Inverse Method and Indentation to Measure Material Parameters

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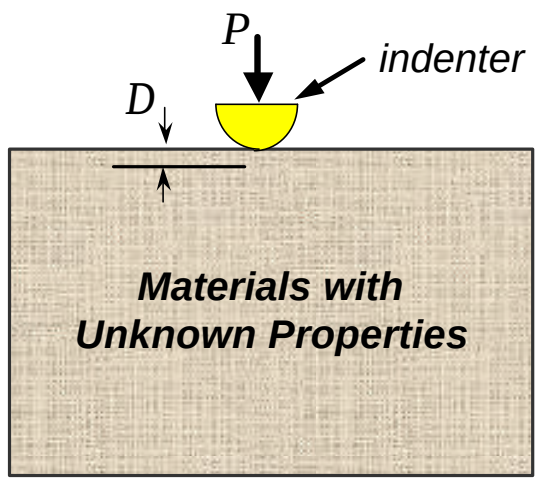
Objective:

Establish a novel data-interpretation procedure based on an **inverse analysis** technique to determine material properties. Utilize experimental measurements of **instrumented micro-indentation**.

Highlights:

- **Kalman filter** for inverse analysis.
- Elastic-plastic **graded** material.
  - *Two phases: PSZ and NiCrAlY*
  - *Tested with FGM specimen*
- Elastic-plastic **anisotropic** material.
  - *Finite element verification*
  - *Proposed experimental procedure*

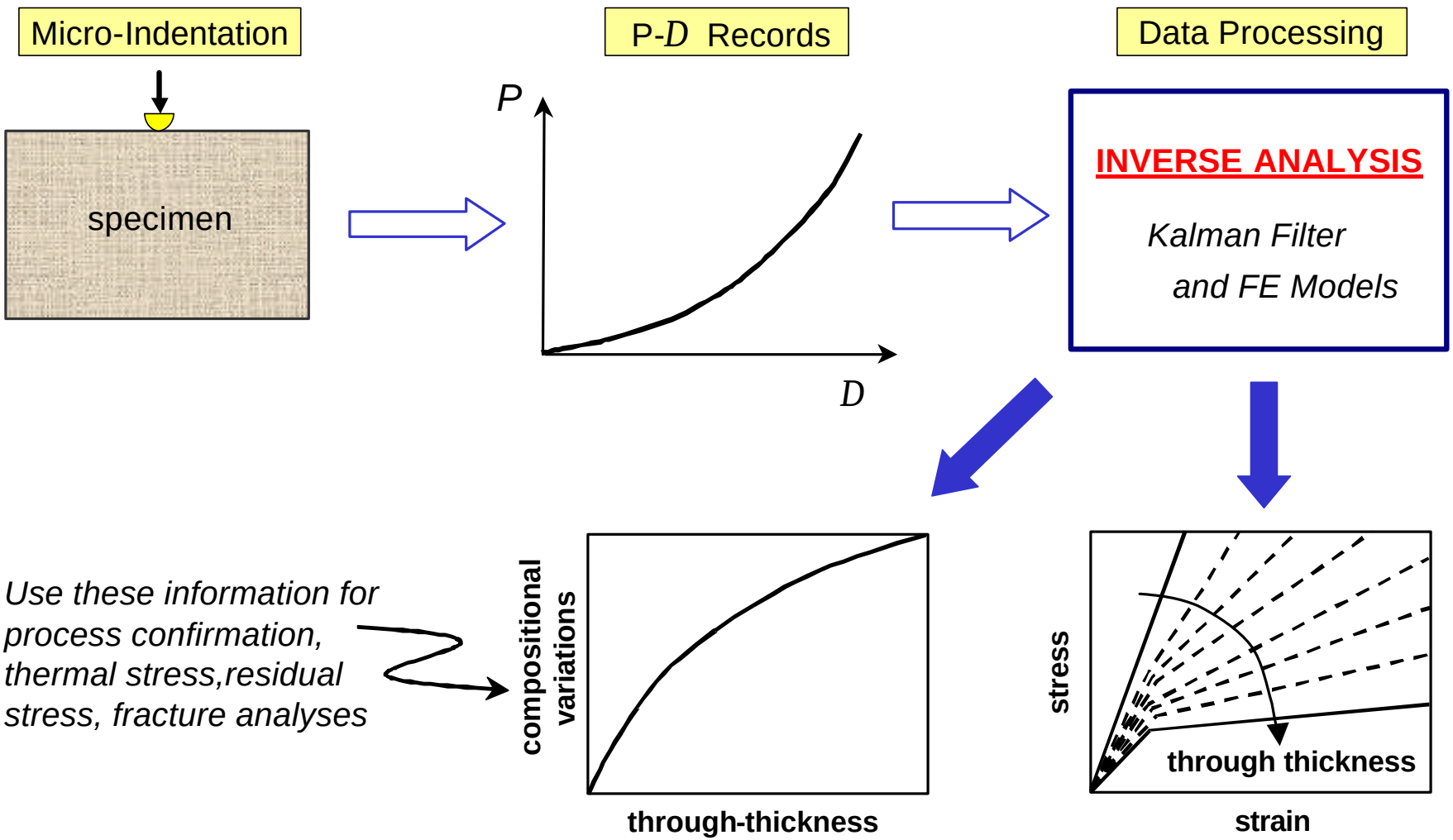


Previous Studies for Related Material Characterization

- Analytical and experimental tools to estimate the elastic properties of graded materials using instrumented micro-indentation. *Suresh et al.* (1997); *Giannakopoulos et al.* (1997).
- Rule-of-mixtures for ceramic/metal system. *Weissenbek et al.* (1997).
- Use elastic wave and progressive-learning neural network to determine material properties of FGM plates. *Liu et al.* (2001).
- Forward-reverse method and instrumented sharp indentation to characterize elastic-plastic properties for thin film and coating. *Dao et al.* (2001).
- Inverse analysis based on Kalman filter and instrumented micro-indentation for characterization of graded materials. *Nakamura et al.* (2000).

Measurement Process with Inverse Analysis

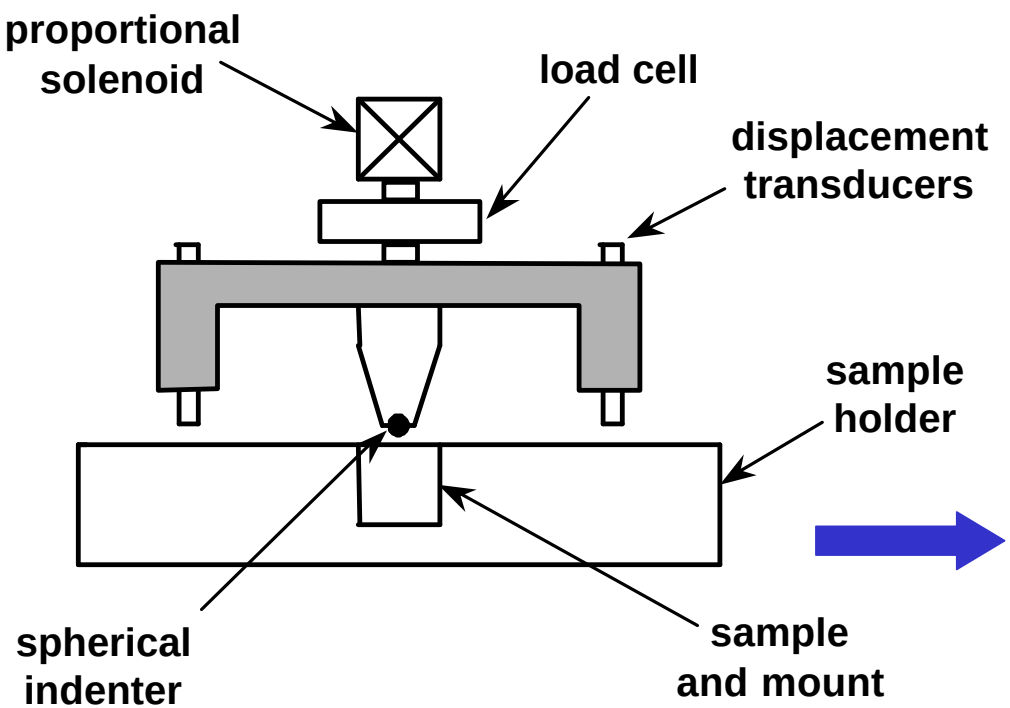
Post-process experimentally obtained records (indirect measurements) by an inverse analysis to extract unknown material parameters

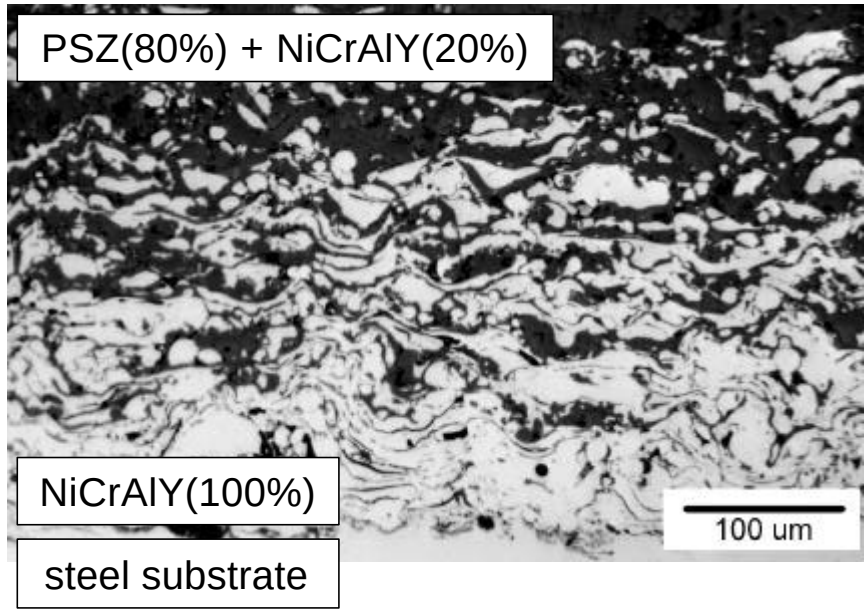


Use these information for process confirmation, thermal stress,residual stress, fracture analyses

Application to Graded Materials

- Specimen fabricated in Thermal Spray Center at Stony Brook.
- Measurements made at **MIT** & **NIST** using instrumented micro-indenters.
- **Different sizes** of indenters: R_1=0.8 mm, R_2=2.4 mm.
- Indented **load-displacement** relations are recorded.



