The role of quantum scalar fields in gravity

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Abstract: The presence of matter as represented by the quantum wave function ψ , for quarks and leptons, causes a gradient on the energy profile of the quantum scalar fields that pervade the Universe, thereby changing the metric tensor g_{uv} for curved spacetime. As another reference frame is considered, with an infinitesimal coordinate transformation, an energy balance is possible between the modified quantum wave functions ψ' and η' and the new metric tensor $g'_{\alpha\beta}$. Gravity then propagates through spacetime as a local change in the quantum scalar fields energy, caused by their interaction with spinors.

Key words : quantum gravity, interaction with spinors, quantum scalar fields, metric tensor, curved spacetime

PACS : 04.20.Fy, 04.60.-m, 04.62.+v, 11.10.Ef, 11.15.-q, 14.80.Bn

1- Introduction

Scalar fields are thought to pervade the Universe and to give the particles of the Standard Model their masses. The yet undiscovered Higgs particles are the quanta of these scalar fields. We might wonder if these scalar fields play any role in the gravitational force on the quantum level. Since they are considered to be present in spacetime across the Universe, these quantum scalar fields may have a role in the process described by general relativity in that matter warps spacetime which in turn acts on matter [1]. Rather than being a direct force between different masses, gravity seems to act in an indirect physical process. Newton himself wrote, in a letter to Bentley, that he considered absurd the assumption that a body some distance from another body could act on it through empty space without some intermediary agency [2].

I propose here to examine the possibility that quantum scalar fields may participate in that process by first interacting with masses and second transmiting the effects of these interactions to other masses along spacetime, resulting in the force of gravity.

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On the quantum level, masses may be represented by quarks and leptons which are spin ½ Dirac particles. I am going to write down a Lagrangian density including these Dirac particles and their interaction with the all pervasive quantum scalar fields and then I will check if it is possible to have local gauge invariance when we include the metric tensor $g_{\mu\nu}$ for curved spacetime.

2- Physical model

The main point in our physical model is that the presence of masses (quarks and leptons) alters the quantum scalar fields energy profile along spacetime and that in turn changes the spacetime metric $g_{\mu\nu}$. In other words, the spinors wave function ψ causes a gradient in the scalar fields energy density, that we may write as :

$$(\partial_\nu \eta^* \partial^\nu \eta)_{;\mu} \qquad (1)$$

Here $;\mu$ is the covariant derivative in curved spacetime

To write the term above as a Lagrangian density term in the appropriate units and including the spinors wave function ψ , we have to multiply it by the spinors current density :

$$-\hbar c\, (\bar{\psi}\gamma^\mu \psi)\, (\partial_\nu \eta^* \partial^\nu \eta)_{;\mu} \qquad (2)$$

where $\bar{\psi} = \psi^* \gamma^0$

As I mentioned above, we are examining the possibility that quantum scalar fields may participate in gravity by interacting with quarks and leptons and then passing on the effects of this interaction to other particles through spacetime. In our physical model the gravitational field is tansmited through spacetime by the change in the quantum scalar field energy density from one spacetime point to another.

As gravity deals with the geometry of spacetime, let's introduce now an infinitesimal coordinate transformation :

$$x'^\mu = x^\mu - \xi'^\mu \qquad (3)$$

where ξ'^μ is the infinitesimal derivative of a scalar function ξ of the coordinates in curved spacetime.

The coordinate transformation (3) effectively adds a phase to any wave function by a factor of $e^{i\xi}$. Here, for convenience, we write this phase difference as $e^{i(k/\hbar c)\xi}$, where k is a real constant. So, in the new spacetime coordinates x'^{μ} the wave functions for spinors and quantum scalar fields necessarily transform as :

$$\psi' = e^{i(k/\hbar c)\xi}\,\psi$$

$$\eta' = \eta\, e^{i(k/\hbar c)\xi}$$

In the new coordinate system with $x'^{\mu} = x^{\mu} - \xi'^{\mu}$, the spacetime metric tensor transforms as [1] :

$$g'_{\mu\nu} = g_{\mu\nu} + \xi_{,\mu;\nu} + \xi_{,\nu;\mu}$$

For the spin connection Γ_{μ} in curved spacetime we will follow the formulation described by H. Arthur Weldon in his article "Fermions without vierbeins in curved spacetime" [3] .

