive Arguments

⌘ Analyzing Real-life Arguments

⌘ The Forces of Persuasion

⌘ Fallacies of Relevance

⌘ Fallacies of Numbers and Statistics

⌘ Fallacies Involving Percentages

4

Logic: The Study of Reasoning

⌘ Many people make decisions for many different reasons.

⌘ Decisions may be made from emotion or from sound reasoning.

⌘ In reasoning, you carefully analyze the potential impacts of your decision on your personal goals.

5

⌘ Reasoning ability, together with personal background and emotional state, shapes beliefs and opinions.

⌘ Reasoning ability is crucial to the individual's evaluation of beliefs, opinions and decisions of others.

⌘ *Quantitative reasoning* is the ability to reason with quantitative or mathematical information.

6

What is Logic?

Logic is the study of the methods and principles used to distinguish correct reasoning from wrong reasoning.

7

Time-out to think:



Do you believe that all decisions should be based on logic? When should logic be used and when should other factors be considered?

8

Logical Arguments

In logic, the word "argument" has a more precise meaning than it does in ordinary English language.

9

Consider the following "argument".

Jose: "Abortion is immoral."
Maria: "No, it is not."
Jose: "Yes it is! Doctors who perform abortions should go to jail."
Maria: "You don't even know what you're talking about!"
Jose: "I know a lot more than you know. I've watched lots of film about it."
Maria: "I can't talk to you. You're an idiot."

10

A "heated conversation" indeed but not a logical argument. It has no structure, and neither Jose nor Maria's conclusion is established from their statements.

A logical argument begins with a set of declarative statements that describe the ideas, facts, or assumptions on which argument is based, called the premises, and ends with one or more declarative statements that represent the conclusion of the argument.

11

Let us restructure Jose's argument.

Opening statement:

Premise: The killing of a human being is immoral.
Premise: Abortion is the killing of a human being.
Conclusion: Abortion is immoral.

Maria's rebuttal:

Premise: The killing of a human being in self-defense is not immoral.
Premise: Killing in circumstances of war is not immoral.
Premises: The death penalty for murderers is not immoral.
Conclusion: The killing of a human being is not necessarily immoral.

12

⌘ The arguments are clear and well conceived, the premises lead logically to the conclusion.

⌘ Logical argument has a great value: through this process each side may better understand the other.

⌘ A logical argument is made up of building blocks called propositions.

13

Propositions

⌘ A proposition (or statement) is a declarative sentence that makes a distinct claim, such as an assertion or denial; it proposes something to be true or false

⌘ Some of the propositions in the example are:

"Abortion is immoral."

"Abortion is not immoral."

"Jose knows a lot about abortion."

"Jose is an idiot."

14

Both premise and conclusion are propositions. Each makes a distinct claim that, depending on your viewpoint, is either true or false.

A proposition must have a *subject* and *predicate*.

A proposition must be capable of being true or false, but not both at the same time, though we may not know which it is.

15

Other examples of propositions:

⌘ "Erap Estrada underwent knee surgery."

Subject: Erap Estrada

Predicate: underwent knee surgery

⌘ "The jeepneys will be phased out."

Subject: The jeepneys

Predicate: will be phased out

16

Types of propositions:

1. Categorical proposition
- a proposition that expresses a relationship between two categories or sets, the subject set S and predicate set P.

Examples:

"All politicians are corrupt."

"No politician is corrupt."

"Some politicians are corrupt."

"Some politicians are not corrupt."

17

The four standard categorical propositions:

A: All S are P.

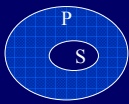
E: No S is P.

I: Some S are P.

O: Some S are not P.

18

Venn Diagrams



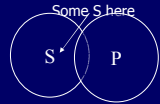
All S are P.



No S are P.



Some S are P.



Some S are not P.

Exercise: Rephrase the following categorical propositions into standard form:

1. All biology courses are easy.
2. Some employees do not pay taxes.
3. Some MMDA traffic enforcers have not even finished high school.
4. Senators never miss committee meetings.