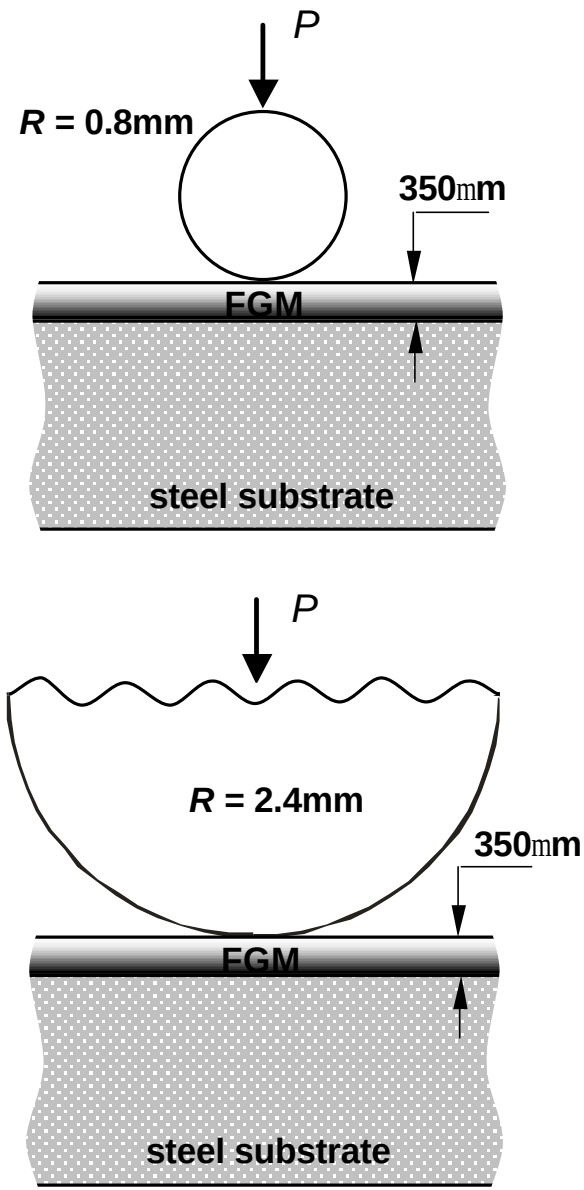


Instrumented indentation at NIST

Cross section of FGM specimen (t = 350㎜)

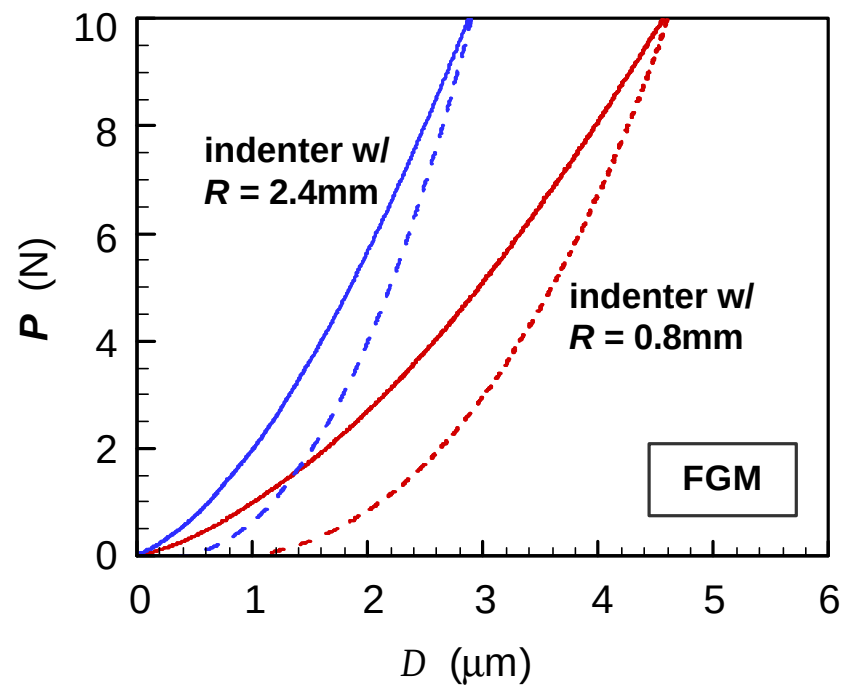
Instrumented-Indentation

Two Differently Sized Spherical Indenters



Measured load-displacement records from micro-indentations

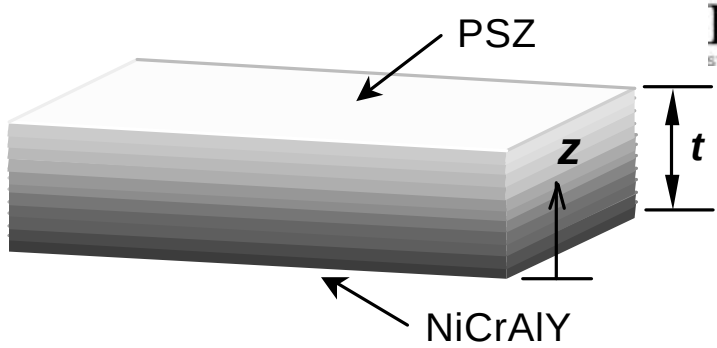
(averaged from 10 records)



Records of two indenters are required for accurate estimates (Nakamura et al, 2000)

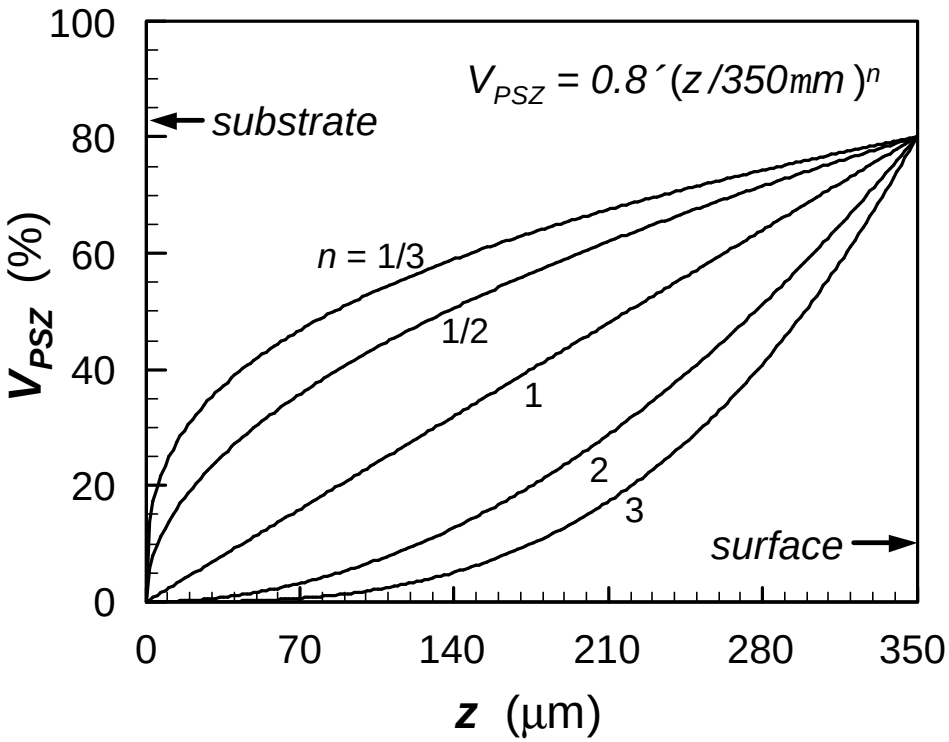
Unknown Parameters of FGM

- *Properties of PSZ and NiCrAlY are characterized separately using homogeneous specimens.*



Compositional Variation

'n' *: defines volume fraction of PSZ via power-low*

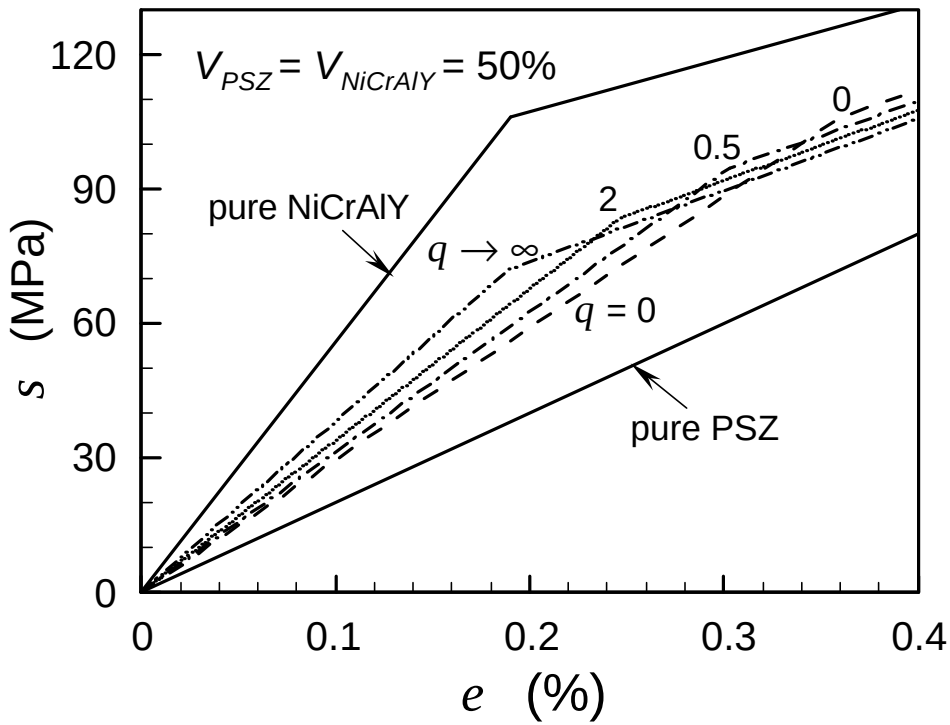


$n < 1 \Rightarrow$ *greater amount PSZ*

$n > 1 \Rightarrow$ *greater amount NiCrAlY*

$s - e$ Relation Through-Thickness

'q' *: stress-strain transfer parameter*



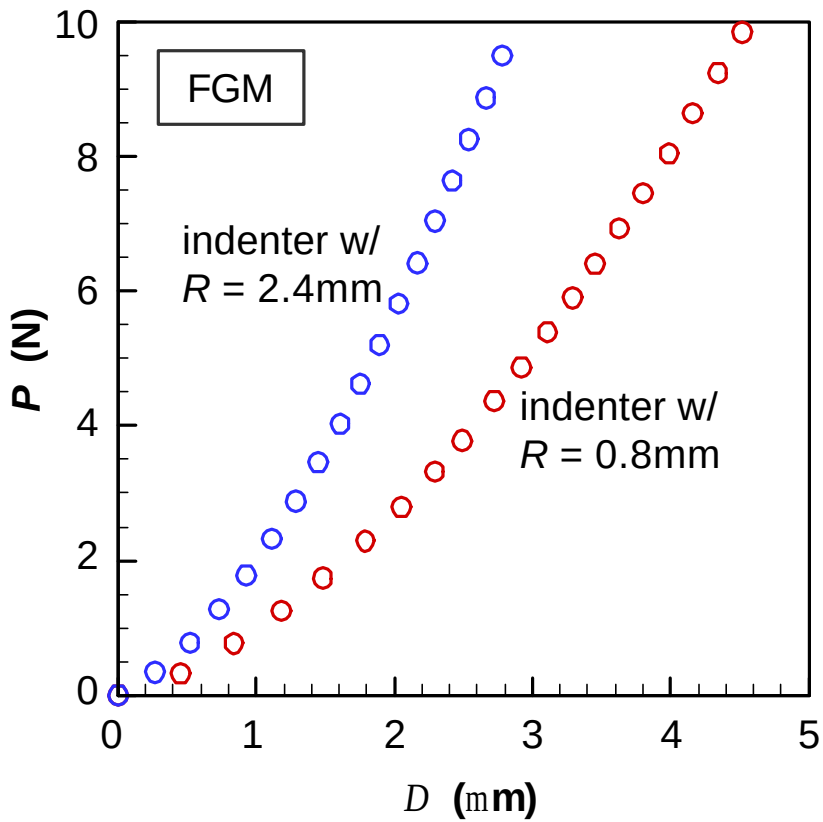
Reuss Model: $q = 0 \Rightarrow E = (V_1/E_1 + V_2/E_2)^{-1}$