As it happens in a non linear physical process such as gravity, the quantum scalar fields respond to their interaction with the spinors by also causing a gradient in the spinor fields energy density. We may represent this in the appropriate units as:

$$j_{(\eta)}\ (i\hbar c\, \bar\psi\gamma^{\mu}(\partial_{\mu} + \Gamma_{\mu})\psi)_{;\nu} = i(\eta\,\partial^{\nu}\eta^{*} - \eta^{*}\partial^{\nu}\eta)\,[(i\hbar c\,\bar\psi\gamma^{\mu}\partial_{\mu}\psi)_{;\nu} + (i\hbar c\,\bar\psi\gamma^{\mu}\Gamma_{\mu}\psi)_{;\nu}] =$$

$$= -\hbar c\,(\eta\,\partial^{\nu}\eta^{*} - \eta^{*}\partial^{\nu}\eta)[(\bar\psi\gamma^{\mu}\partial_{\mu}\psi)_{;\nu} + (\bar\psi\gamma^{\mu}\Gamma_{\mu}\psi)_{;\nu}] \qquad (4)$$

Where the current density for complex quantum scalar fields [4] is :

$$j_{(\eta)} = i(\eta\,\partial^{\nu}\eta^{*} - \eta^{*}\partial^{\nu}\eta)$$

The gradient of the transformed η' quantum scalar fields energy density in curved spacetime is given by :

$$(\partial_{\nu}\eta'^{*}\,\partial^{\nu}\eta')_{;\mu}$$

Now, introducing the gauge ξ with $\eta' = \eta\, e^{i(k/\hbar c)\xi}$ it follows that :

$$(\partial_{\nu}\eta'^{*}\,\partial^{\nu}\eta')_{;\mu} = \{\partial_{\nu}\eta^{*}\,\partial^{\nu}\eta + (k/\hbar c)^{2}\,\eta^{*}\eta\ \partial_{\nu}\xi\ \partial^{\nu}\xi + \eta\,\partial_{\nu}\eta^{*}\,(ik/\hbar c)\,\partial^{\nu}\xi +$$

$$+ \eta^{*}\,\partial^{\nu}\eta\,(-ik/\hbar c)\,\partial_{\nu}\xi\}_{;\mu} \qquad (5)$$

Our assumption here is that we have the infinitesimal coordinate transformation $x'^{\mu} = x^{\mu} - \xi'^{\mu}$. Then $\partial^{\nu}\xi = \xi'^{\nu}$ is an infinitesimal and from (5) we may write:

$$(\partial_\nu\eta'^{*}\,\partial^\nu\eta')_{;\mu} = (\partial_\nu\eta^{*}\,\partial^\nu\eta\,)_{;\,\mu} + (ik/\hbar c)\,((\eta\,\partial^\nu\eta^{*} - \eta^{*}\,\partial^\nu\eta)\,\partial_\nu\xi)_{;\,\mu} =$$

$$= (\partial_\nu\eta^{*}\,\partial^\nu\eta\,)_{;\,\mu} + (ik/\hbar c)\,(\eta\,\partial^\nu\eta^{*} - \eta^{*}\,\partial^\nu\eta)_{;\,\mu}\,\partial_\nu\xi + (ik/\hbar c)(\eta\,\partial^\nu\eta^{*} - \eta^{*}\,\partial^\nu\eta)\xi_{,\nu\,;\mu} \qquad (6)$$

In the infinitesimal spacetime coordinates interval ξ'^{ν} , the quantum scalar field current density $j_{(\eta)}$ does not change, so the second term in equation (6) vanishes and we can write :

$$(\partial_\nu\eta'^{*}\,\partial^\nu\eta')_{;\mu} = (\partial_\nu\eta^{*}\,\partial^\nu\eta\,)_{;\,\mu} + (ik/\hbar c)(\eta\,\partial^\nu\eta^{*} - \eta^{*}\,\partial^\nu\eta)\,\xi_{,\nu\,;\mu} \qquad (7)$$

