Optics

Interference

Introduction

- Huygen suggested- light propagates in the form of waves

  nature of light?

  Whether transverse or longitudinal?

  Young (1801): First experimental evidence of wave nature given by Interference experiment

  He used superposition principle and Young’s concept of waves to explain the interference such as observed in double slit experiment

Superposition of waves

What happens when a number of disturbances reaches at a point simultaneously?

When two or more waves overlap, the resultant displacement at any point and any instant may be found by adding the instantaneous displacements

It means that the resultant is simply the sum of the disturbances

Interference: Superposition of coherent waves

Interference

Two or more light waves of the same frequency overlap at a point, the resultant effect depends on the phases of the waves as well as their amplitudes

The resultant wave: Governed by principle of superposition

Combined effect at each point of superposition- algebraic sum of the amplitudes of individual waves

If two waves are in phase, the amplitude of the resultant wave

$$A_R = A + A = 2A$$

Intensity of the resultant wave is

$$I_R = A_R^2 = 2^2 A^2 = 4I$$

Interference produced at these points: **Constructive Interference**

- If two waves are opposite in phase

$$A_R = A - A = 0$$

The intensity of the resultant wave is zero

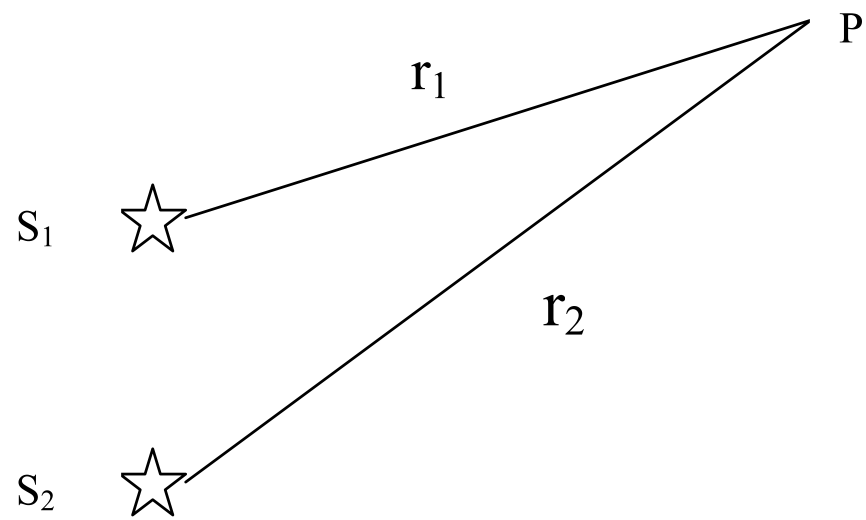
Interference produced at these points: Destructive **Interference**

The phenomenon of redistribution of energy due to superposition of light waves from two or more coherent sources is known as Interference

Collision of waves When two waves traveling in opposite directions through the same medium collide, the amplitude of the resulting wave will be the sum of the two initial waves. This is called interference and there are of two types:

Constructive interference is when the amplitudes of the initial waves are in the same direction. The resulting wave will be larger than the original waves. The highest point of a constructive interference is called an antinode.

Destructive interference is when the amplitudes of the initial waves are opposite. The amplitude of the resulting wave will be zero. The point in the middle of a destructive interference is called a node and it never moves


P
r1
S1
r2
S2

Whether we get brightness or darkness at point P due to superposition of two waves reaching from S_1 and S_2?

If the optical path difference $\Delta = (\mu_2 r_2 - \mu_1 r_1)$ is zero or integral multiple of λ waves arrives in phase at P and crest to crest correspondences

$$\Delta = m\lambda$$

where m is an integer and m= 1, 2, 3, 4……. etc.

Constructive interference or brightness is observed at the point P

if path difference $\Delta = (2m + 1)\lambda/2$ destructive interference or darkness is observed at point P

- Theory of interference

1. Analytical Treatment

   Let the electric field components of the two waves reaching at point P vary with time as

$$E_A = E_1 \sin \omega t$$

$$E_B = E_2 \sin(\omega t + \delta)$$

The resultant electric field

$$E_R = E_A + E_B$$
$$= (E_1 + E_2 \cos \delta) \sin \omega t + E_2 \sin \delta \cos \omega t$$

let

$$E_1 + E_2 \cos \delta = E \cos \phi$$
$$E_2 \sin \delta = E \sin \phi$$

Where E is the amplitude of the resultant wave and ϕ is the new initial phase angle

$$E^2 = E_1^2 + E_2^2 + 2E_1E_2\cos\delta$$

Intensity Distribution

$$I \propto E^2$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

whenever the phase difference between the waves is zero, i.e. $\delta = 0$ we get maximum amount of light

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$when\, I_1 = I_2 = I_0$$

$$I_{\max} = 4I_0$$

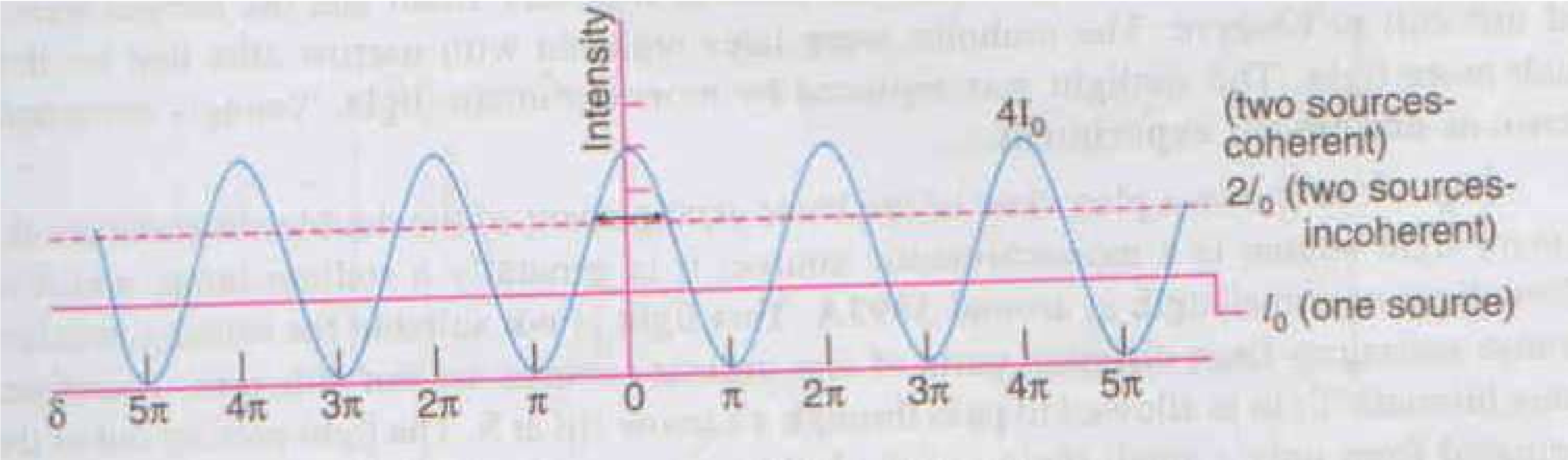
When the phase difference is $\delta = 180^0$ we have a minimum amount of light

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

At the points that lie between the maxima and minima, when $I_1 = I_2 = I_0$

$$I = 4I_0 \cos^2 \frac{\delta}{2}$$

The intensity can be shown as follows



Intensity
4I₀
(two sources-
coherent)
2I₀ (two sources-
incoherent)
I₀ (one source)
δ
5π
4π
3π
2π
π
0
π
2π
3π
4π
5π

Superposition of incoherent waves

Incoherent waves: waves that don't maintain a constant phase difference

$$I_{ave} = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle\cos\delta\rangle$$

the average value of the cosine over a large interval will be zero

$$I_{ave} = I_1 + I_2$$

♣♣ superposition of incoherent waves does not produce interference but gives a uniform illumination