Voigt Model: $q \to \infty \Rightarrow E = V_1E_1 + V_2E_2$

modified rule-of-mixtures: *Suresh & Mortensen (1998)*

Estimation Process

Input



Use measured load-displacement records at 20 load increments

Data Interpretation

INVERSE ANALYSIS

Kalman Filter and FE Models

Best Estimates

Identifies two unknown parameters

- ***Stress-strain relation (q)***
- ***Compositional variation (n)***

Kalman Filter Procedure

- ***Incremental process to obtain best estimates***
- ***Straightforward implementation in Fortran code***

Set $t = 0$

Assign initial estimates

$\boldsymbol{x}_o = (m_1, m_2)^T$

(state vector containing unknown parameters m_1, m_2)

Set $t = t + 1$

Compute Kalman Gain Matrix

$$\boldsymbol{K}_t = \boldsymbol{P}_t d_t^T \boldsymbol{R}_t^{-1}$$

where *matrix of covariance* (of estimated $\boldsymbol{x}_t$) is

$$\boldsymbol{P}_t = \boldsymbol{P}_{t-1} - \boldsymbol{P}_{t-1} d_t^T (d_t \boldsymbol{P}_{t-1} d_t^T + \boldsymbol{R}_t)^{-1} d_t \boldsymbol{P}_{t-1}$$

and *gradient of displacement* wrt to m_1 and m_2 is

$$d_t = \frac{\partial D_t(\boldsymbol{x}_{t-1})}{\partial \boldsymbol{x}}$$

Read in (measured) D_t^{meas} at increment t

Update $\boldsymbol{x}_t = (m_1, m_2)^T$

$$\boldsymbol{x}_t = \boldsymbol{x}_{t-1} + \boldsymbol{K}_t [D_t^{\text{meas}} - D_t(\boldsymbol{x}_{t-1})]$$

Check $t = t_{\max}$

Yes → Set $\boldsymbol{x}_{\text{est}} = \boldsymbol{x}_t$

No → Set $t = t + 1$

Reference Data Set

$\Rightarrow$ *to match the measured record with reference solutions*

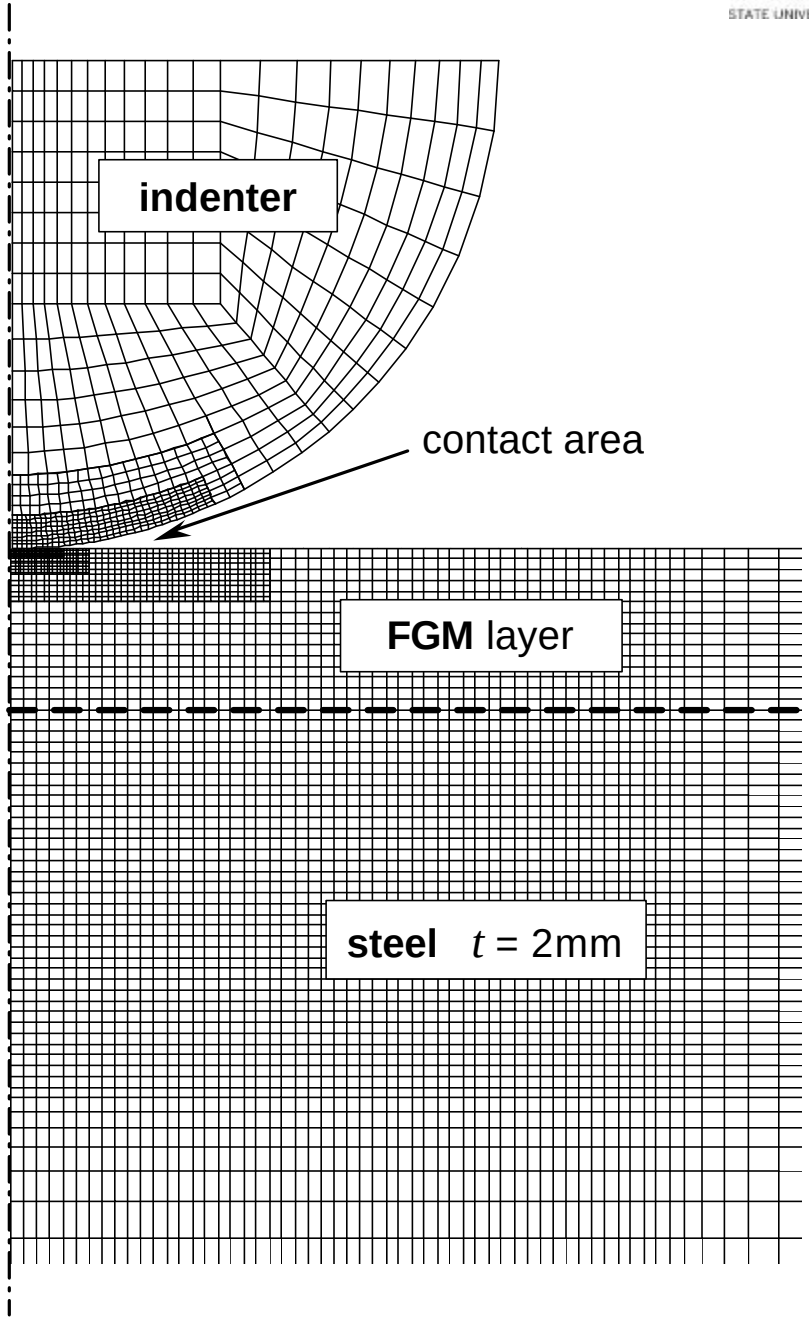
Measurement vector $D = (D_1, D_2)^T$ and its gradient d for a given set of m_1, m_2 at every time increment $\boldsymbol{t}$

$$D_t = D_t(m_1, m_2) \quad \text{and} \quad d_t = \begin{pmatrix} \dfrac{\partial D_t(m_1, m_2)}{\partial m_1} \\ \dfrac{\partial D_t(m_1, m_2)}{\partial m_2} \end{pmatrix}$$

Choose suitable values for *covariance matrix* of observation $\boldsymbol{R}_t$, (constant with t) and Initial *covariance matrix* $\boldsymbol{P}_o$ of estimated parameters.

Computational Analysis

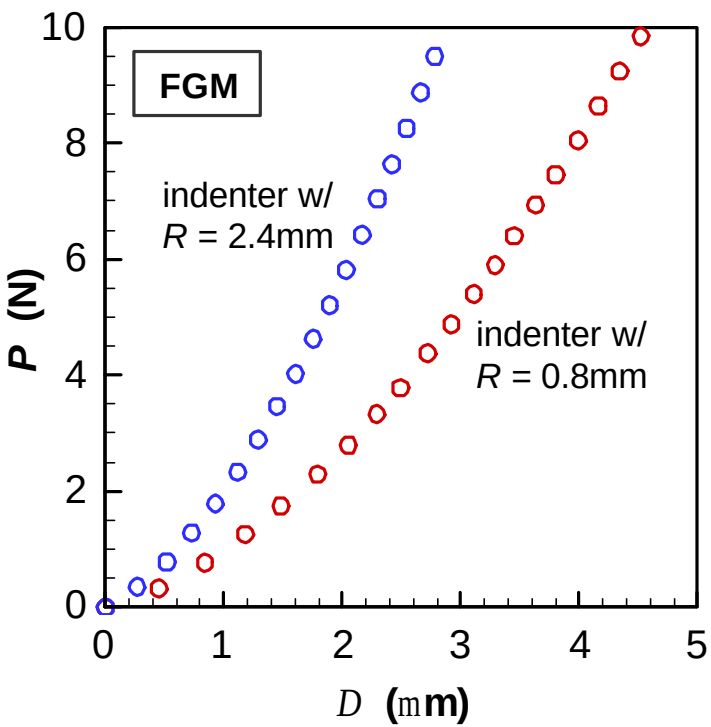
- Generates reference data set since no closed-form solution available.
- 6,600 axisymmetric elements.
- Displacement controlled loading.
- Contact condition w/ no friction between indenter and FGM layer.
- Large deformation analysis.
- Carried out 16 separate calculations for different values of *q* & *n*.
- Bi-cubic Lagrangian interpolation functions to determine *P-D* relations for any values of *q* & *n*.



