

$$\begin{array}{ccccc}
 & & \text{copies of } \mathbb{Z}/2\mathbb{Z} & & \\
 E = & \frac{C}{C \curvearrowright [A,A]} & \longrightarrow & \frac{A}{[A,A]} & \longrightarrow & \frac{B}{[B,B]} = \mathbb{Z}/2\mathbb{Z} \\
 & \uparrow & & \uparrow & & \uparrow \\
 & C & \longrightarrow & A & \xrightarrow{f} & B = S(n) \\
 & \uparrow & & \uparrow & & \uparrow \\
 & C \curvearrowright [A,A] & \longrightarrow & [A,A] & \longrightarrow & [B,B] = A(n), \text{ the alternating group}
 \end{array}$$

E is a subgroup of $C/[C,C]$ = the first homology group of the moduli space.

E is non-trivial, because there are reversals of even signature, which are sent in 0 in the first row. This will imply that $C/[C,C]$ has $\mathbb{Z}/2\mathbb{Z}$ - torsion.

Note: C is the notation for the fundamental group (π in Latex)

The diagram is exact: the morphisms are monomorphisms or surjections.