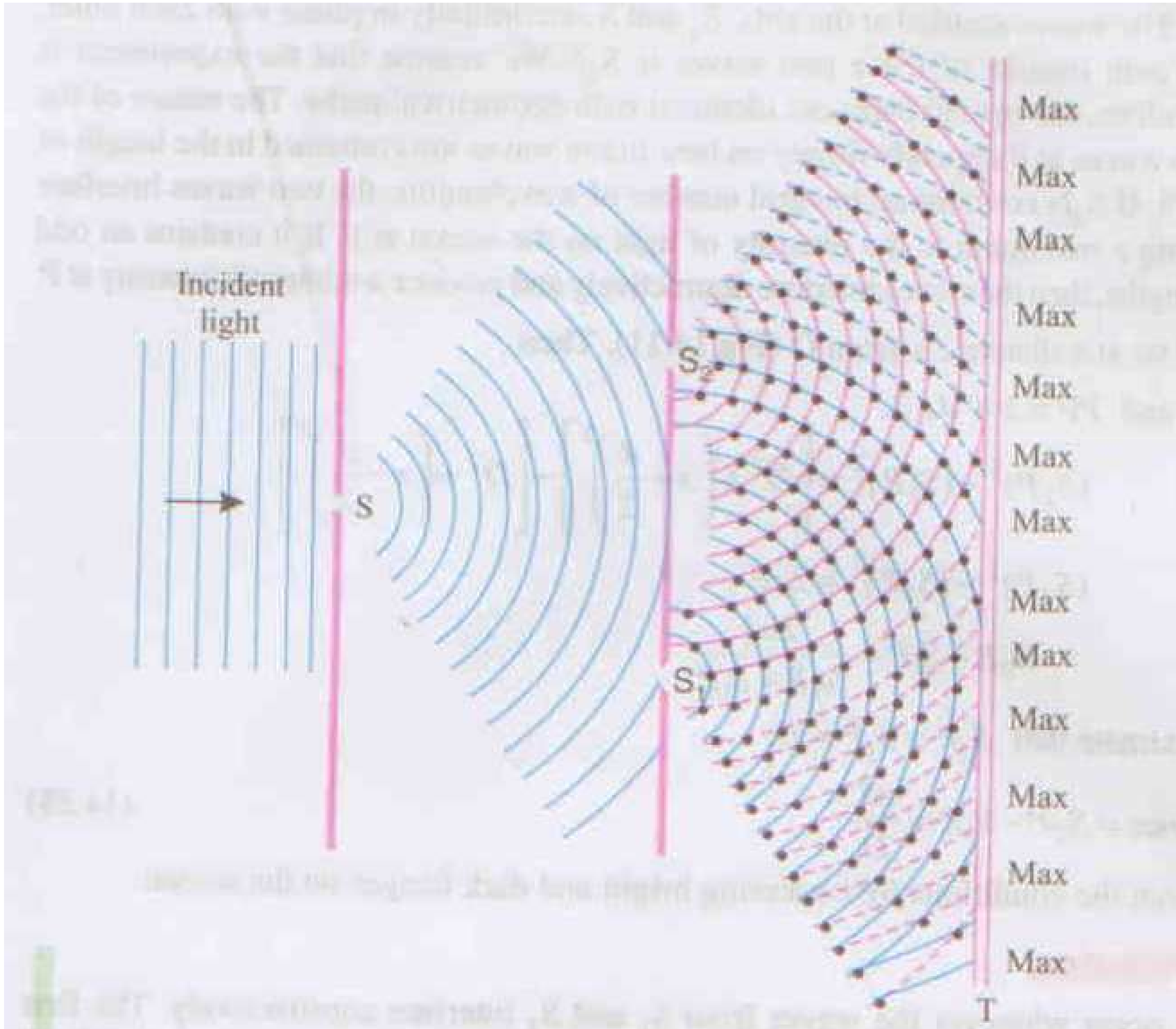
Superposition of many coherent waves

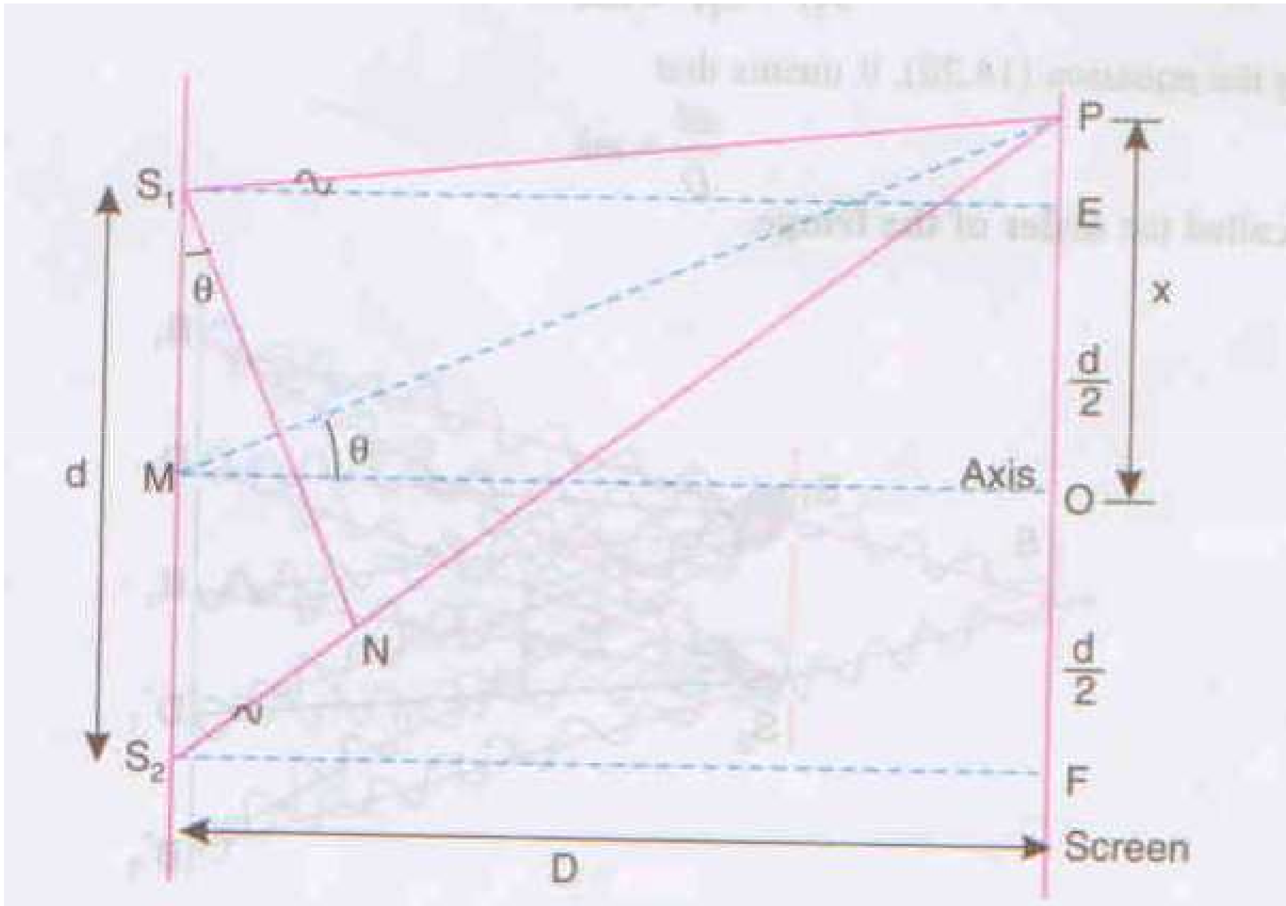
resultant intensity due to two coherent waves $I_{\max} = 2^2 I_0$

The resultant maximum intensity due to N coherent waves will be therefore

$$I_{\max} = N^2 I_0$$

Young's double slit experiment:





P
S₁
α
E
θ
x
d
2
θ
d
M
Axis
O
N
d
2
α
S₂
F
D
Screen

Optical path difference between the waves at P

let the point P be at a distance x from O, then PQ = x - d/2 and PR = x + d/2

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

$$(S_2P)^2 - (S_1P)^2 = 2xd$$

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

we can approximate $S_2P \cong S_1P \cong D$

So path difference = $S_2P - S_1P = \dfrac{xd}{D}$

Bright Fringes: waves from S_1 and S_2 interfere constructively

Centre point O- what happens at O?

waves from S_1 and S_2 travel the same optical path length to O and arrive in phase – A bright fringe is observed

The condition for finding a bright fringe at P is

$$S_2P - S_1P = m\lambda$$

it means that $\dfrac{xd}{D} = m\lambda$

Dark Fringes: The waves are in opposite phase

$$\frac{xd}{D} = (2m+1)\frac{\lambda}{2}$$

Separation between neighbouring fringes: **Fringe Width**

The m[th] order fringe occurs when

$$x_m = \frac{m\lambda D}{d}$$

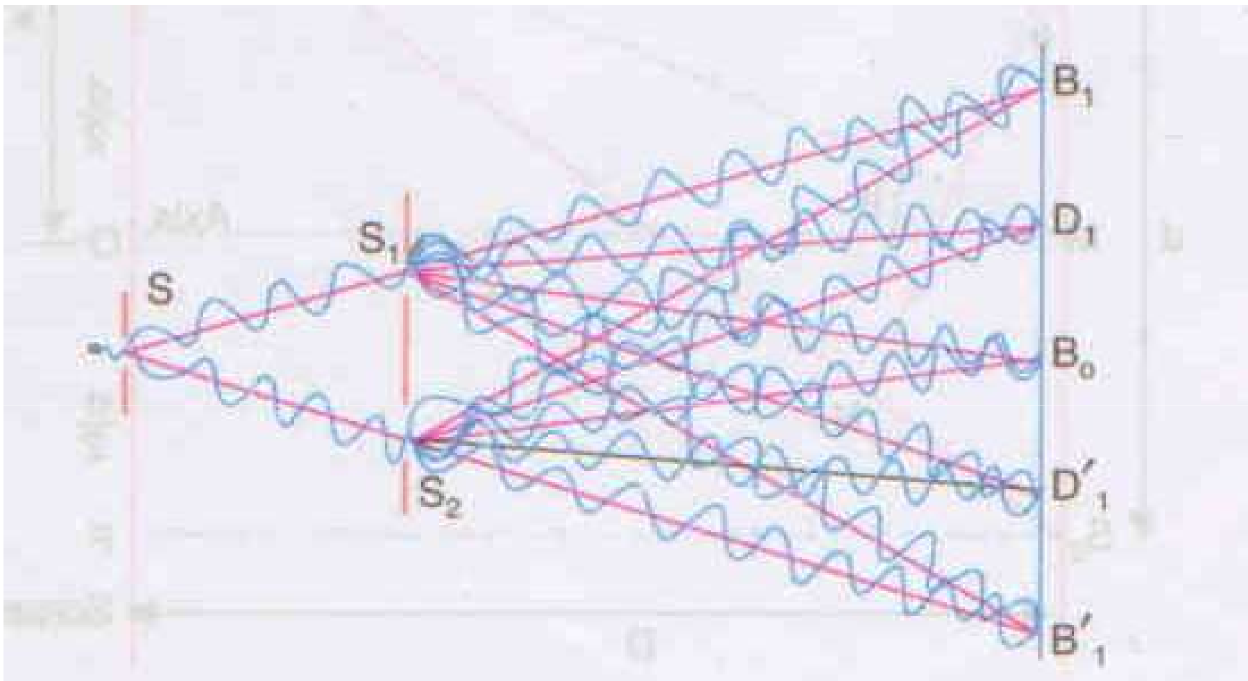
The (m+1)[th] order fringe occurs when

$$x_{m+1} = \frac{(m+1)\lambda D}{d}$$

The fringe separation, β is given by

$$\beta = x_{m+1} - x_m = \frac{\lambda D}{d}$$

Fringe Width: The distance between any two consecutive bright and dark fringes is known as the fringe width and is same for bright as well dark fringes.


S
S1
S2
B1
D1
B0
D′1
B′1

Coherence

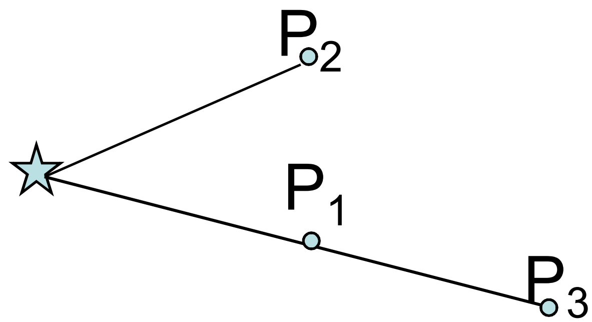
Connection between the phase of light waves at one point and time, and the phase of the light waves at another point and time.

Coherence Time: It is the average time during which the wave remains sinusoidal and phase of the wave packet can be predicted reliably

Coherence effects are mainly divided into two parts;

1. Temporal coherence
2. Spatial coherence

Temporal coherence: The temporal coherence is directly related to the finite bandwidth of the source. It is also known as **longitudinal coherence.**



Coherence Length: The length of the wave over which it may be assumed to have sinusoidal character and predictable phase

$$l_{coh} = c\,\Delta t$$

$$t_{coh} = \Delta t$$

$$\therefore l_{coh} = c\,t_{coh}$$

A wave train consists of a group of waves, which have a continuous spread of wavelength over a finite range. According to Fourier analysis the frequency bandwidth is given by

$$\Delta\nu = \frac{1}{\Delta t}$$

where Δt is the average lifetime of the excited state of the atom.

$$\Delta\nu = \frac{1}{\Delta t} = \frac{1}{t_{coh}}$$

$$\Delta\nu = \frac{c}{l_{coh}}$$

Relation between coherence length and bandwidth

The frequency and wavelength of a light wave are related as

$$\nu = \frac{c}{\lambda}$$

$$\Delta\nu = -\frac{c}{\lambda^2}\Delta\lambda$$

$$\frac{c}{l_{coh}} = -\frac{c}{\lambda^2}\Delta\lambda$$

$$so$$

$$l_{coh} = \frac{\lambda^2}{\Delta\lambda}$$

Spatial Coherence

Spatial coherence refers to the continuity and uniformity of a wave in a direction perpendicular to the direction of propagation. If the phase difference for any two fixed points in a plane normal to the wave propagation does not vary with time, then the wave is said to exhibit spatial coherence.