5. Fidel Ramos is a good president. (S consists of one element only. S is called a singular set.)
6. My final grades last semester are acceptable.
7. Movie magazines are always fun to read.
8. Every real number has a square root.
9. Some real numbers do not have square roots.
10. All my students never miss the class.

A proposition always makes a claim of truth. This claim however is not necessarily true. Although a proposition is capable of being either true or false, determining which it is may not be possible.

Claims of truth:

A proposition maybe

1. unambiguous - no one can reasonably disagree with its truth or falsity
2. unverifiable - would require impossible or impractical procedures to determine truth or falsity
3. matter of opinion - truth can be argued endlessly

2. Compound proposition

- a proposition that consists of two or more simple (or prime) propositions joined together by *logical connectors* like *and* (conjunction) and *or* (disjunction), *if* and *then*, and *if and only if*.

Let p and q be propositions. If both p and q are true, then the compound proposition **p and q** (denoted $p \wedge q$) is true. Otherwise $p \wedge q$ is false.

Or is inclusive when it means "either or both".

Or is exclusive when it means "either but not both".

In logic, *or* is always inclusive.

Let p and q be propositions. If at least one of p or q is true, then the compound proposition **p or q** (denoted $p \vee q$) is true. Otherwise, $p \vee q$ is false.

Conditional proposition

- a proposition joined by the words *if* and *then*

Form: *If p then q.*

p is called the *antecedent*, while q is called the *consequent*.

The conditional proposition *if p then q* (denoted $p \rightarrow q$) is true unless p is true and q is false.

25

Examples:

- ⌘ If FPJ wins the election, we will migrate to the U.S.
- ⌘ Further debates on this issue will cause confusion.
- ⌘ He's a teacher, so he must know everything.
- ⌘ The ad implies that the price should only be 200 pesos.

26

Biconditional proposition

p if and only if q

The proposition *p if and only if q* (denoted $p \leftrightarrow q$) is true if p and q have the same truth values. It is false if p and q have opposite truth values.

Example: $ab=0$ if and only if $a=0$ or $b=0$.

27

Deductive arguments

Goal: to investigate how arguments actually proceed from premises to conclusions. This process is called *inference* (we infer the conclusion from the premises).

Two basic types of inferential processes:

- *deductive inference* - a specific conclusion is deduced (or logically derived) from more general premises
- *inductive inference* - a conclusion is formed by generalizing from specific premises

28

Deductive arguments with categorical propositions

Argument 1:

Premise: All UP students are intelligent.

Premise: All intelligent people will succeed in life.

Conclusion: All UP students will succeed in life.

Draw the Venn diagram.

29

Question: Is the argument valid?

An argument is *valid* if its conclusion necessarily follows from its premises - even though we may not agree that its premises are true or that its conclusion is true.

In logic, there is a distinction between *validity* and *truth*. Validity is concerned only with the *logical structure* (or *form*) of an argument, not the truth of its premises or conclusions.

Four possibilities:

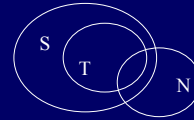
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1. An argument is a *sound argument* if it is a valid argument and its premises are true. The conclusion must also be true.
2. An argument may be *valid but not sound* if its premises are false, and necessarily its conclusion is also false.
3. An argument is *invalid* if the premises are true but the conclusion is false. There is a flaw in its logical structure. The argument suffers from a so-called *formal fallacy* (a fallacy of structure or form).
4. An argument has a true conclusion but is invalid because of a flaw in the logical structure (formal fallacy). Its conclusion does not follow from the premises.

31

Argument 2

Premise: All tabloids are sensational.
 Premise: Some newspapers are tabloids.
 Conclusion: Some newspapers are sensational.

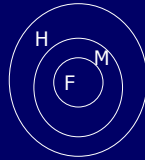


Answer: valid argument (is it sound?)

32

Argument 3.

Premise: All fish are mammals.
 Premise: All mammals are human beings.
 Conclusion: All fish are human beings.



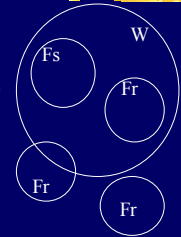
Is the argument valid? Is it sound?