From equation (4) , we need to calculate $(\bar{\psi}'\gamma^{\mu}\partial_\mu\psi')_{;\nu}$ and $(\bar{\psi}'\gamma^{\mu}\Gamma_\mu\psi')_{;\,\nu}$ in the coordinate system where $x'^{\mu} = x^{\mu} - \xi'^{\mu}$:

$$(\bar{\psi}'\,\gamma^{\mu}\partial_\mu\psi')_{;\nu} = (\,\bar{\psi}\,\gamma^{\mu}\partial_\mu\psi + (\bar{\psi}\,\gamma^{\mu}\,\psi)\,(ik/\hbar c)\;\partial_\mu\xi\,)_{;\,\nu} =$$

$$= (\,\bar{\psi}\,\gamma^{\mu}\partial_\mu\psi\,)_{;\,\nu} + (ik/\hbar c)(\,\partial_\mu\xi\,(\bar{\psi}\,\gamma^{\mu}\,\psi)_{;\,\nu} + (\bar{\psi}\,\gamma^{\mu}\,\psi)\,\xi_{,\mu\,;\nu}\,)$$

In the infinitesimal spacetime coordinates interval ξ'^{μ} the current density for the spinors does not change : $\bar{\psi}'\gamma^{u}\,\psi' = \bar{\psi}\,\gamma^{u}\,\psi$. So, $\partial_\mu\xi\,(\bar{\psi}\,\gamma^{\mu}\,\psi)_{;\,\nu} = 0$ and we can write :

$$(\bar{\psi}'\,\gamma^{\mu}\partial_\mu\psi')_{;\nu} = (\bar{\psi}\,\gamma^{\mu}\partial_\mu\psi)_{;\,\nu} + (ik/\hbar c)(\bar{\psi}\,\gamma^{\mu}\,\psi)\,\xi_{,\mu\,;\nu} \qquad (8)$$

According to reference [3] the spin connection Γ_μ is not affected when a gauge is applied to ψ , so we can neglect the gradient $(\bar{\psi}'\gamma^{\mu}\Gamma_\mu\psi')_{;\,\nu}$ in the infinitesimal spacetime coordinates interval ξ'^{ν}.

As a consequence of equations (7) and (8) the interaction Lagrangian in the new spacetime coordinates x'^{μ} is :

$$L'_{\mathbf{int}} = -\,\hbar c\{(\eta'\,\partial^\nu\eta'^{*} - \eta'^{*}\,\partial^\nu\eta')(\bar{\psi}'\,\gamma^{\mu}\partial_\mu\psi')_{;\nu} + (\bar{\psi}'\gamma^{\mu}\,\psi')\,(\partial_\nu\eta'^{*}\,\partial^\nu\eta')_{;\mu}\,\} =$$

$$= -\,\hbar c(\eta\,\partial^\nu\eta^{*} - \eta^{*}\,\partial^\nu\eta)(\bar{\psi}\,\gamma^{\mu}\partial_\mu\psi)_{;\nu} - k\,(\bar{\psi}\,\gamma^{\mu}\,\psi)\,i(\eta\,\partial^\nu\eta^{*} - \eta^{*}\,\partial^\nu\eta)\,\xi_{,\mu\,;\nu} -$$

$$-\;\hbar c(\bar{\psi}\,\gamma^{\mu}\psi\,)\,(\partial_\nu\eta^{*}\,\partial^\nu\eta)_{;\mu} - k\,(\bar{\psi}\,\gamma^{\mu}\,\psi)\,i(\eta\,\partial^\nu\eta^{*} - \eta^{*}\,\partial^\nu\eta)\,\xi_{,\nu\,;\mu} =$$

$$= L_{\mathbf{int}} - k(\bar{\psi}\,\gamma^{\mu}\,\psi)\,i(\eta\,\partial^\nu\eta^{*} - \eta^{*}\,\partial^\nu\eta)\,\xi_{,\mu\,;\nu} - k\,(\bar{\psi}\,\gamma^{\mu}\,\psi)\,i(\eta\,\partial^\nu\eta^{*} - \eta^{*}\,\partial^\nu\eta)\,\xi_{,\nu\,;\mu} \qquad (10)$$

where L_{int} is the Lagrangian density for the interaction between spinors and quantum scalar fields in the old spacetime coordinates x^{μ} . But in the new spacetime coordinates x'^{μ} the spacetime metric tensor [1] is $g'_{\mu\nu} = g_{\mu\nu} + \xi_{,\mu;\nu} + \xi_{,\nu;\mu}$

It is easy to see that in order to make the Lagrangian L'_{int} in equation (10) invariant, we need to add a term with the current densities of the fields ψ and η that participate in the interaction and the spacetime metric tensor $g_{\mu\nu}$:

$$k\,(\bar{\psi}\gamma^{\mu}\psi)\, i(\eta\,\partial^{\nu}\eta^* - \eta^*\partial^{\nu}\eta)\, g_{\mu\nu}$$