It is also known as *lateral coherence*

Techniques of Obtaining Interference

- Wavefront Splitting: Dividing a light wavefront emerging from a narrow slit, by passing it through two slits closely spaced. Method useful only with narrow sources. The two parts travel through different paths and reunite to produce interference fringes.

  Examples: Young's Double Slit, Fresnel's double mirror, Fresnel's Bi-prism etc.

- Amplitude Splitting: Amplitude (intensity) of a light wave is divided into two parts by reflection at a surface (reflected and transmitted components are obtained from this process). The two parts travel through different paths and reunite to produce interference fringes.

  Beam splitters, Mirrors are used to produce amplitude division.

  Examples: Newton's rings, Michelson interferometer etc

Fresnel Bi-Prism

The Bi-prism consists of two prisms of very small refracting angles joined base to base.

In practice, a thin glass plate is taken and one of its face is ground and polished till a prism is formed with an obtuse angle of about 179^0 and two side angles of the order of $30'$.

When a light ray is incident on an ordinary prism, the ray is bent through an angle called the *angle of deviation*. As a result, the ray emerging out of the prism appears to have emanated from a source located at a small distance above the real source. Thus we can say that prism produces a *virtual image* of the source.

A bi-prism in the same way produces two virtual sources from a single source.

The two virtual sources are the images of the same source S produced by refraction and are hence coherent.

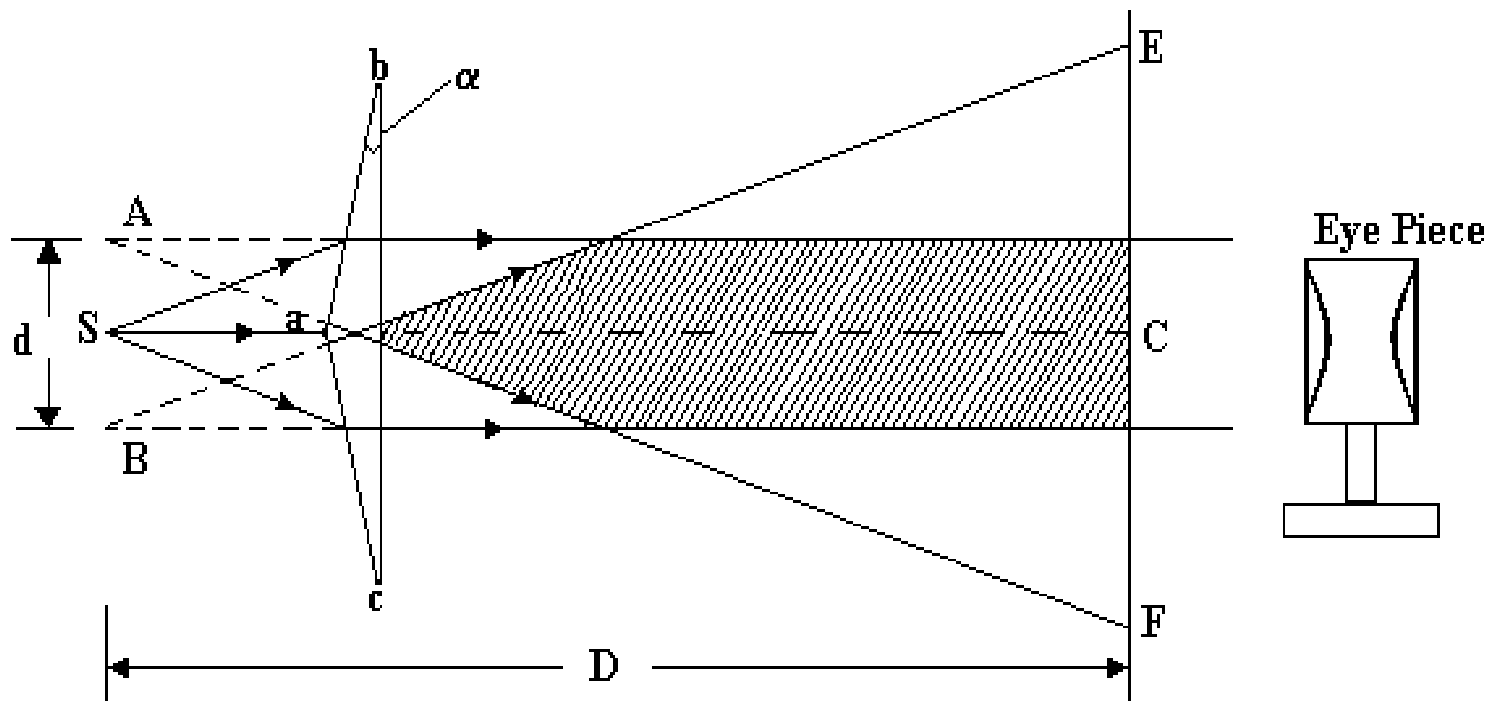


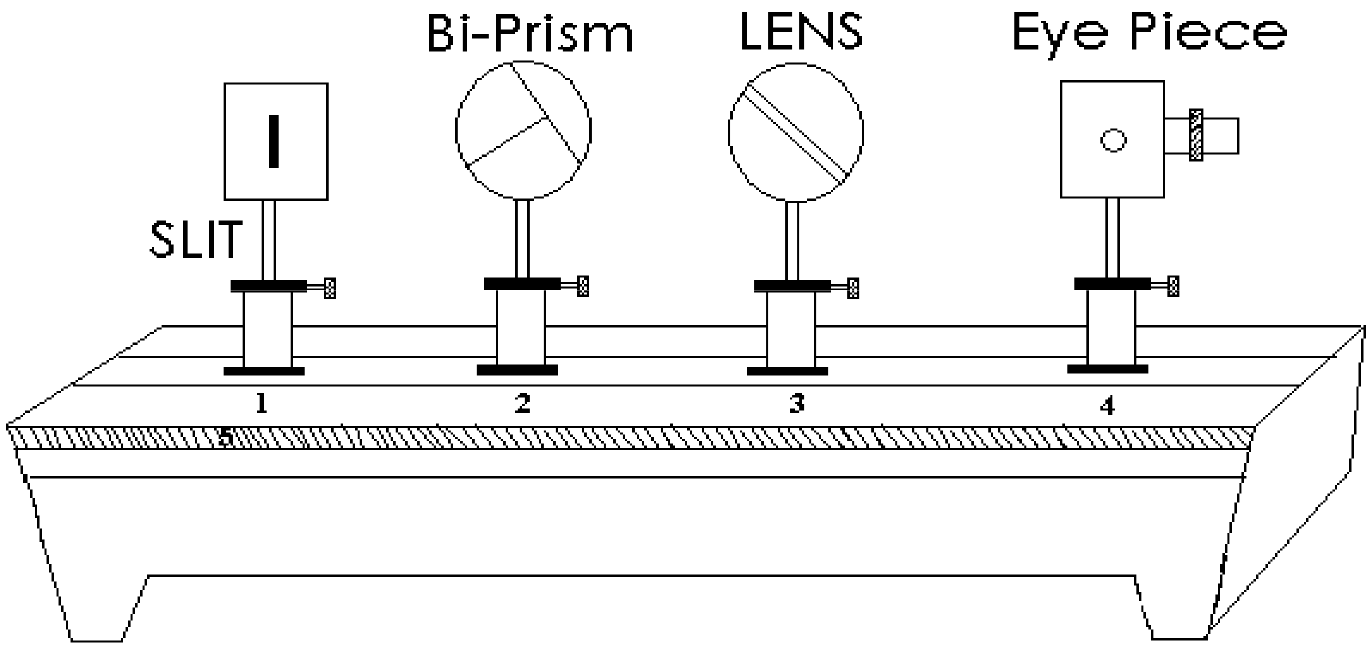
Fig. 1

When light falls from source S on the lower portion of the biprism it is bent upwards and appears to come from the virtual source B. Similarly light falling from S on the upper portion of the prism is bent downwards and appears to come from the virtual source A. Therefore A and B act as two coherent sources. Suppose the distance between A and B = d.

If a screen is placed at C, interference fringes of equal width are seen in the field of view of eye piece.

The point C is equidistant from A and B. On both sides of C, alternately bright and dark fringes are produced. The width of the bright fringe or dark fringe $\beta = \lambda D/d$ where D is the distance between light source and eye piece.

Experimental Arrangement:



Theory:

The wavelength of the sodium light is given by

$$\lambda = \frac{\beta\, d}{D}$$

where β = fringe width,

D = distance between the slit and eye piece

d = distance between the two virtual sources

$= \sqrt{d_1 d_2}$

where d_1 = distance between the two images formed by the convex lens 1st position.

d_2 = distance between the two images formed by the convex lens the 2nd position.

Determination of *'d'*

(i) A convex lens is moved back and forth near the bi-prism till a sharp pair of images of the slit is obtained in the field of view of the eyepiece. Let the distance between the images be d_1.

if u is the distance of the slit and v that of the eyepiece from the lens, then the magnification is

$$v/u = d_1/d$$

The lens is then moved to a position nearer to the eyepiece, where again a pair of images of the slit are seen. Now, let the distance between the two images be d_2. The magnification is given by

$$u/v = d_2/d$$

From above two equations $d = \sqrt{d_1 d_2}$

Alternate Method to Find *‘d’* (Theoretical method)