Estimated FGM Parameters

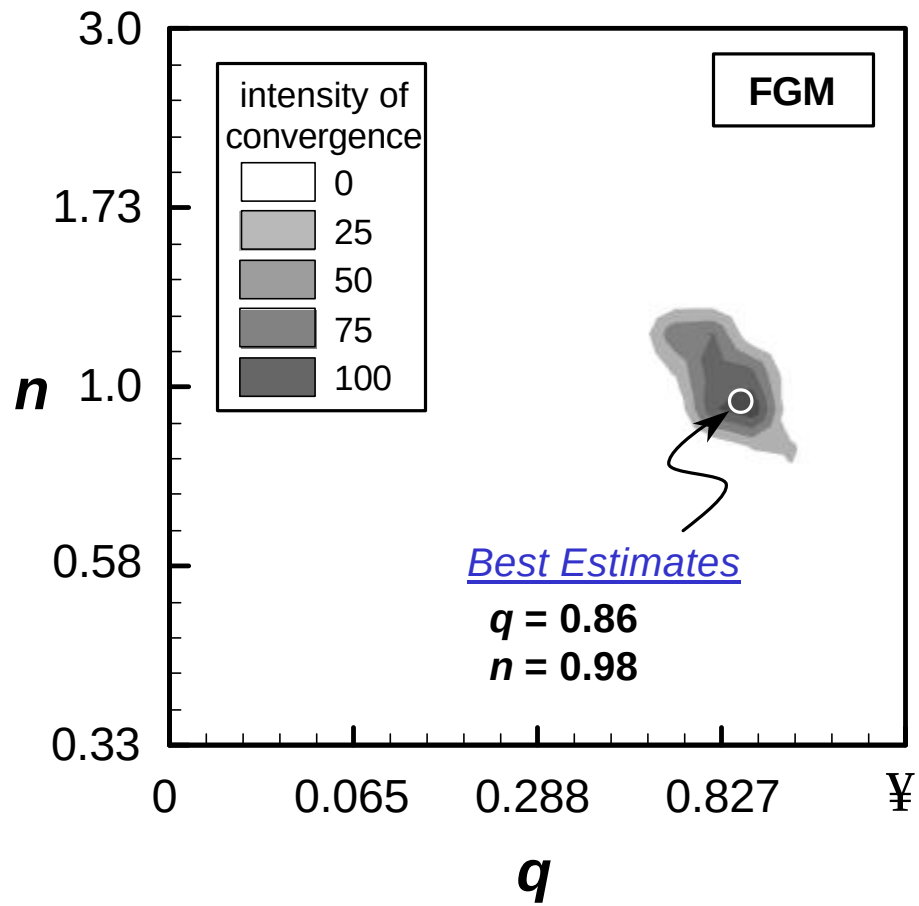
- ***Kalman Filter*** was carried out for 1,600 sets of initial estimates.
- In inverse analysis, 20 load increments are chosen as measurement data.

two load-displacement records



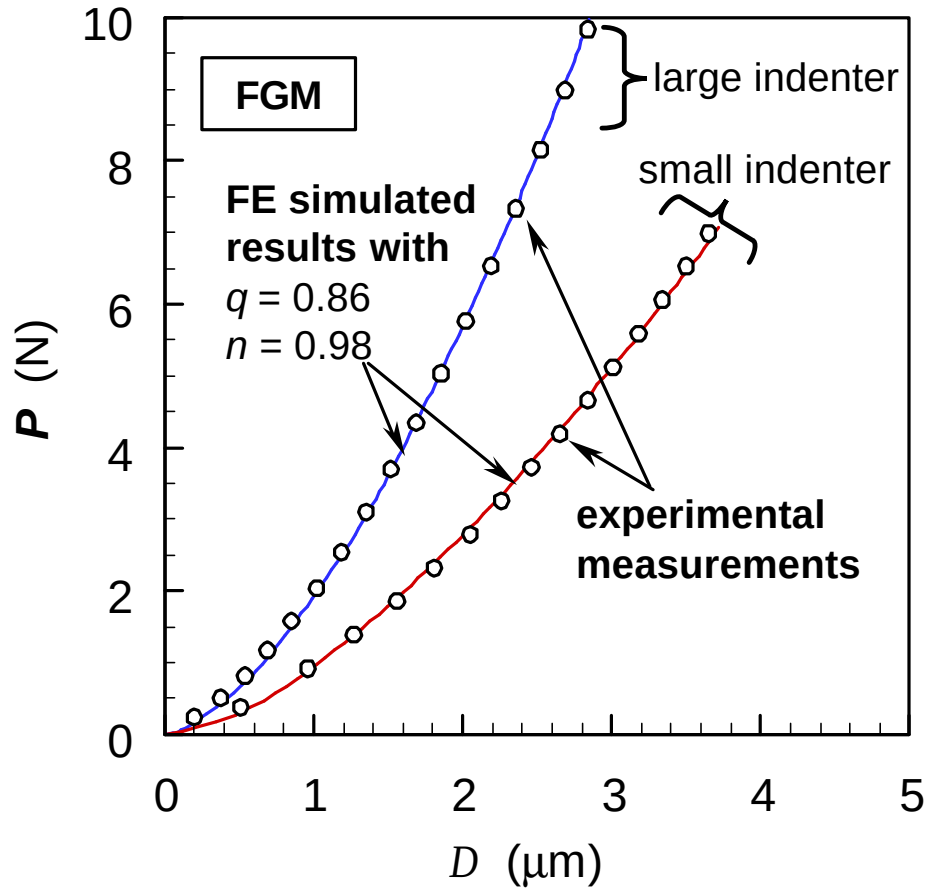
Kalman Filter

Intensity of converged initial estimates



Re-Verification with Estimated Parameters

Load-displacement from FE analysis and experimental record



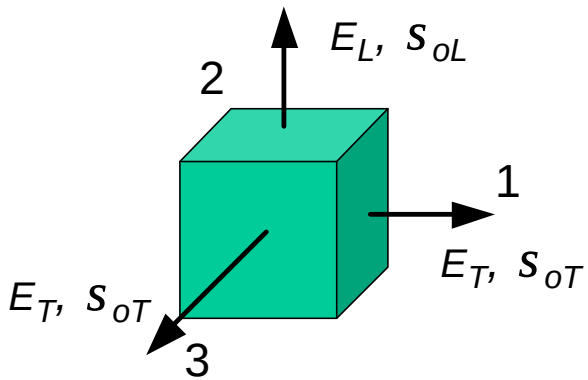
Excellent match between experimental records and FE simulation using the parameters extracted from inverse analysis shows good accuracy.

Application to Anisotropic/Transversely Isotropic Materials

Many coating and thin films exhibits anisotropy due fabrication processes

Three Unknown Material Parameters

- **Ratio of Young's Moduli** in two directions
- **Yield Stress** in longitudinal direction
- **Yield Stress** in transverse direction



Non-Dimensionalized Parameters:

$$E_L^* = \frac{E_L}{E_T} - 0.5$$

$$s_{oT}^* = \frac{s_{oT}}{s_o} - 1$$

$$s_{oL}^* = \frac{s_{oL}}{s_o} - 1$$

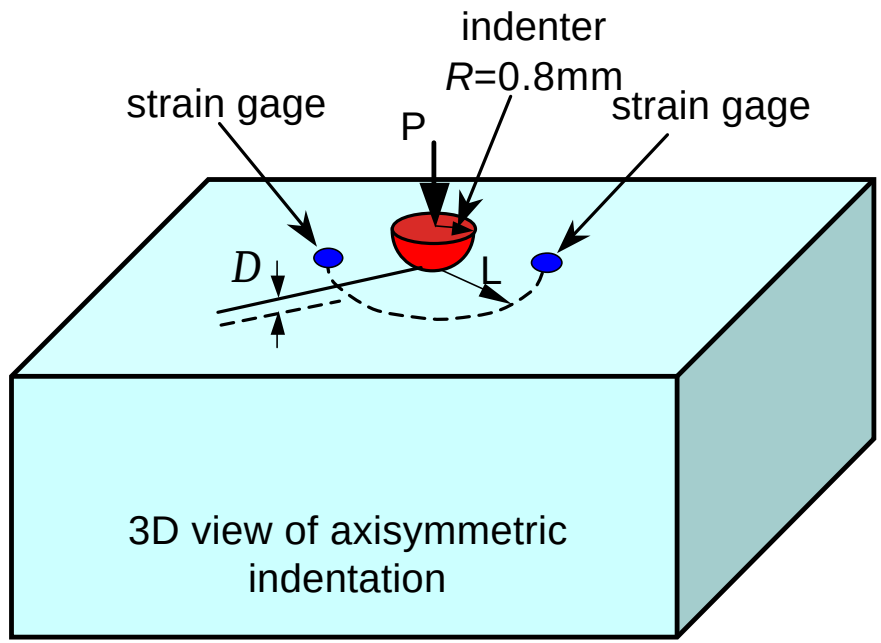
L : longitudinal direction

T : transverse direction

Proposed Measurement Procedure

- *Since there are more unknowns, P-D record is not sufficient to estimate the parameters, and* ***additional measurements*** *are required to make good estimates.*

- *Without making the experiments more complex,we choose to* ***add strain gages*** *to provide more information to Kalman Filter.*

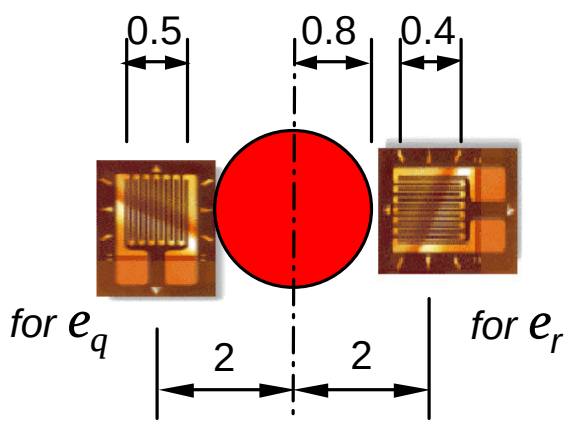


Required Measurements

- One **load-displacement** record
- Two **strain-displacement** records

Strain gages must be close enough to the indentation to detect changes due to anisotropic and elastic-plastic responses.