Now the terms in $\xi_{,\mu;\nu}$ and $\xi_{,\nu;\mu}$ due to the transformed metric tensor $g'_{\mu\nu}$ cancel out the two last terms in equation (10) and the interaction lagrangian is invariant :

$$L' = -\hbar c(\eta\,\partial^{\nu}\eta^* - \eta^*\partial^{\nu}\eta)(\bar{\psi}\gamma^{u}\partial_{\mu}\psi)_{;\nu} - k\,(\bar{\psi}\gamma^{\mu}\psi)\, i(\eta\,\partial^{\nu}\eta^* - \eta^*\partial^{\nu}\eta)\,\xi_{,\mu;\nu} -$$

$$-\hbar c(\bar{\psi}\gamma^{\mu}\psi)\,(\partial_{\nu}\eta^*\,\partial^{\nu}\eta)_{;\mu} - k\,(\bar{\psi}\gamma^{\mu}\psi)\, i(\eta\,\partial^{\nu}\eta^* - \eta^*\partial^{\nu}\eta)\,\xi_{,\nu;\mu} +$$

$$+ k\,(\bar{\psi}\gamma^{\mu}\psi)\, i(\eta\,\partial^{\nu}\eta^* - \eta^*\partial^{\nu}\eta)\, g_{\mu\nu} + k\,(\bar{\psi}\gamma^{\mu}\psi)\, i(\eta\,\partial^{\nu}\eta^* - \eta^*\partial^{\nu}\eta)\,\xi_{,\mu;\nu} +$$

$$+ k\,(\bar{\psi}\gamma^{\mu}\psi)\, i(\eta\,\partial^{\nu}\eta^* - \eta^*\partial^{\nu}\eta)\,\xi_{,\nu;\mu} = L$$

So, the complete Lagrangian for the interaction between spinors and quantum scalar fields is :

$$L = -\hbar c\{(\eta\,\partial^{\nu}\eta^* - \eta^*\partial^{\nu}\eta)\,(\bar{\psi}\gamma^{u}\partial_{\mu}\psi)_{;\nu} + (\bar{\psi}\gamma^{\mu}\psi)\,(\partial_{\nu}\eta^*\,\partial^{\nu}\eta)_{;\mu}\} +$$

$$+ k\,(\bar{\psi}\gamma^{\mu}\psi)\, i(\eta\,\partial^{\nu}\eta^* - \eta^*\partial^{\nu}\eta)\, g_{\mu\nu} \qquad (11)$$

The Lagrangian (11) describes the interaction of spinors (quarks and leptons) with quantum scalar fields which results in a non-Euclidian spacetime metric tensor $g_{\mu\nu}$.

3 – Conclusions

We have described above the interaction of spinors with quantum scalar fields and how this interaction changes the spacetime metric tensor $g_{\mu\nu}$. Of course, as the number of spinors increases to make a certain amount of mass of an object, the sum of their interactions with quantum scalar fields amounts to a much greater contribution to change the local spacetime metric tensor and thereby the curvature of spacetime.

The fact that the gauge terms in ψ' and η' , which are due simply to the change in the coordinate system from x^{μ} to x'^{μ} , cancel out so well with the corresponding terms in the transformed spacetime metric tensor $g'_{\mu\nu}$, is a strong indication of the consistency of the physical model described here. Gravity then propagates through spacetime as a local change (gradient) of the all pervasive quantum scalar fields energy, caused by their interaction with quarks and leptons.

Finally, adding a free lagrangian for the curved spacetime geometry [1,5] , the complete lagrangian becomes:

$$L = - \hbar c \{(\eta\, \partial^\nu \eta^* - \eta^* \partial^\nu \eta)\, (\bar{\psi}\, \gamma^u \partial_\mu \psi)_{;\nu} + (\bar{\psi}\, \gamma^\mu\, \psi)\, (\partial_\nu \eta^*\, \partial^\nu \eta)_{;\mu} \} +$$

$$+ k\, (\bar{\psi}\, \gamma^\mu\, \psi)\, i(\eta\, \partial^\nu \eta^* - \eta^* \partial^\nu \eta)\, g_{\mu\nu} + (1/16\pi)\, g^{\alpha\beta}\, R_{\alpha\beta}\, (-g)^{1/2}$$

where $R_{\alpha\beta}$ is the Ricci tensor and g is the determinant of the metric tensor.

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