The deviation produced in the path of ray by a thin prism is given by

$$\delta = (\mu - 1)\alpha$$

where α is the refracting angle of the prism

From figure $\delta \approx \frac{d/2}{a}$

$$d = 2a(\mu - 1)\alpha$$

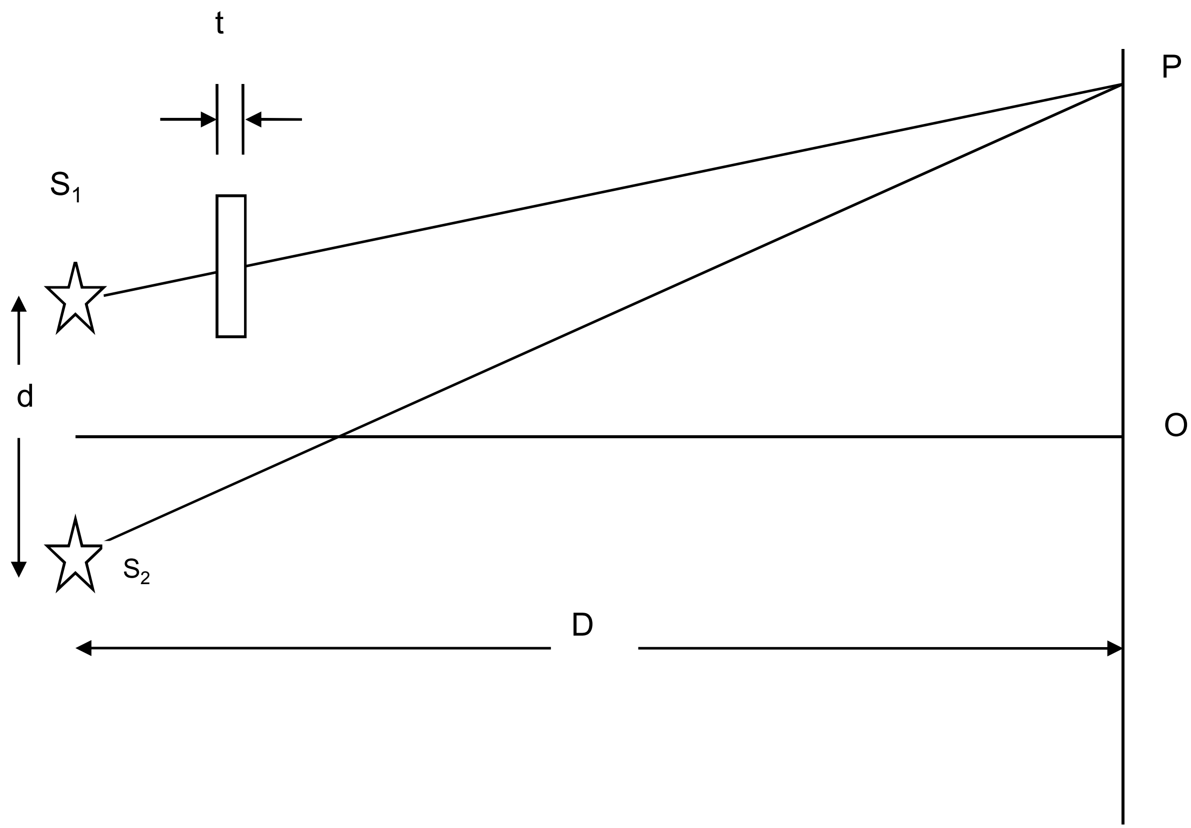
Interference Fringes with White Light

In the bi-prism experiment if the slit is illuminated by white light, the interference pattern consists of a central white fringe flanked on its both sides by a few coloured fringes and general illumination beyond the fringes.

The central white fringe is the zero-order fringe.

With monochromatic light all the bright fringes are of the same colour and it is not possible to locate the zero order fringe.

Lateral Displacement of Fringes



The bi-prism experiment can be used to determine the thickness of a given thin sheet of transparent material such as glass or mica

The light waves from S_1 to P travel partly in air and partly in the sheet G;

The optical path $\Delta_{S_1P} = (S_1P - t) + \mu t$

The optical path $\Delta_{S_2P} = S_2P$

The optical path difference at P is $\Delta_{S_1P} - \Delta_{S_2P}$

In the presence of thin sheet, the optical path lengths S_1P and S_2P are equal and central fringe is obtained at P

$$\Delta_{S_1P} = \Delta_{S_2P}$$

$$(S_1P - t) + \mu t = S_2P$$

So, path difference

$$S_2P - S_1P = (\mu - 1)t$$

But

$$S_2P - S_1P = \frac{xd}{D}$$

Where x is the lateral shift of the central fringe due to the introduction of the thin sheet

$$(\mu - 1)t = \frac{xd}{D}$$

and

$$t = \frac{xd}{D(\mu - 1)}$$

Interference in Thin Films

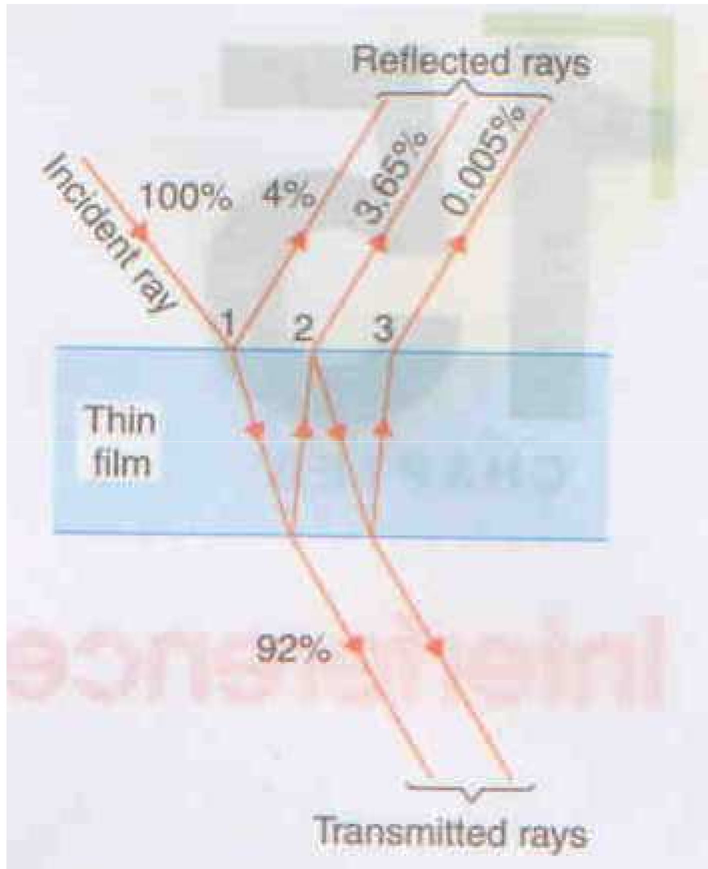
Thin Film- An optical medium of thickness about the order of 1 wavelength of light in visible region. A film of thickness in the range 0.5 μm to 10 μm may be considered as a thin film

Example: Thin sheet of transparent material as glass, mica, an air film enclosed between two transparent material, soap bubble etc.

When light is incident on such a film, a small part of light is reflected from the top surface and a major part is transmitted into the film. Again a small part of the transmitted light is reflected back into the film by the bottom surface and this process continues

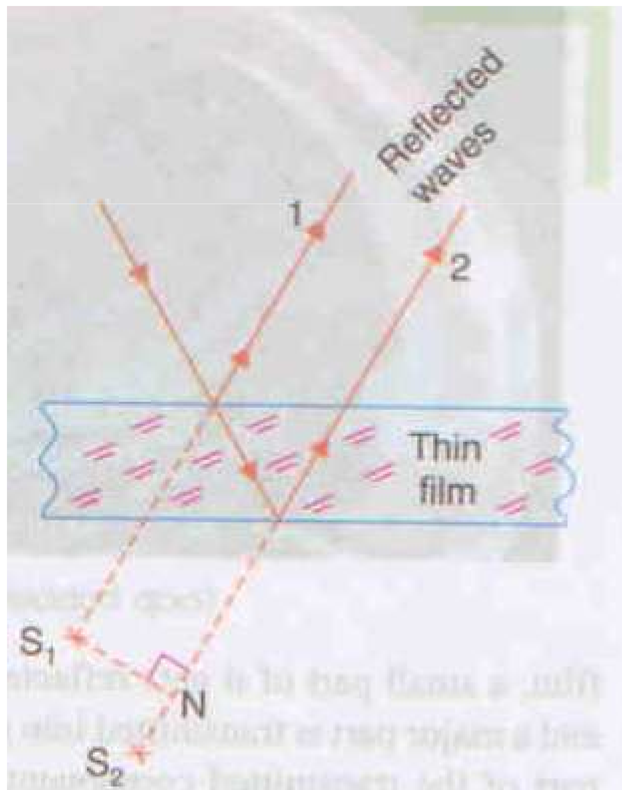
Soap bubble.

Multiple reflections from a thin film

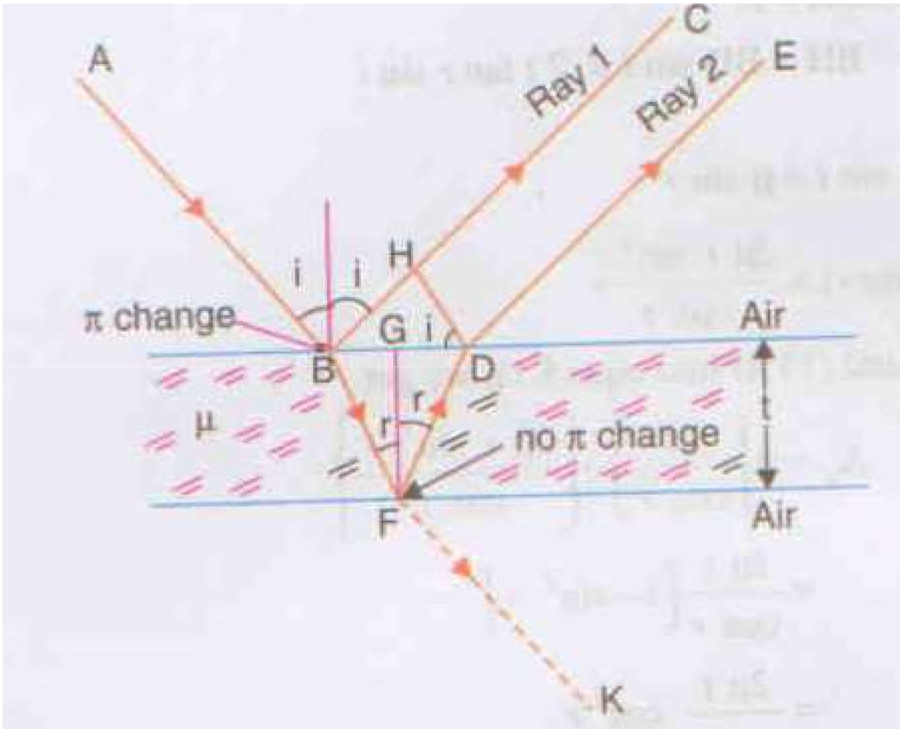


Plane Parallel Film

A transparent thin film of uniform thickness bounded by two parallel surfaces is known as a plane parallel thin film

