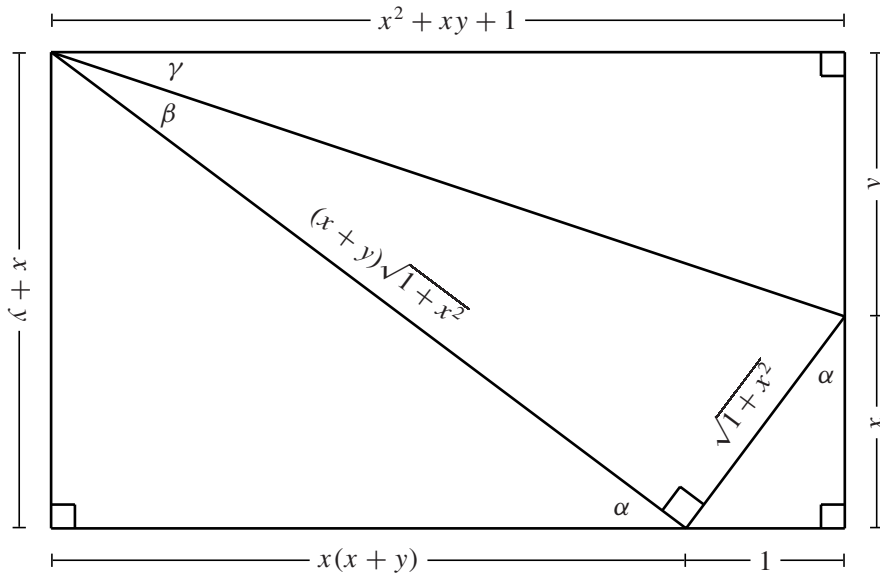


- 14. G. F. Simmons, *Differential Equations*, McGraw-Hill, 1972.
- 15. A. J. Simoson, The gravity of Hades, *this MAGAZINE*, **75**:5 (2002) 335–350.
- 16. Dava Sobel, *Galileo’s Daughter*, Penguin, 2000.
- 17. M. R. Spiegel, *Schaum’s Outline of Vector Analysis*, McGraw-Hill, 1959.
- 18. J. R. R. Tolkien, The Two Towers, *The Lord of the Rings*, Vol. II, Houghton Mifflin, 1994.

Proof Without Words: Euler’s Arctangent Identity

$$\tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}\left(\frac{1}{x+y}\right) + \tan^{-1}\left(\frac{y}{x^2+xy+1}\right)$$



$$\alpha = \beta + \gamma$$

This is one of the many elegant arctangent identities discovered by Leonhard Euler. He employed them in the computation of π . For $x = y = 1$, we have Euler’s Machin-like formula, $\pi/4 = \tan^{-1}(1/2) + \tan^{-1}(1/3)$. For $x = 2$ and $y = 1$, $\tan^{-1}(1/2) = \tan^{-1}(1/3) + \tan^{-1}(1/7)$. Substitute this into the previous identity, we obtain Hutton’s formula, $\pi/4 = 2 \tan^{-1}(1/3) + \tan^{-1}(1/7)$. In conjunction with the power series for arctangent, Hutton’s formula was used as a check by Clausen in 1847 in computing π to 248 decimal places.

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REFERENCES

- 1. P. Beckmann, *A History of π* , Golem Press, 1971, p. 154.
- 2. R. B. Nelsen, Proof without words: An arctangent identity and series, *this MAGAZINE* **64** (1991), 241.

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