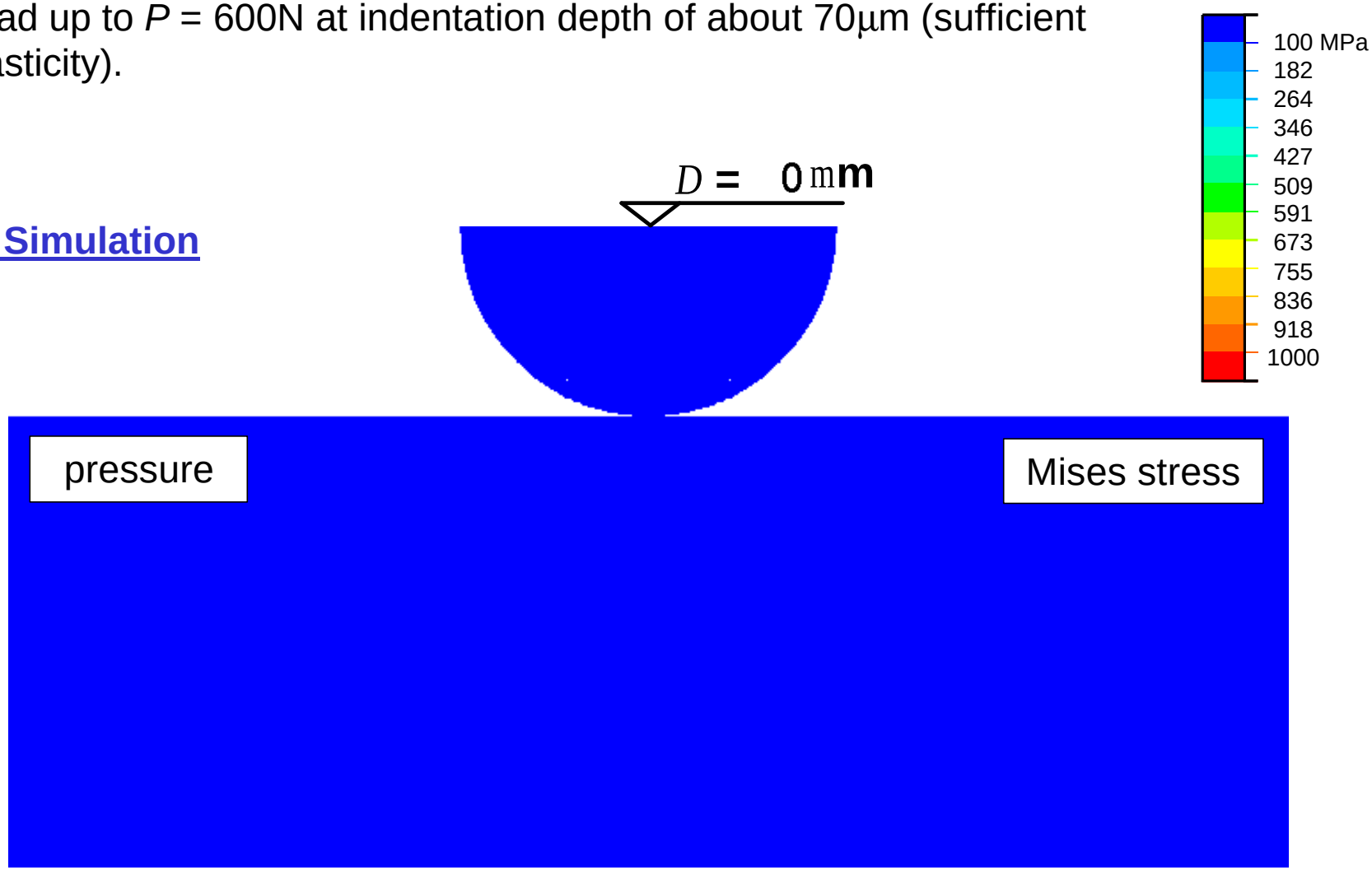
strain gage placements (unit in mm)



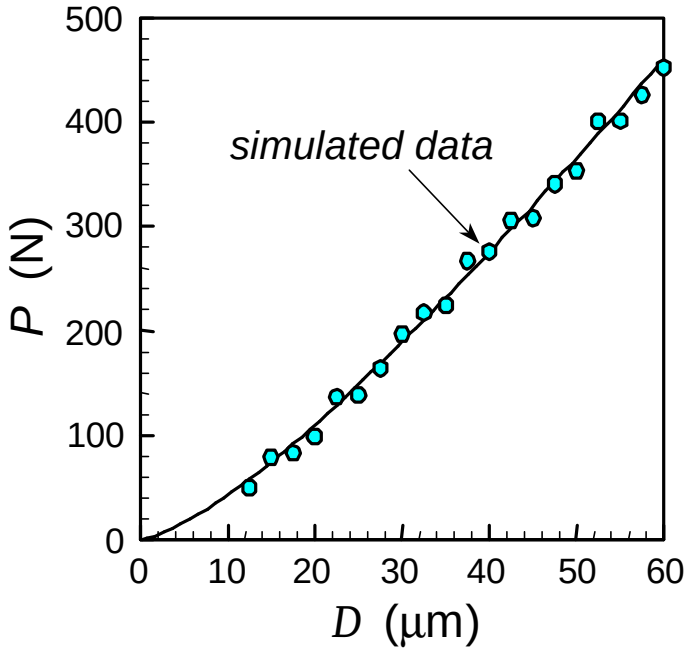
Planned Experiment Setup

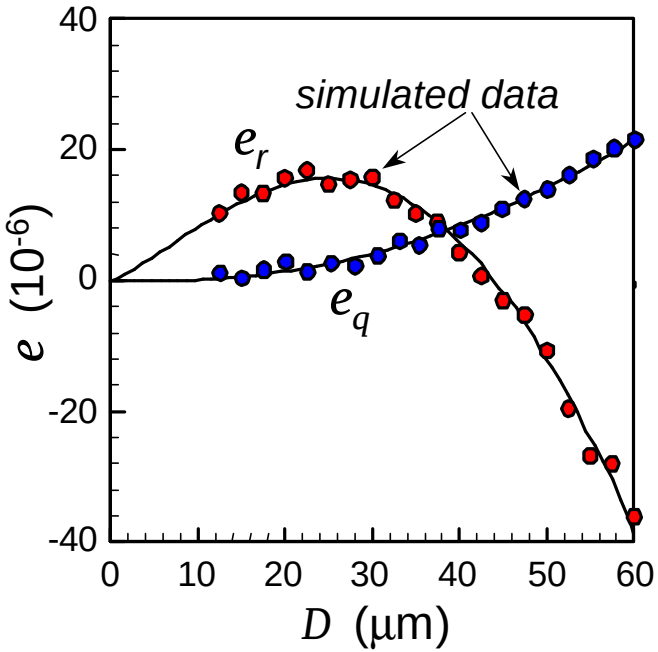
- Thermally sprayed 1.6mm thick NiCrAlY on 2mm thick steel substrate
- Use 1/16 inch (R = 0.8mm) diameter indenter.
- Load up to P = 600N at indentation depth of about 70μm (sufficient plasticity).

FE Simulation



FE Verification

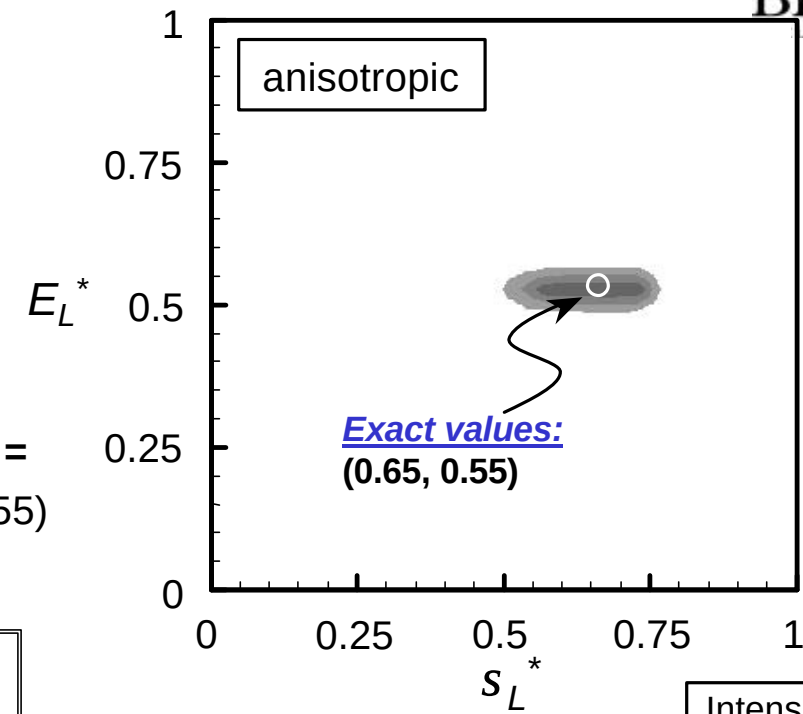


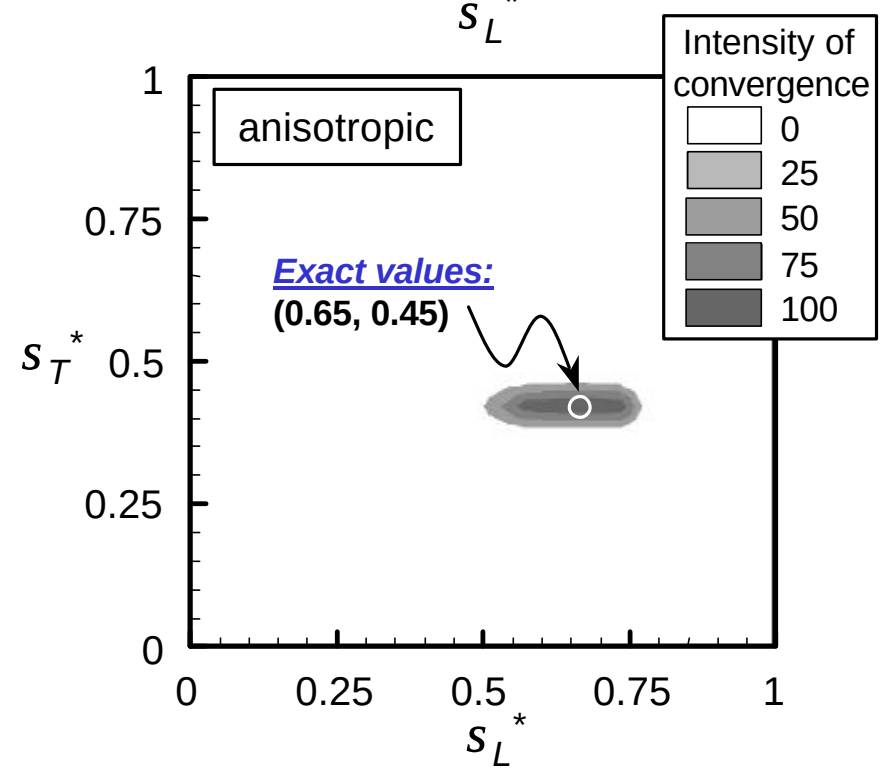


Exact values:

$(s^*_L,\ s^*_T,\ E^*_L)$ = (0.65, 0.45, 0.55)