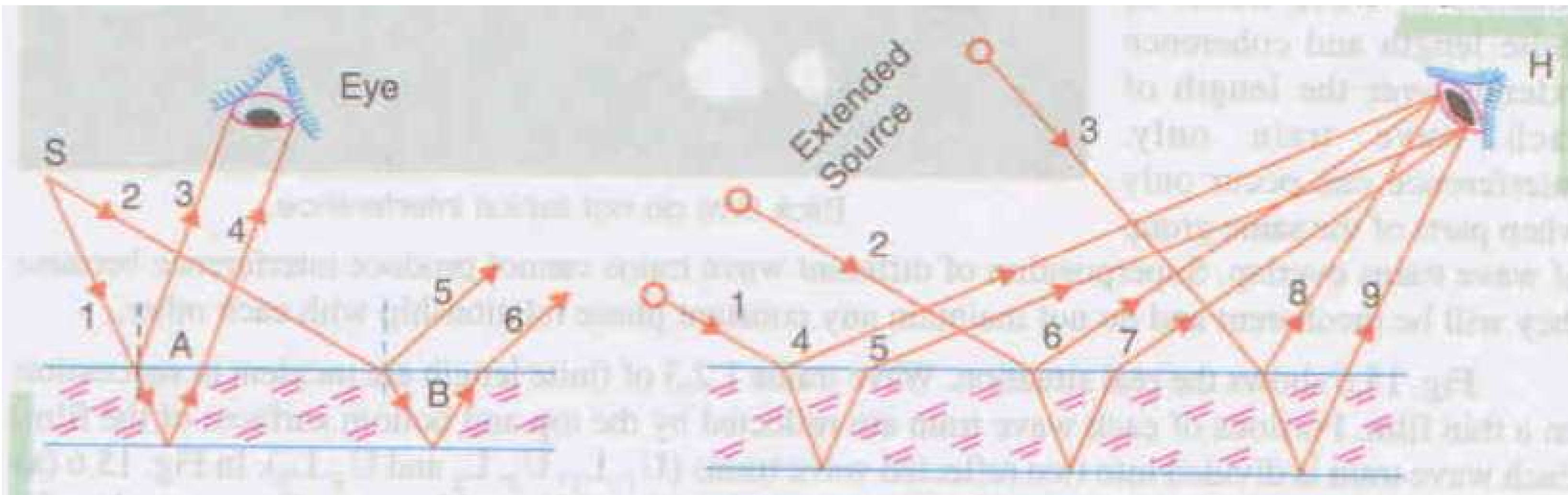


Interference due to reflected light



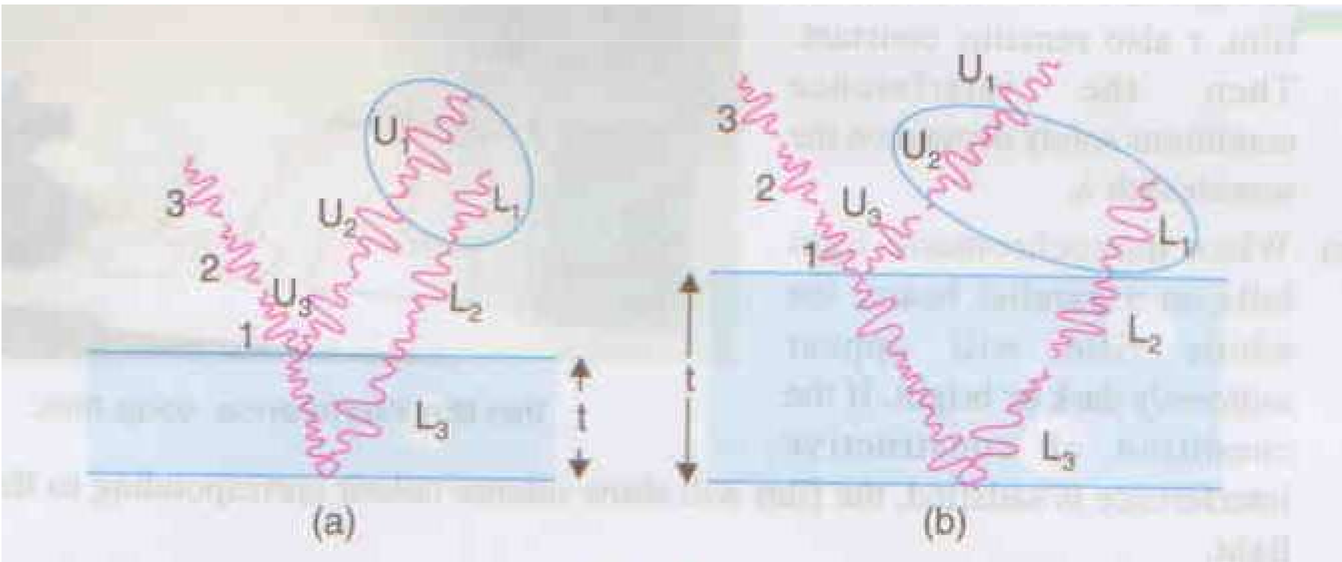
$$BF = t/\cos r$$
$$BG = t.\tan r$$
$$BH = 2t.\tan r \,.\, \sin i$$
$$\Delta = 2\mu t \cos r - \lambda/2$$

Narrow light source versus extended light source



Restriction on thickness of the film

Interference colours are only observed in thin films but not in thick plates like window panes or glass slabs etc. - Light waves can interfere only when both the conditions of temporal and spatial coherence are satisfied



Interference occurs only when the optical path difference between the superposing waves is less than the coherence length

i.e. $$\Delta << l_{coh}$$

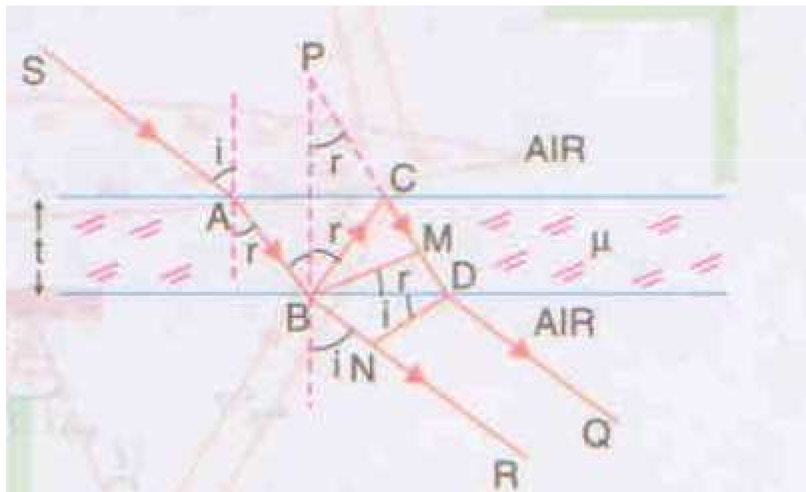
$$2\mu t \cos r - \lambda/2 << l_{coh}$$

$$(2\mu t \cos r - \lambda/2) << \lambda^2/\Delta\lambda$$

Interference in thin films will take place only when the thickness of the film is less than the coherence length of the incident light. Interference is seen with the films of thickness of the order of few hundred microns only

Interference due to Transmitted Light

Consider a thin film of thickness ‘t’

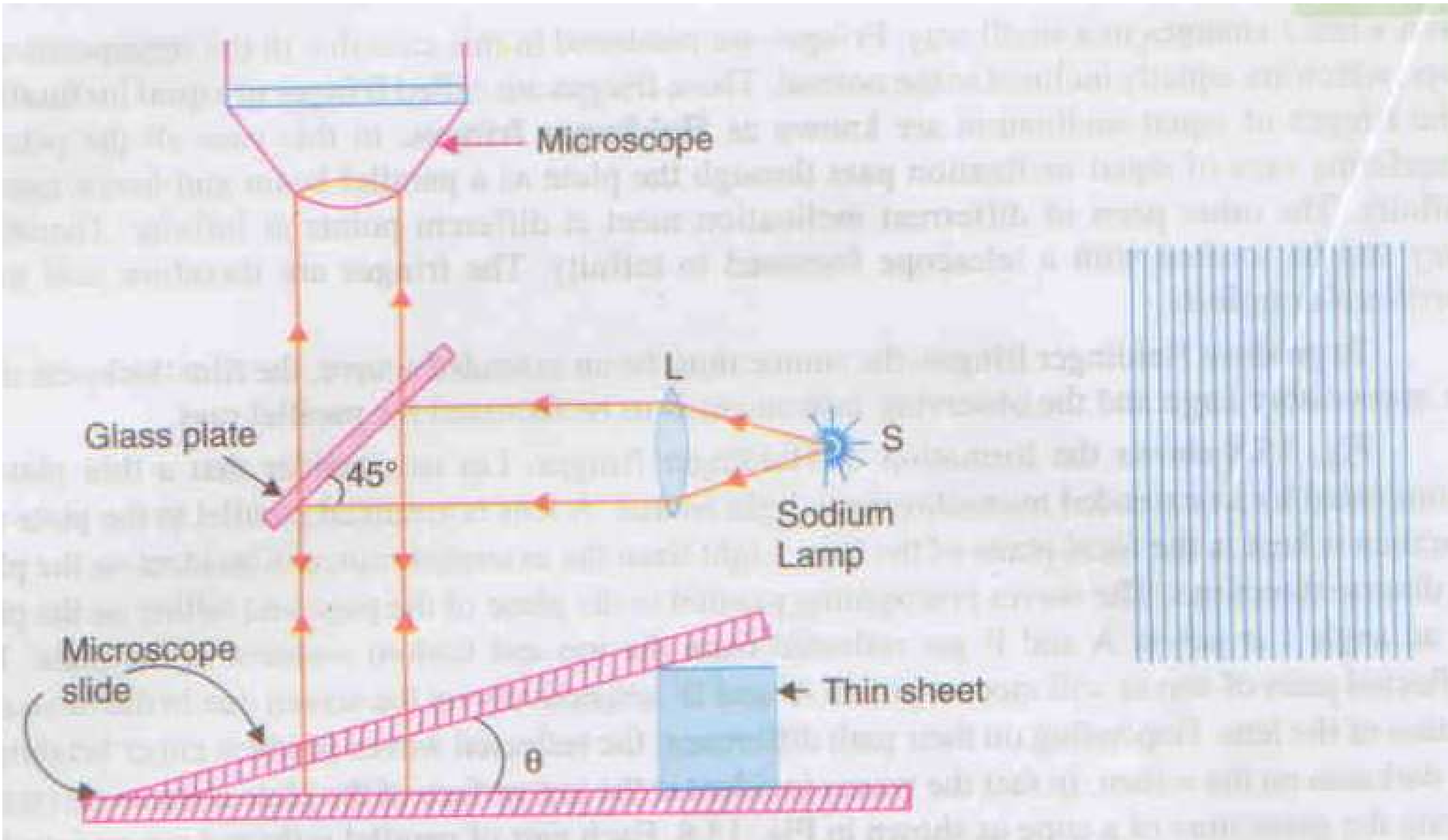


Haidinger Fringes:

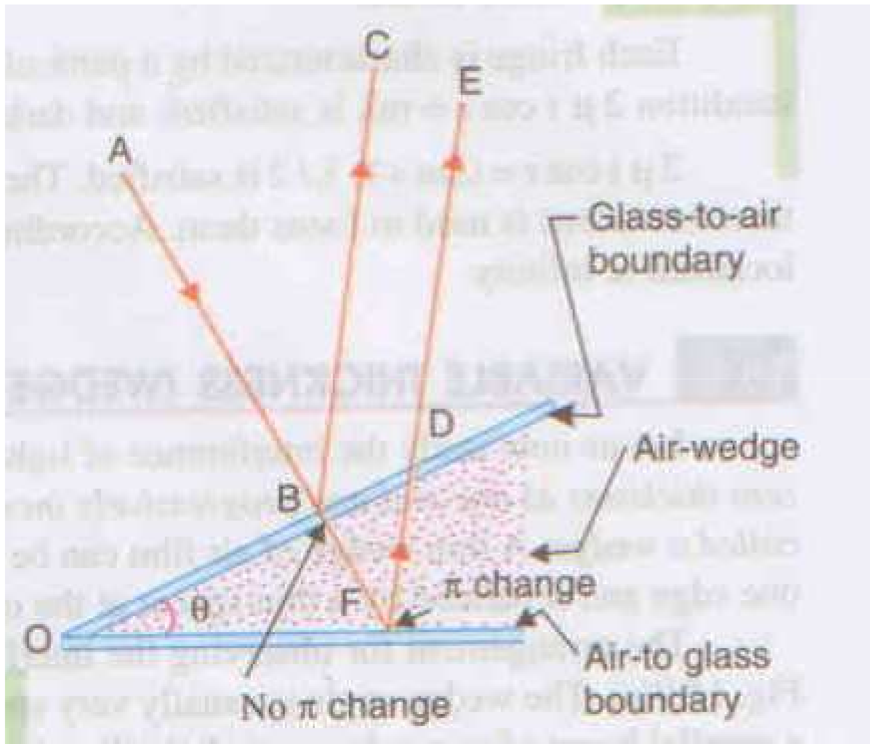
When the film is of uniform (constant) thickness, the change in path difference is only due to the change in ‘r’. If the thickness of the film is large, the path difference will change appreciably even when ‘r’ changes in a small way. The fringes are produced due to the superposition of rays, which are equally inclined with the normal.