Answer: valid (the conclusion follows from the premises) but not sound (premises are false, conclusion also false)

33

Argument 4.

Premise: Fish live in the water.
 Premise: Frogs are not fish.
 Conclusion: Frogs do not live in the water.



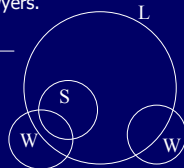
Is this argument valid?

Answer: invalid (it does not necessarily show that frogs are not water dwellers), premises true, yet conclusion false. This happens only when there is a structure flaw.

34

Argument 5.

Premise: All Supreme Court judges are lawyers.
 Premise: Some women are lawyers.
 Conclusion: Some women are Supreme Court judges.



Is the argument valid?

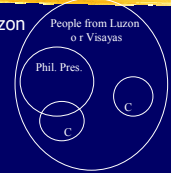
Answer: invalid (there is flaw in structure) but true conclusion. The argument suffers from a formal fallacy because the conclusion, though true, does not follow from the premises.

Replace women with legislators (same structure remains, premises still true, but conclusion is false.)

35

Argument 6. (Argument with a singular proposition).

Premise: All Philippine Presidents were from Luzon or Visayas.
 Premise: Cory Aquino was from Luzon.
 Conclusion: Cory Aquino was a Philippine president.



Is the argument valid?

Answer: invalid, the conclusion may be true but does not follow from the premises.

Replace Cory Aquino with another name. Is the conclusion still true?

36

Deductive Arguments with one conditional premise (the four basic conditional arguments)

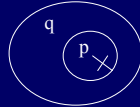
1. Affirming the antecedent (*modus ponens*):

Form: Premise: *If p, then q.*
 Premise: *p.*
 Conclusion: *q.*

This is a valid argument.

Example:

Premise: If it is Monday, then traffic is heavy.
 Premise: It is Monday.
 Conclusion: Traffic is heavy.

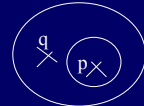


2. Affirming the consequent

Form: Premise: *If p then q.*
 Premise: *q.*
 Conclusion: *p.*

Example:

Premise: If it rains heavily, then Ermita will be flooded.
 Premise: Ermita was flooded.
 Conclusion: It rained heavily.
 This is an invalid argument. The conclusion does not follow from the premises.



3. Denying the consequent (*modus tollens*)

If p then q.
Not q

Not p

This is a valid argument.

Example:

If fuel prices increase, jeepney operators will ask for a fare adjustment. Jeepney operators did not ask for a fare adjustment. Therefore, fuel prices did not increase.

4. Denying the antecedent

If p then q.
Not p

Not q

This is a fallacy.

Example:

If you like the book, then you'll love the movie. You did not like the book. Therefore, you will not love the movie.

Deductive Arguments with a chain of conditionals

If p then q.
If q then r.

If p then r.

Example:

"If there is a typhoon in the North, then there will be a shortage of cabbages. A shortage of cabbages means high prices for cabbages. Therefore, a typhoon in the north means high prices for cabbages."

Time-Out To Think:



Suppose that cabbage prices are high. Can you conclude that the high prices were caused by a typhoon in the North? Why or why not?

Is the following argument valid? sound?

"Ping Lacson advocates peace and order and the eradication of corruption, which will benefit my children's future. Therefore if I vote for Lacson as President in 2004, my children will live in a more secure and stable country."

Answer: valid, but soundness is a matter of opinion.

Inductive Arguments

Consider the following example of an inductive argument.

- Premise: Birds fly up into the air but eventually come back down.
- Premise: People or animals that jump into the air fall back down.
- Premise: Rocks thrown into the air come back down.
- Premise: Balls thrown into the air come back down.
- Conclusion: What goes up must come down.

Note: Each premise represents a specific case or example of something that goes up then comes back down. The conclusion represents a **generalization** of these specific cases.