Kalman Filter



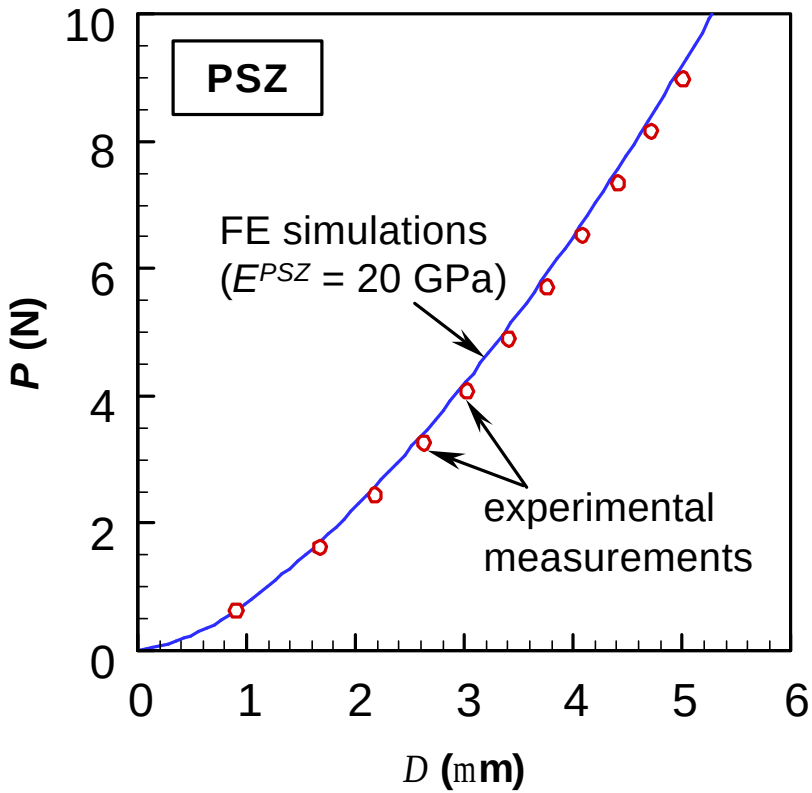


Summary

- A new procedure based on inverse analysis is introduced to measure elastic-plastic material properties of graded and anisotropic materials.
- The method uses load-displacement records of instrumented indentation.
- Parameters of graded material were determined using experimentally measured load-displacement records and Kalman Filter. Estimated properties agrees with the experimental records very well.
- Simulated study shows properties of elastic-plastic transversely isotropic materials can be estimated with single indentation when strain measurements near indentation were included.
- Kalman filter procedure can be extended/modified to determine other unknown parameters including geometrical dimensions (e.g., thickness).

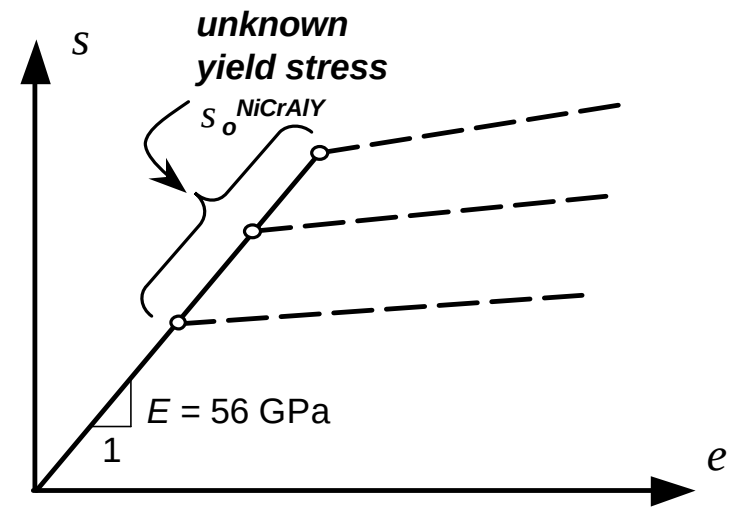
Material Properties of PSZ and NiCrAlY

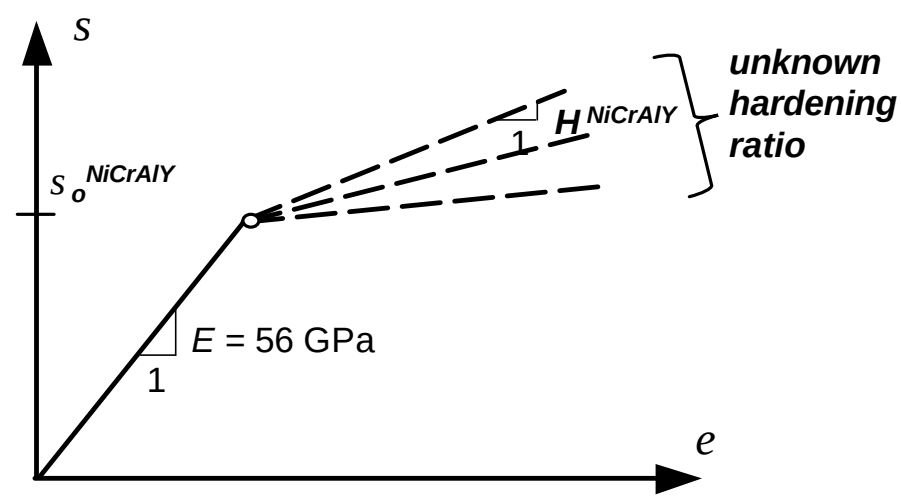
Linear Elastic PSZ



force-displacement results from both simulated analysis and experimental measurements

Elastic-Plastic NiCrAlY

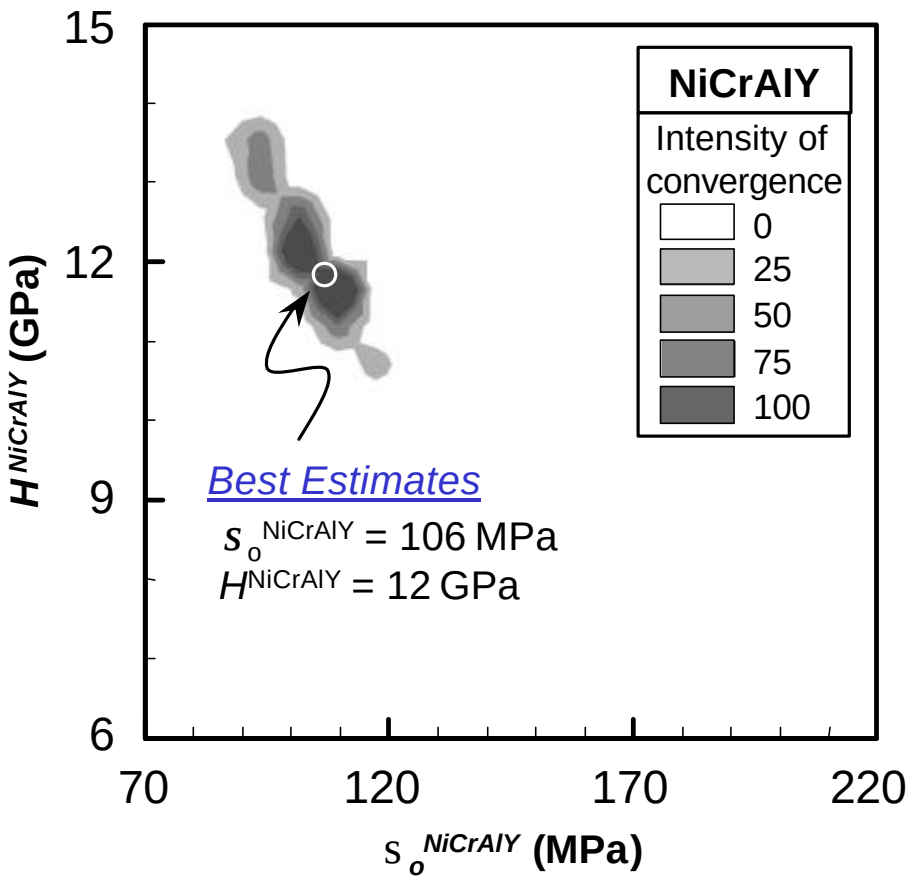




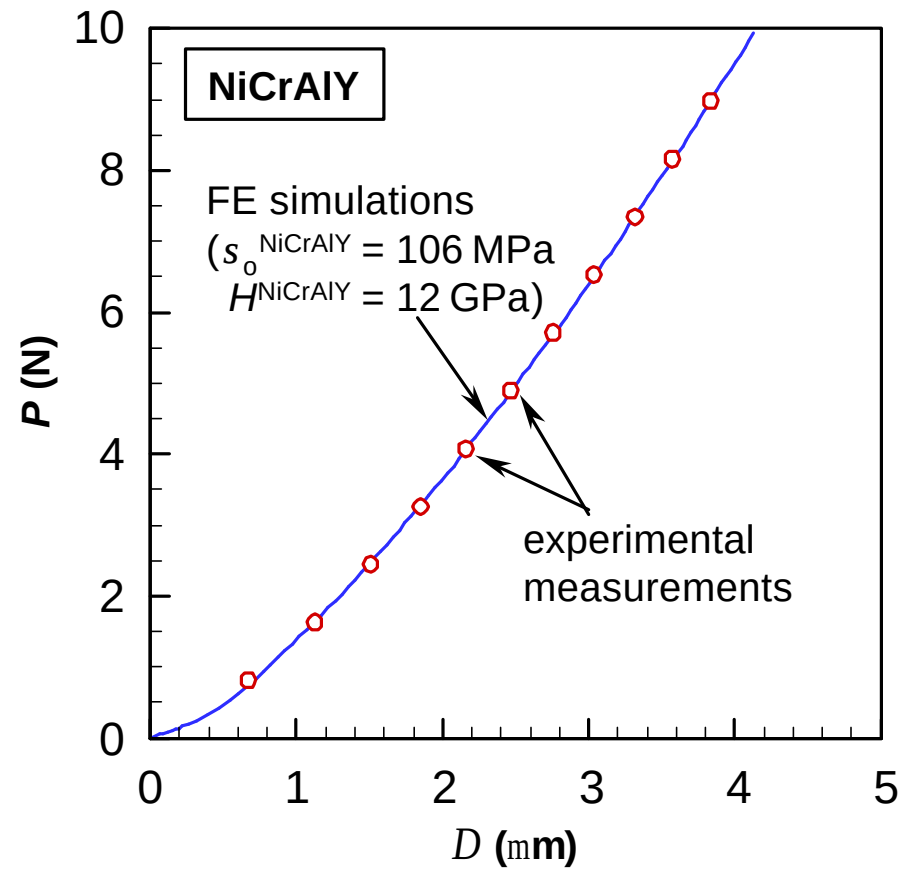
two unknown parameters

Characterization of Elastic-Plastic NiCrAlY

- ***Kalman Filter*** was carried out for 1,600 sets of initial estimates.
- In inverse analysis, 20 load increments are chosen as measurement data.