These fringes are called *fringes of equal inclination* or *Haidinger Fringes.* These fringes are said to be localized at infinity

Variable Thickness Film (Wedge-Shape Film)



Wedge Shape Film




E
λ/2
C
θ
D
A
θ
β
B
β
t1
t2
θ
O
K
Dark
L
Dark
M
Dark

Fringe width for the wedge shape film is

$$\beta = \frac{\lambda}{2\mu\theta}$$

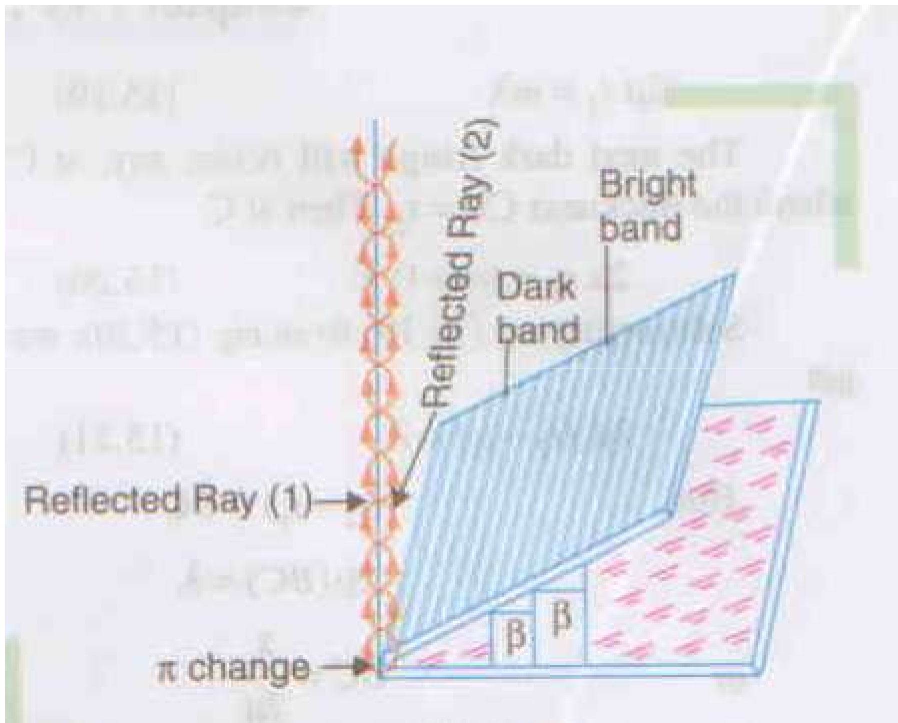
thus, fringe width is constant for a given wedge angle

The interference pattern has following salient features

1. Fringes at the apex is dark- At the apex the two glass plates are in contact so the thickness of the air-film at the contact edge is negligible ($t \cong 0$). So the optical path difference becomes

$$\Delta = 2\mu t - \lambda/2 = -\lambda/2$$

It implies that path difference of $\lambda/2$ occurs between the reflected waves. Two waves interfere destructively so the fringe at the apex is always dark



Reflected Ray (2)
Bright band
Dark band
Reflected Ray (1)
β
β
π change

2. Straight and Parallel Fringes

3. Equidistant Fringes

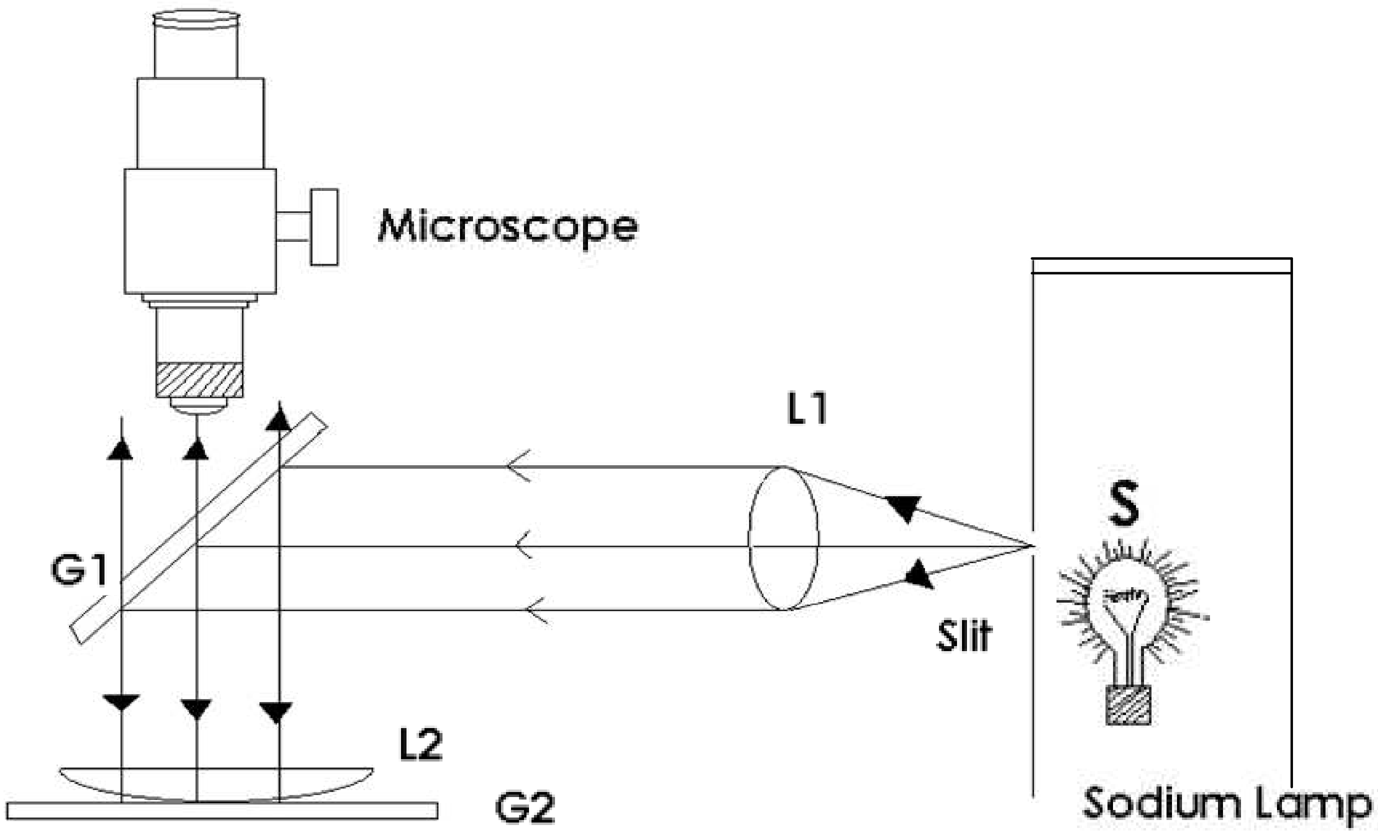
4. Localized Fringes

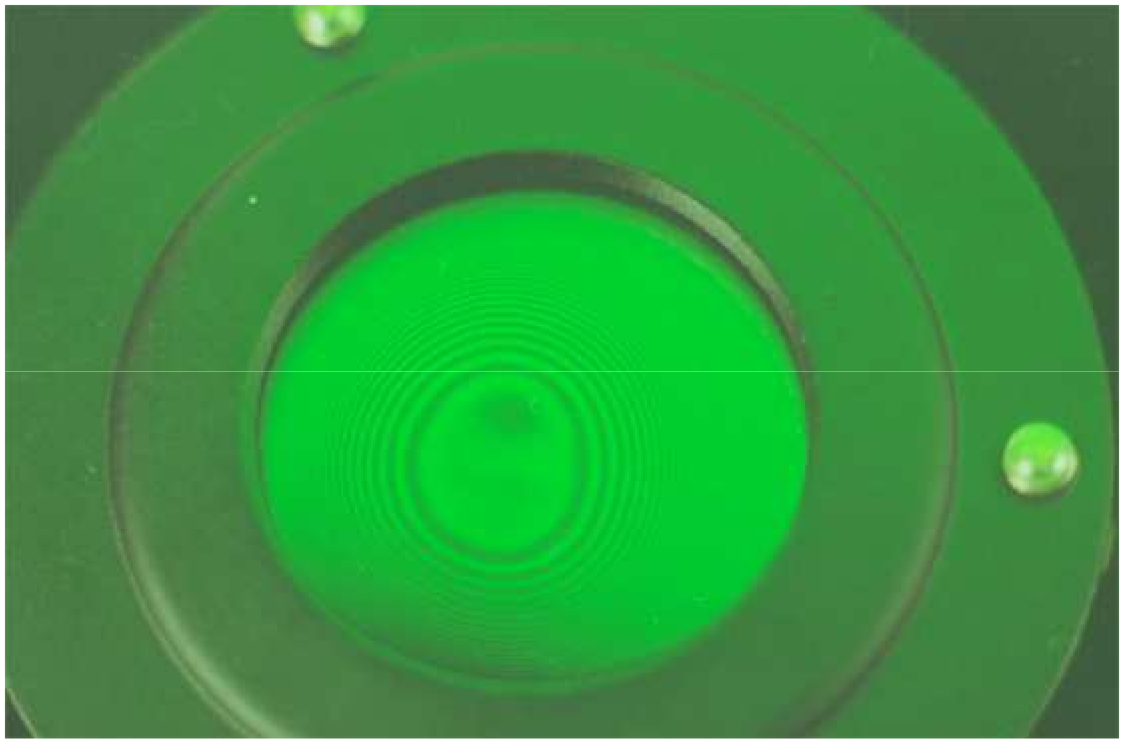
5. Fringes of equal thickness

Fizeau Fringes:

If a parallel beam of light is incident perpendicularly or nearly perpendicularly on a variable thickness film, then dark and bright fringes are seen in reflected light. These fringes are fringes of equal thickness, each fringe corresponds to lines of equal optical thickness. These localized fringes of equal thickness are known as **Fizeau Fringes**

Newton's Rings

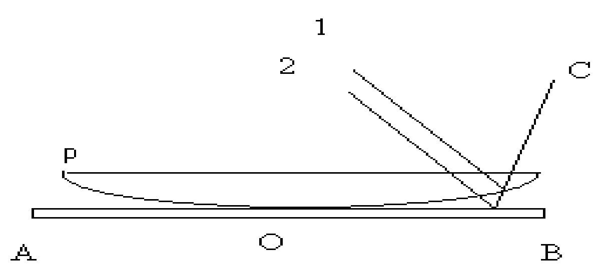




Newton’s rings are an example of fringes of equal thickness.

The combination of plano-convex lens and glass plate forms a thin circular air film of variable thickness in all directions around the point of contact of the lens and the glass plate.

