A **cubic equation** is of the form $a \cdot x^3 + b \cdot x^2 + c \cdot x + d = 0$ where a, b, c, d are some given numbers and $a \neq 0$.

Since we required that $a \neq 0$ we are aloud to divide every term of the cubic equation by a and get a simpler cubic equation

$$x^3 + \frac{b}{a} \cdot x^2 + \frac{c}{a} \cdot x + \frac{d}{a} = 0$$

To simplify the notation we write b_1 for $\frac{b}{a}$ and c_1 for $\frac{c}{a}$ and d_1 for $\frac{d}{a}$.

Our next step is to get rid of the quadratic term $b_1 \cdot x^2$. to do so we use a substitution $u = x + \frac{b_1}{3}$ and so $x = u - \frac{b_1}{3}$. After substituting each x in the last cubic equation by $u - \frac{b_1}{3}$ the equation becomes:

$$0 = (u - \frac{b_1}{3})^3 + b_1 \cdot (u - \frac{b_1}{3})^2 + c_1 \cdot (u - \frac{b_1}{3}) + d_1 = u^3 + [-\frac{b_1^2}{3} + c_1] \cdot u + [\frac{2 \cdot b_1^3}{27} - \frac{c_1 \cdot b_1}{3} + d_1]$$

Again, to simplify the notation we write p for $-\frac{b_1^2}{3} + c_1$ and q for $\frac{2 \cdot b_1^3}{27} - \frac{c_1 \cdot b_1}{3} + d_1$ and the cubic equation becomes

$$u^3 + p \cdot u + q = 0$$

where p and q are some numbers.

The next step in simplifying the cubic equation is to substitute

$$u = \sqrt{\frac{-p}{3}} \cdot (s + \frac{1}{s})$$

which will change the cubic equation to

$$0 = [\sqrt{\frac{-p}{3}} \cdot (s + \frac{1}{s})]^3 + p \cdot [\sqrt{\frac{-p}{3}} \cdot (s + \frac{1}{s})] + q = \sqrt{\frac{-p^3}{27}} \cdot s^3 + \sqrt{\frac{-p^3}{27}} \cdot \frac{1}{s^3} + q$$

Multiplying the last equation by s^3 gives us:

$$\sqrt{\frac{-p^3}{27}} \cdot s^3 \cdot s^3 + q \cdot s^3 + \sqrt{\frac{-p^3}{3}} = 0$$

Finally, substituting $w = s^3$ and thus $s = \sqrt[3]{w}$ gives us a quadratic equation:

$$\sqrt{\frac{-p^3}{27}} \cdot w^2 + q \cdot w + \sqrt{\frac{-p^3}{27}} = 0$$

We solve this quadratic equation to find w:

$$w = \frac{-q \pm \sqrt{q^2 + 4 \cdot \frac{p^3}{27}}}{2 \cdot \sqrt{\frac{-p^3}{27}}}$$

Now we go back along our formulae to find x. So

$$s = \sqrt[3]{w}$$

$$u = \sqrt{\frac{-p}{3}} \cdot (s + \frac{1}{s}) = \sqrt{\frac{-p}{3}} \cdot (\sqrt[3]{w} + \frac{1}{\sqrt[3]{w}})$$

$$x = u - \frac{b_1}{3} = \sqrt{\frac{-p}{3}} \cdot (\sqrt[3]{w} + \frac{1}{\sqrt[3]{w}}) - \frac{b_1}{3}$$

And we get the ultimate **Cubic Formula:**

$$x = \frac{1}{\sqrt{3}} \cdot [\frac{\sqrt[3]{-q \cdot \sqrt{27} \pm \sqrt{27 \cdot q^2 + 4 \cdot p^3}}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2} \cdot p}{\sqrt[3]{-q \cdot \sqrt{27} \pm \sqrt{27 \cdot q^2 + 4 \cdot p^3}}}] - \frac{b}{3 \cdot a}$$

where

$$p = -\frac{b^2}{3 \cdot a^2} + \frac{c}{a} \ \mid \ q = \frac{2 \cdot b^3}{27 \cdot a^3} - \frac{c \cdot b}{3 \cdot a^2} + \frac{d}{a}$$

By the way, we know that a Cubic Equation has exactly three roots, counting with repetitions. But our Formula seems to suggest six possible roots, since taking a Cubic Root produces three answers. Actually, in the set of six answers which our Formula produces there will be identical answers produced by different ways. And all-together only three answers will be produced. The reason for this twisting lies in the properties of symmetries of roots of various equations and is studied in depth by the **Galois Theory**.

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