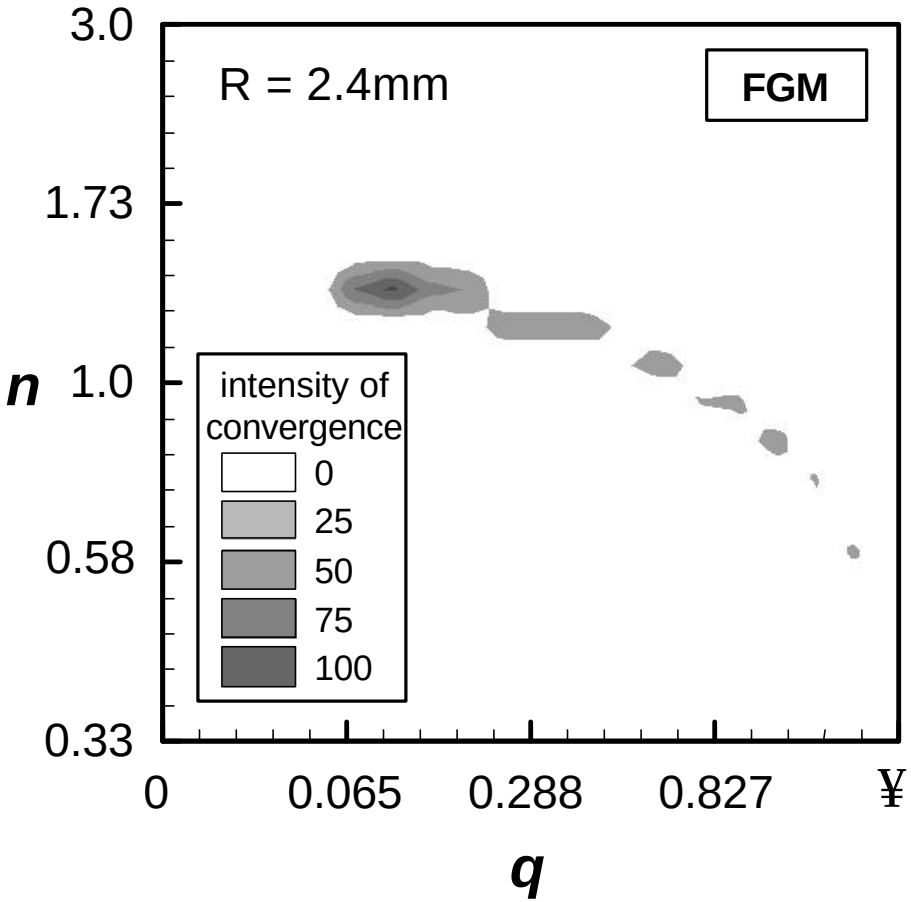


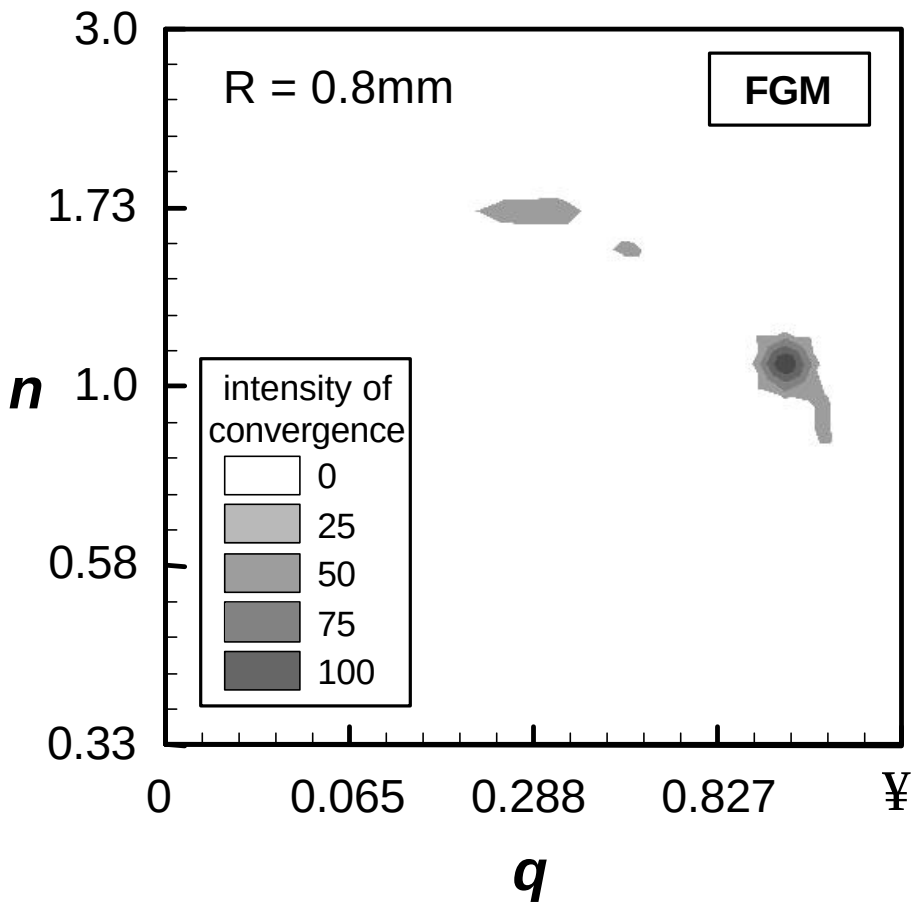
intensity of converged initial estimates for NiCrAlY



force-displacement results from both simulated analysis and experimental measurements

Compare to the Estimation Using Single Indenter





Scattered regions of convergence → Inaccurate.