Locus of all points corresponding to specific thickness of air film falls on a circle whose centre is at O, the point of contact of plano-convex lens and glass plate.



Condition for Bright and Dark Rings

The optical path difference between the rays is given by $\Delta = 2\mu t\cos r - \lambda/2$ since $\mu = 1$ for air and $\cos r = 1$ for normal incidence of light,

$$\Delta = 2t - \lambda/2$$

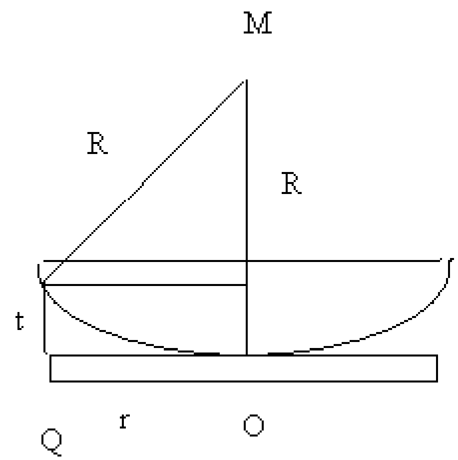
For Bright Fringe $2t - \lambda/2 = m\lambda$

$$2t = (2m+1)\,\lambda/2$$

For dark Fringe $2t - \lambda/2 = (2m+1)\,\lambda/2 = m\lambda$

Circular Fringes: In Newton's ring arrangement, a thin air film is enclosed between a plano-convex lens and a glass plate. The thickness of the air film at the point of contact is zero and gradually increases as we move outward. The locus of the points where the air film has the same thickness then fall on a circle whose centre is the point of contact. The fringes are therefore circular

Radii of Dark Fringes



let R be the radius of curvature of the lens and a dark fringe be located at Q. Let the thickness of the air film at Q be PQ = t. Let the radius of the circular fringe at Q be r.

By Pythagorus theorem

$$R^2 = r^2 + (R - t)^2$$

$$r^2 = 2Rt - t^2$$

as $R >> t, 2Rt >> t^2$

$$r^2 = 2Rt$$

The condition for darkness at Q is that

$$2t = m\lambda$$

$$r_m = \sqrt{m\lambda R}$$

Thus, the radii of the dark rings are proportional to under root of the natural numbers.

Diameter of the dark ring is $D_m = 2\sqrt{m\lambda R}$

Determination of Wavelength of Light: We have diameter of dark rings

$$D_m^2 = 4m\lambda R$$

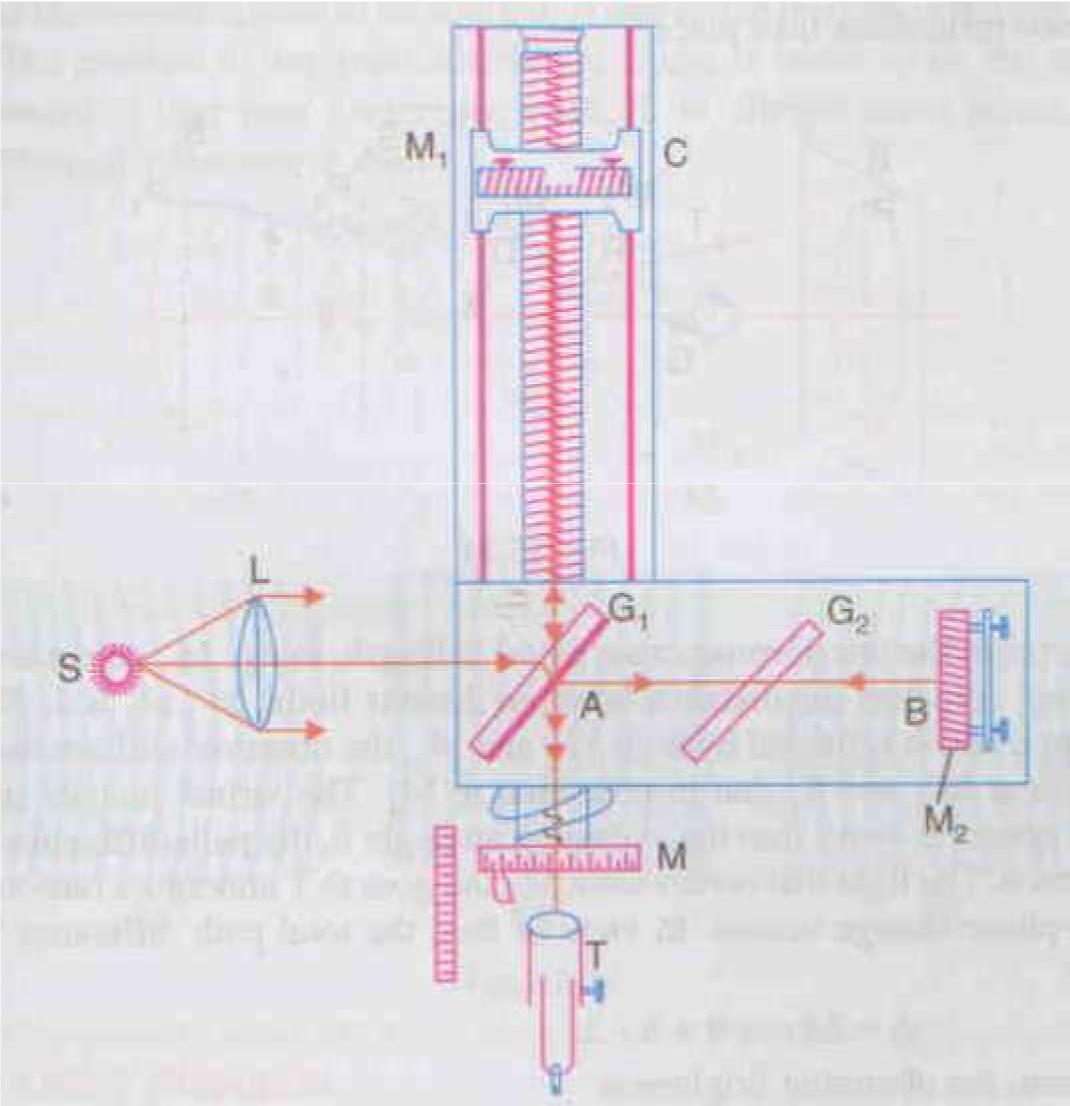
For the (m+p)th ring $\quad D_{m+p}^2 = 4(m+p)\lambda R$

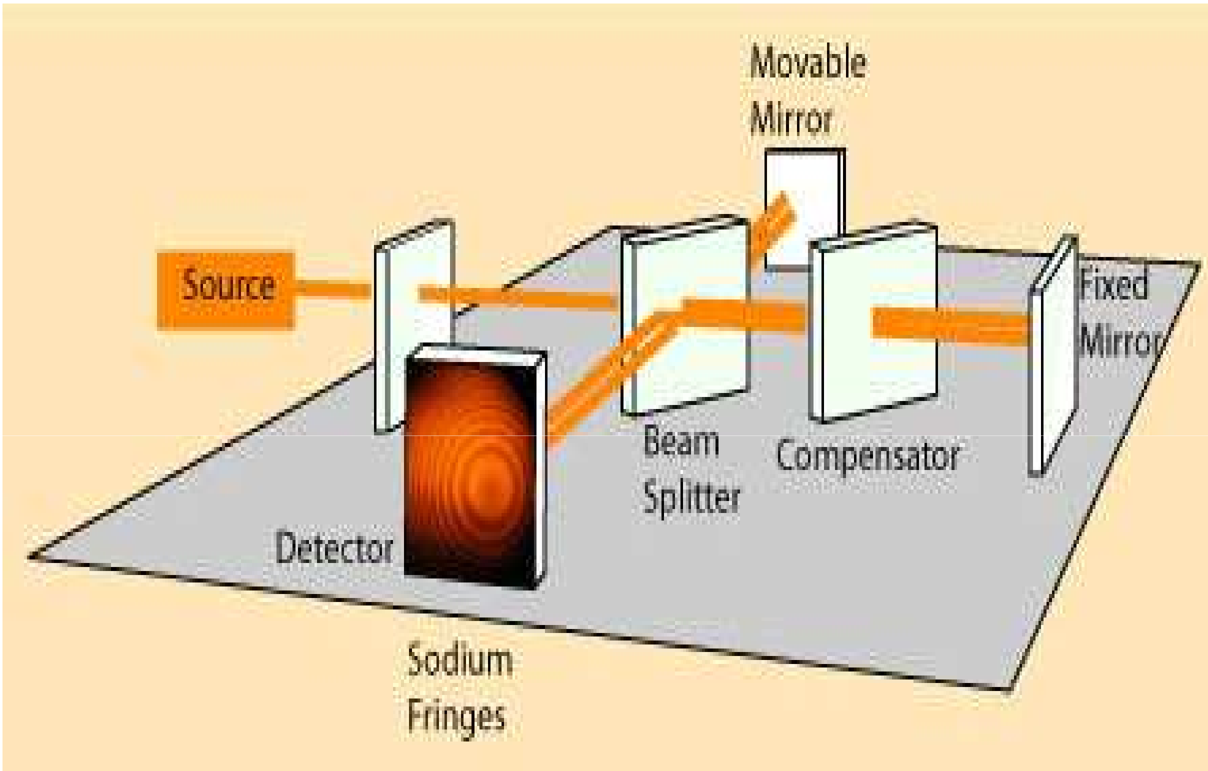
$$\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR}$$

Refractive index of a liquid

The liquid, whose refractive index is to be determined, is filled between the lens and plane glass plate. Liquid substitutes the air-film.