A Comparison of Deductive and Inductive Arguments

Deductive	Inductive
The conclusion is more specific than the premises.	The conclusion usually is more general than the premises.
In a valid deductive argument, the conclusion necessarily follows from the premises.	There is no such thing as a valid inductive argument. Inductive arguments can be analyzed only in terms of their strength, that is, we make a subjective judgment about how well the premises support the generalization in the conclusion. The conclusion of a strong inductive argument seems likely to follow from its premises but it does not necessarily do so.
Validity concerns only logical structure; an argument can be valid even when its conclusion is blatantly false.	Although an inductive argument cannot be valid, it can be <i>invalidated</i> by a single premise that counters its conclusion.
A deductive argument is <i>sound</i> if it is valid and its premises are true. Because people may disagree about the truth of its premises, they may also disagree about the soundness of a deductive argument.	Because an inductive argument cannot be valid, neither can it be sound. At best, the conclusion of a strong inductive argument <i>probably</i> is true.

Induction and Deduction in Everyday Life

⌘ People usually form reasoned opinions and decisions through inductive reasoning. It helps a person to organize knowledge and suggest possible truths.

⌘ But *proof* requires deduction, in which a conclusion is necessarily established from a set of premises. Deduction allows a person to prove or disprove those possible truths.

Induction and Deduction in Mathematics

- ⌘ **Theorems** are statements of mathematical truth which requires proof which is possible only through deductive logic.
- ⌘ **Axioms** are the starting points for mathematical proof, the "givens", assumed to be true without proof.
- ⌘ Although proof is deductive, induction also plays a role: ideas for theorems usually come through inductive reasoning.
- ⌘ Example: Goldbach Conjecture (1742)

"Every even number (except 2) can be expressed as a sum of two prime numbers."

Usefulness of seeking inductive evidence:

- ⌘ A mathematical rule can be tested inductively. Although test cases constitute inductive evidence only, and not proof, they often are enough to satisfy yourself of a rule's truth.
- ⌘ Example: For all numbers a and b , $a \times b = b \times a$.

⌘ A proposed rule can be invalidated even by one failed test case.

⌘ Example:

"Is it true that for any number a , $2/3 = (2+a)/(3+a)$?"

Fermat's Last Theorem (Pierre Fermat, 1601-1665)

For any natural number n besides 1 or 2, it is impossible to find natural numbers a , b , and c that satisfy the relationship

$$a^n + b^n = c^n$$

49

Analyzing Real-life Arguments

- ⌘ The most important skill in argument analysis is clear and organized thinking.
- ⌘ In analyzing a real-life argument, we "pick it apart" and figure out how it all fits together.

⌘ *How do we do this?*

Identify all the stated propositions and determine which are premises and which are conclusions. Make a flowchart or diagram of the argument. Evaluate the argument. Is it deductive, is it valid? If it is inductive, how strong is it?

50

Example 1:

- (1) Birds are animals and they are mortal.
- (2) Fishes are animals and they are mortal.
- (3) Spiders are animals and they are mortal.
- (4) Human being are animals and they are mortal.
- (5) All animals are mortal.

Note:

No single premise is essential to reaching the conclusion, but each additional premise strengthens the argument. The premises *independently* support the conclusion. This happens in *inductive* arguments. How does the flowchart look like?

51

Example 2:

- (1) All carcinogens (cancer-causing materials) are dangerous.
- (2) Asbestos is carcinogen.
- (3) Asbestos is dangerous.

Note:

The two premises support the conclusion only when they are *linked together*, or *combined additively*. This happens in *deductive arguments*.

52

Exercise: Analyze the following argument.

"Buying the mobile phone was a good idea because the price was reasonable and it will be easier to communicate with my friends."

*What are the premises? the conclusion?
Is this a deductive argument? Is it valid?
Or is it an inductive argument? Is it a strong one?*

53

Exercise:

Suppose that you are planning a trip six months in advance and discover that you have two options in purchasing an airline ticket:

- (A) The lowest fare is P1100, but 25% of the fare is non-refundable if you change or cancel the ticket.
- (B) A fully-refundable ticket is available for P1900.

Analyze this situation and explain how you would make a decision.

54