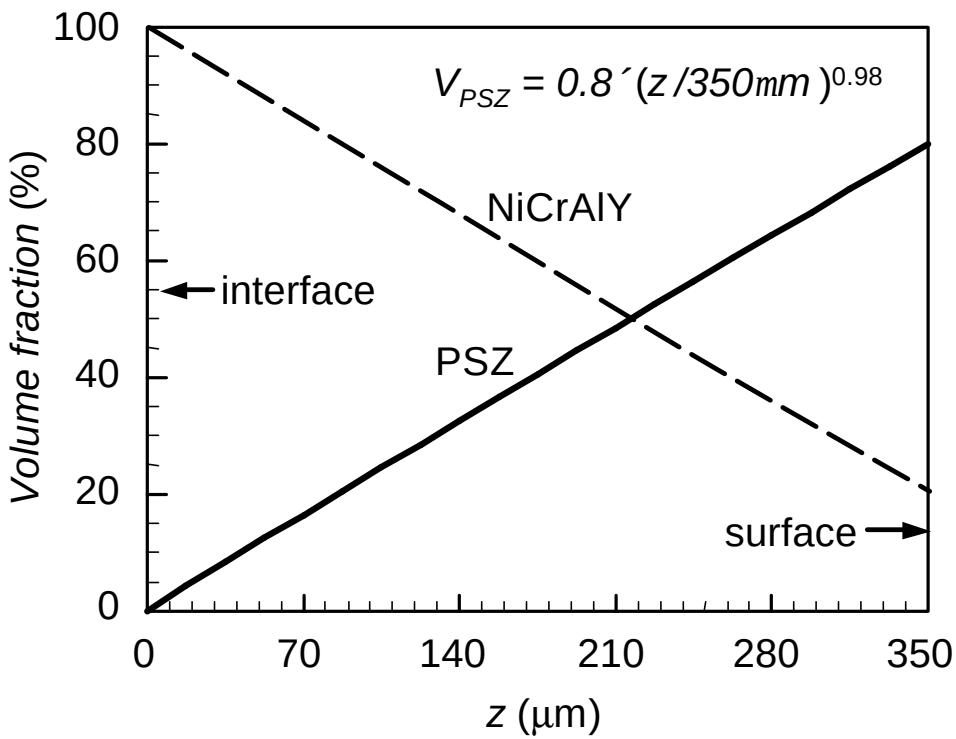
Measured Properties with Estimates

Estimated Parameters
q = 0.86, n = 0.98

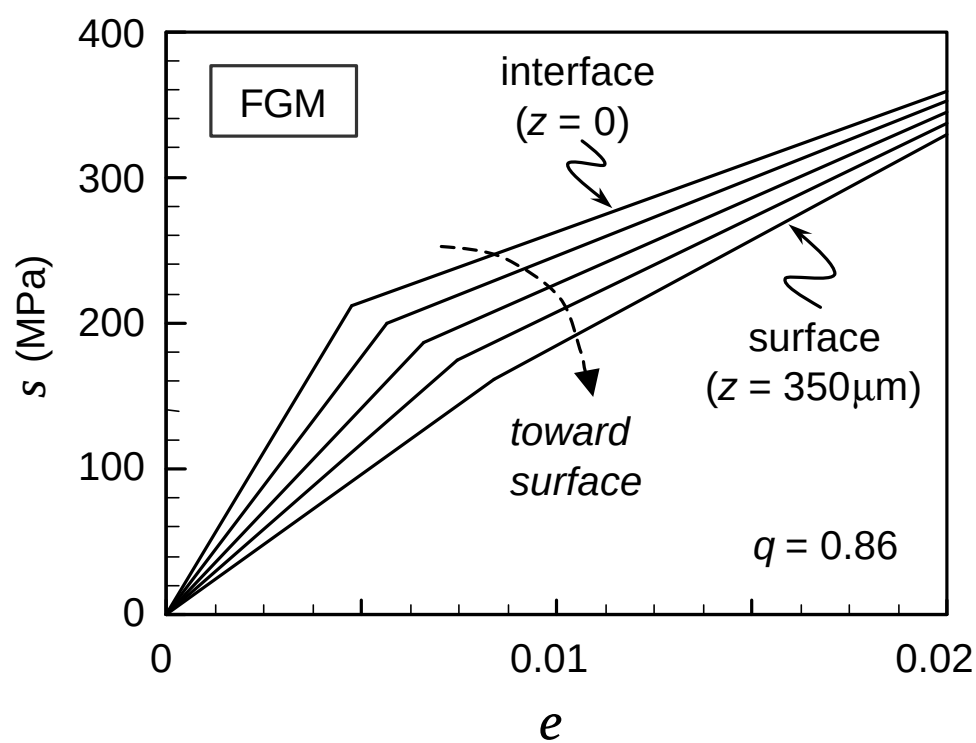
→

Material properties through thickness

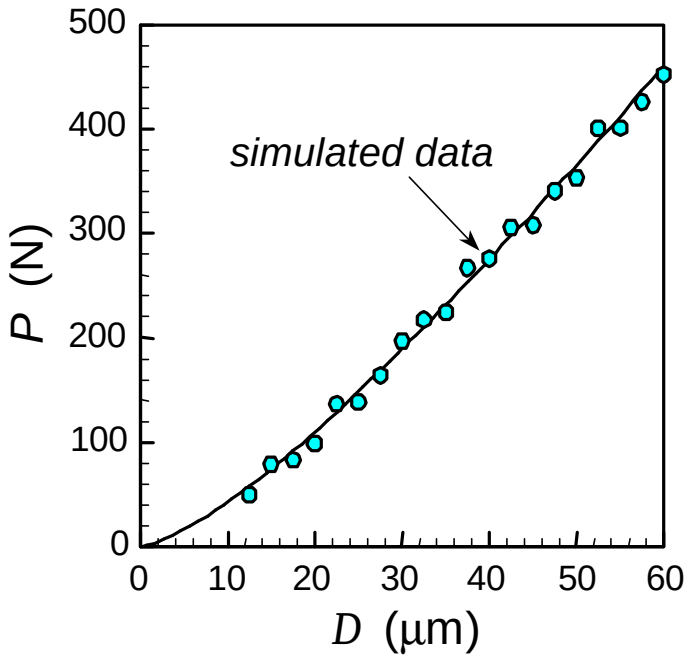
Variation of V_{psz} through thickness

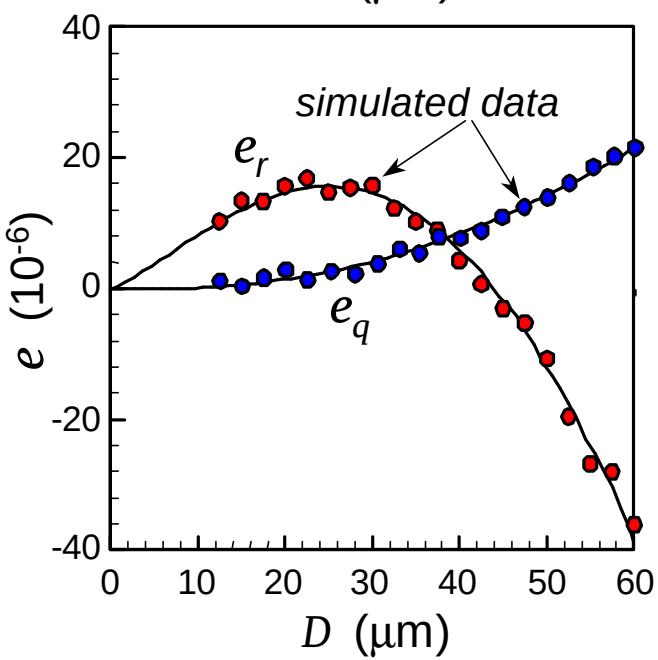


Stress-strain relation through thickness



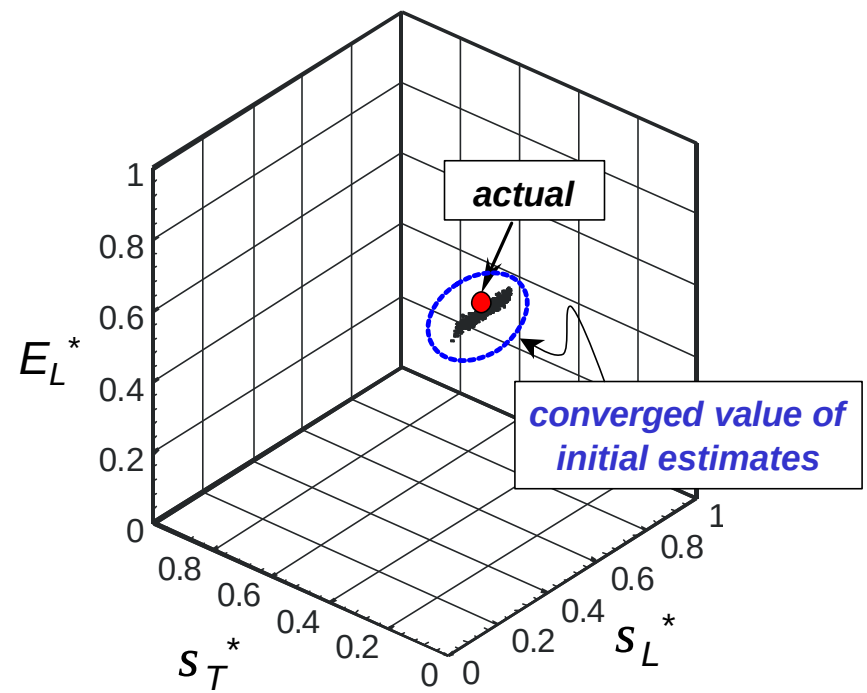
FE Verification of Proposed Procedure





Kalman Filter

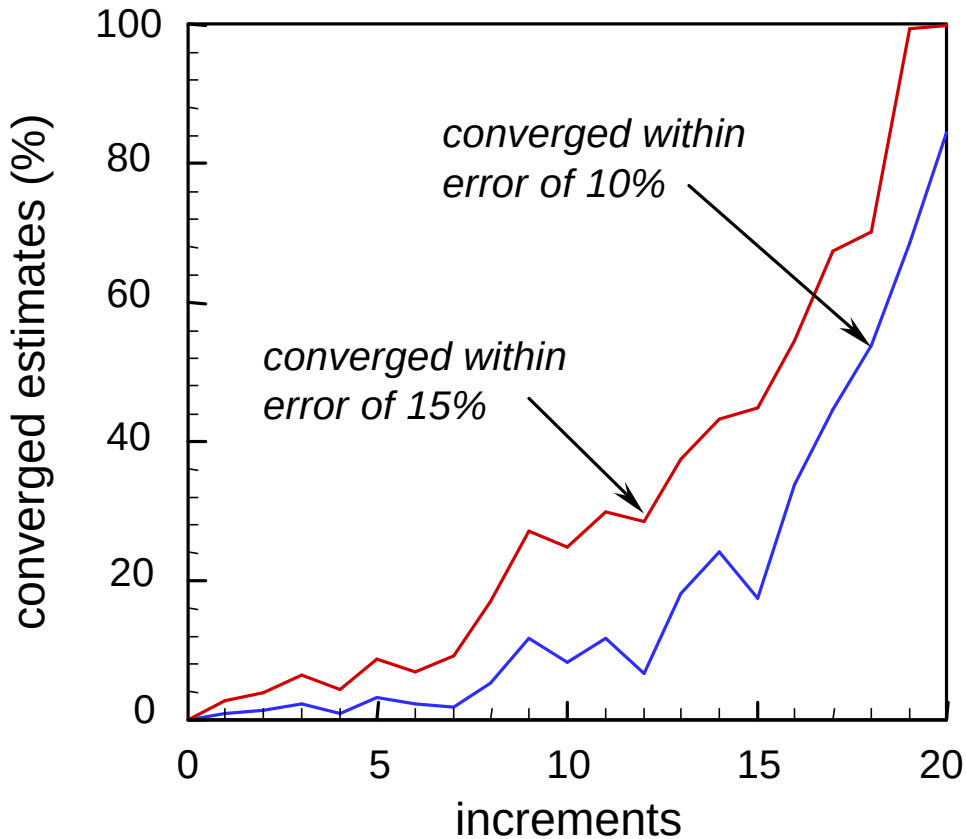
Final locations of all initial estimates: shown in 3 unknown parameter space



Actual values:
(s^*_L, s^*_T, E^*_L) = (0.65, 0.45, 0.55)

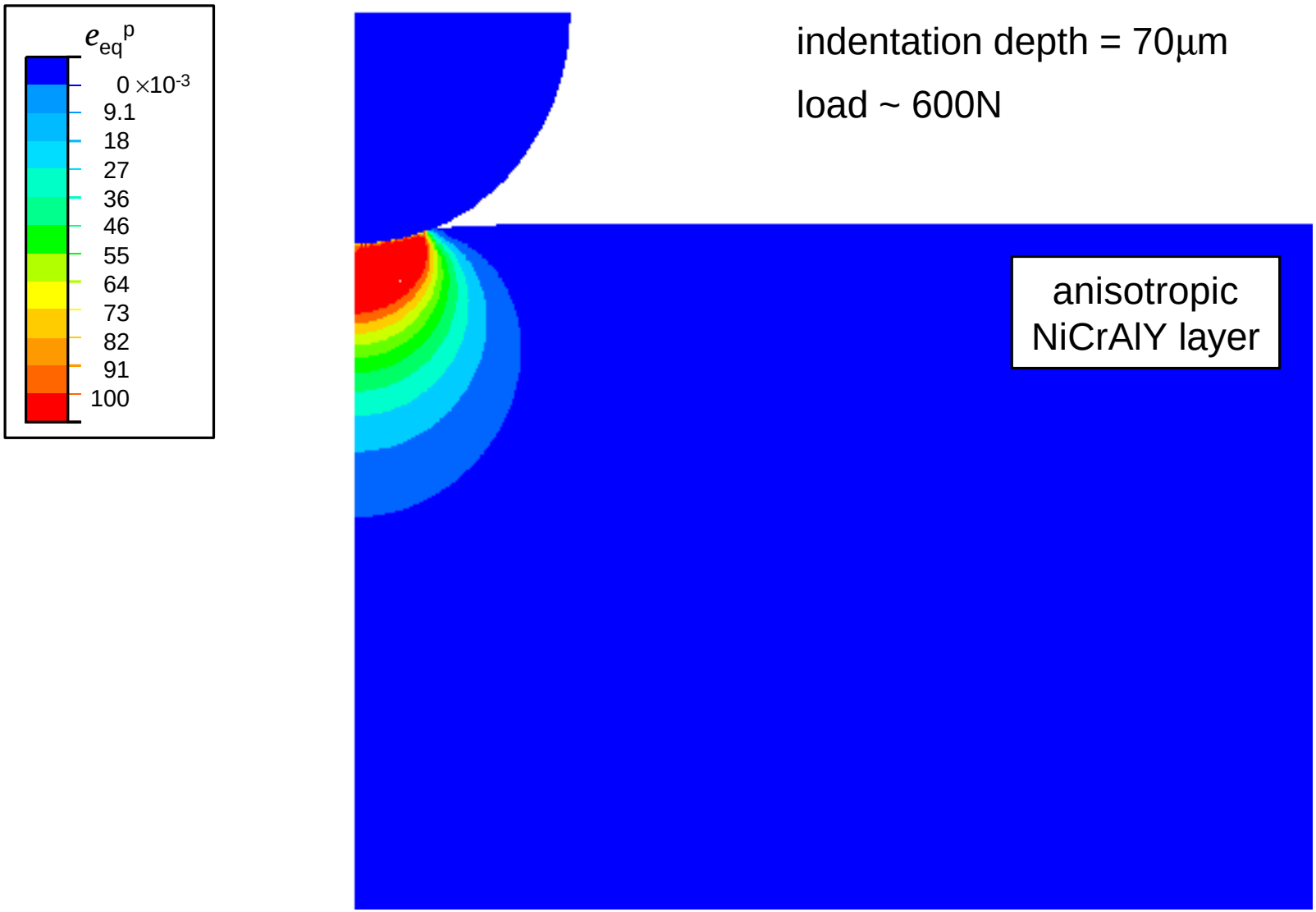
Schematic of the Convergence during the Inverse Analysis

- Input of **three measurements** gives excellent rate of convergence.
- *Kalman Filter* performs well with **large measurement error/noise**.



Percentage of initial estimates which converged close to the actual values

Equivalent Plastic Strain



FE Simulation

indentation depth = 70μm

load ~ 600N