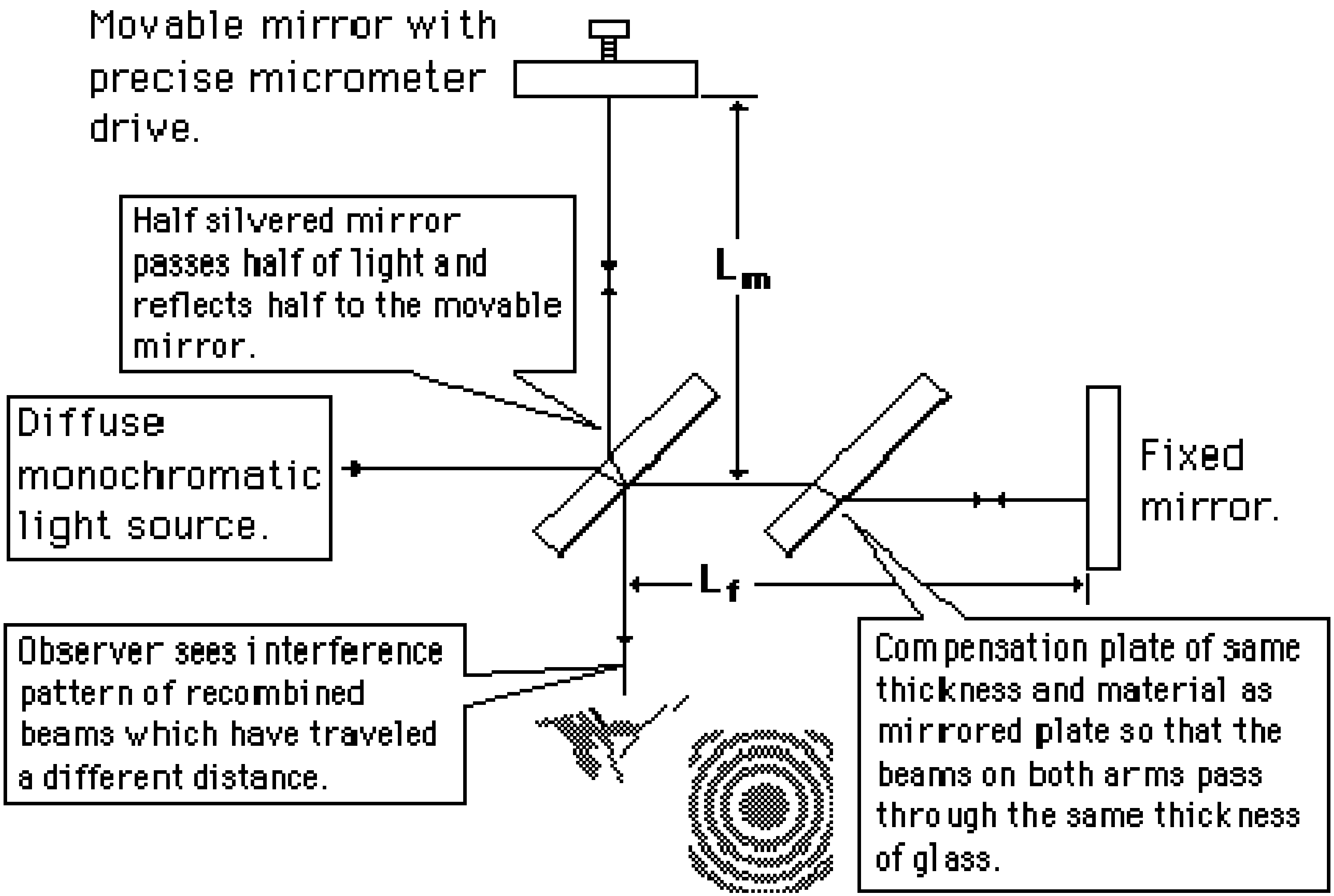
Michelson Interferometer

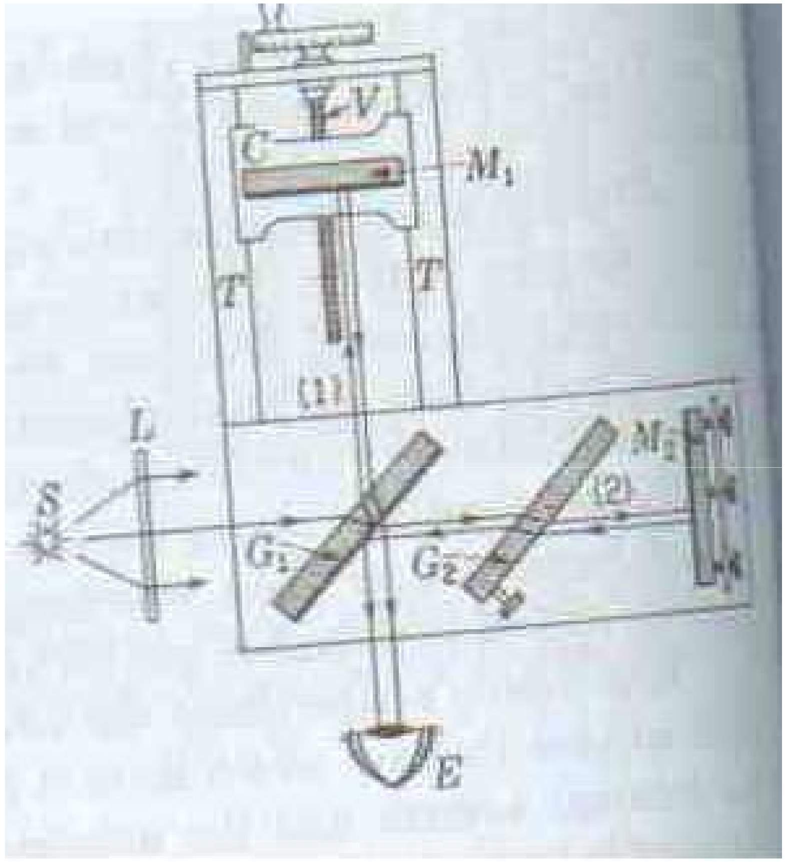



Movable
Mirror
Source
Fixed
Mirror
Beam
Splitter
Compensator
Detector
Sodium
Fringes

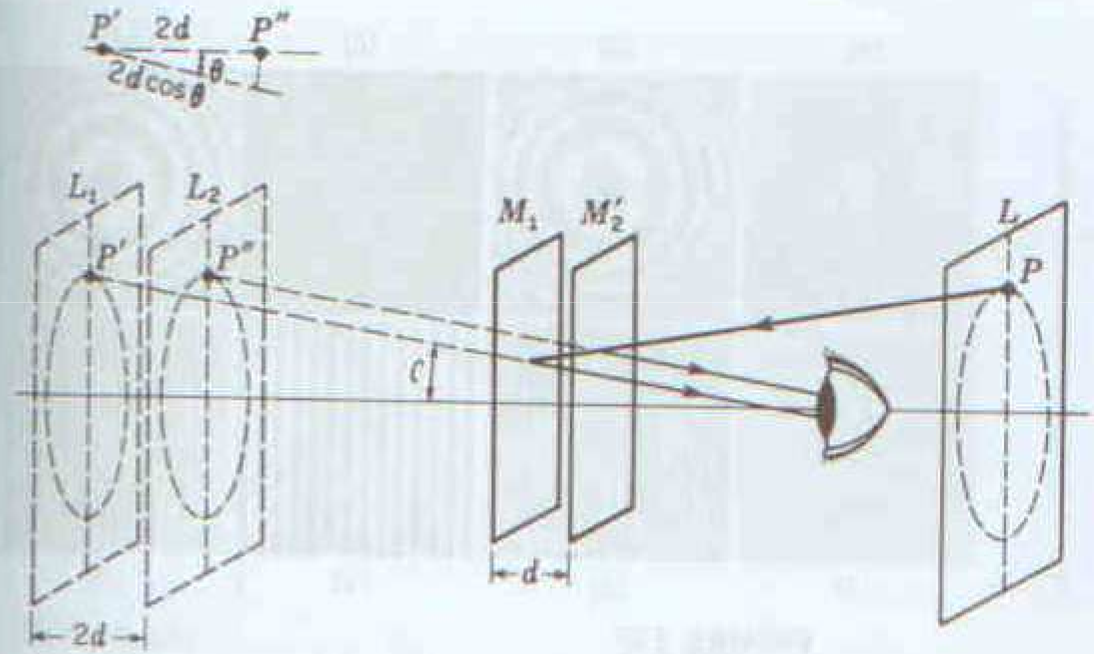
- **Michelson Interferometer**

  The Michelson interferometer produces interference fringes by splitting a beam of monochromatic light so that one beam strikes a fixed mirror and the other a movable mirror. When the reflected beams are brought back together, an interference pattern results.

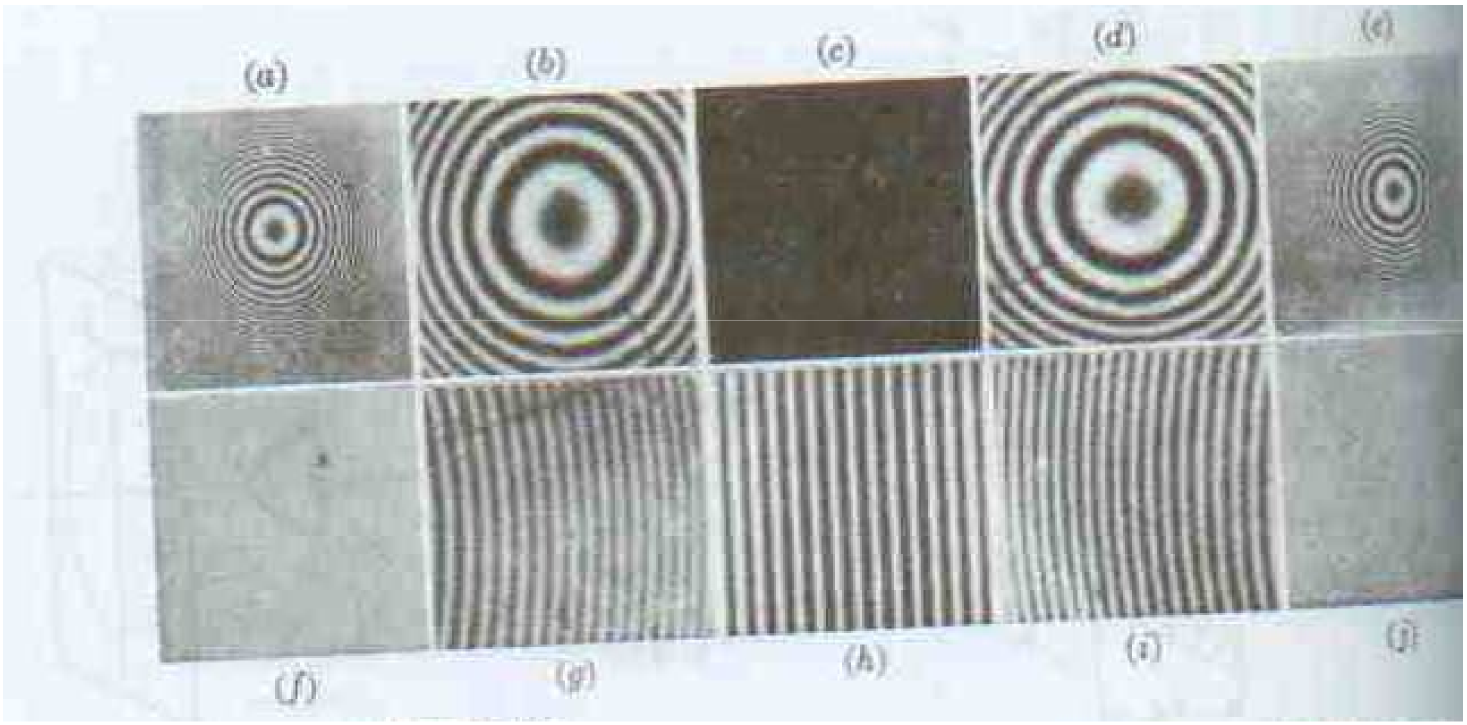

Movable mirror with precise micrometer drive.
Half silvered mirror passes half of light and reflects half to the movable mirror.
Lm
Diffuse monochromatic light source.
Fixed mirror.
Lf
Observer sees interference pattern of recombined beams which have traveled a different distance.
Compensation plate of same thickness and material as mirrored plate so that the beams on both arms pass through the same thickness of glass.



V
C
M_1
T
T
(1)
L
S
G_1
G_2
M_2
(2)
E



P′
2d
P″
2d cos θ
θ
L1
L2
P′
P″
M1
M′2
L
P
θ
d
2d


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)