Vectored-DMT: A FEXT Canceling Modulation Scheme for Coordinating Users

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Abstract— **This paper describes a modulation scheme achieving Far-End Crosstalk (FEXT) cancellation, assuming that joint signal processing among the users can be performed at either the receiver or at the transmitter side. The multi-user channel is modeled as a Multiple-Input-Multiple-Output (MIMO) system, and the development starts with the zero-forcing Generalized Decision Feedback Equalizer (GDFE) ([1]). Structures are derived, which combine the Discrete Multi-Tone (DMT) transmission technique ([2]) with either successive cancellation at the receiver or precoding at the transmitter. A promising application of the proposed scheme is in FEXT-limited Digital Subscriber Line (DSL) systems. Computer simulations demonstrate that significant performance improvements can be realized in such scenarios.**

I. Introduction

Crosstalk, rather than additive noise, represents the major limitation in several multi-user digital communication systems. Currently, conservative and sub-optimum techniques are used to fight crosstalk (e.g. guard bands in TDMA, power control in CDMA, 1% worst-case designs in DSL). Employing techniques that alleviate the impact of crosstalk holds the key in delivering further improvements in the performance of such systems.

The crosstalk problem has been previously studied in various contexts. In [3] and [4], "multichannel" signal processing was introduced and the corresponding Minimum Mean Square Error (MMSE) linear equalizer was specified. In the absence of user coordination, wider than Nyquist transmitters have been shown to provide a performance advantage over Nyquist-limited transmitters ([5], [6]). A DMT-based scheme was proposed in [7] for the multiple-antenna scenario assuming spatial signal processing at both the transmitter and the receiver. In the BLAST prototype ([8]), the (narrowband) incoming signals are separated by joint signal processing of the signals of the receiving antennas.

This paper introduces a multi-user modulation scheme (Vectored-DMT) achieving FEXT removal, suitable for scenarios where either the receivers or the transmitters are co-located as shown in Figures 1 and 2. The channel model is given in Section II with the basic presumption of block transmission. Section III describes the zero-forcing version of the GDFE equalizer ([1]), which serves as the foundation for the derivation in Section IV of the VDMT crosstalk canceling transmission structures. Section V shows simulation results exhibiting the potential performance improvements for DSL applications and quantifying the effect of channel identification errors. Finally, Section VI provides the conclusions of this contribution.

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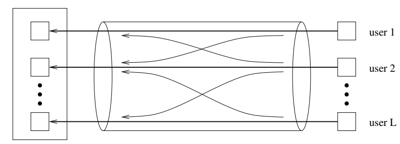

Fig. 1. System with co-located receivers.

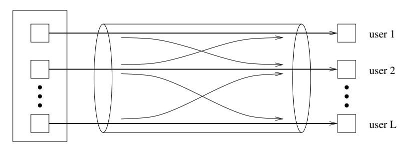

Fig. 2. System with co-located transmitters.

II. Channel Model

In the following, the superscripts ^{T} and * denote the transpose and conjugate-transpose operations respectively.

The sampled output for a certain user in any of the two scenarios shown in Figures 1 and 2 depends on the present and past input symbols of both the intended user and the other crosstalking users. A block of N output samples for user i satisfies:

$$\mathbf{y}_i = H^c_{i,1}\mathbf{x}^p_1 + \cdots + H^c_{i,i}\mathbf{x}^p_i + \cdots + H^c_{i,L}\mathbf{x}^p_L + \mathbf{n}'_i \qquad (1)$$

where $H^c_{i,1}, \ldots, H^c_{i,L}$ are convolution matrices derived from the channel impulse response matrix, $\mathbf{y}_i$ is the vector of N output samples of receiver i, $\mathbf{x}^p_k$ is the vector of $N+\nu$ input symbols of user k, and $\mathbf{n}'_i$ is the vector of N noise samples of receiver i. Evidently, ν represents the maximum memory of the transfer and crosstalk coupling functions expressed in number of symbols. Also, the noise samples are white and have (without loss of generality) unit variance.

Assuming that all L users are frame-synchronized and employ a cyclic prefix of length ν (as outlined in [9]), (1) becomes:

$$\mathbf{y}_i = H_{i,1}\mathbf{x}_1 + \cdots + H_{i,i}\mathbf{x}_i + \cdots + H_{i,L}\mathbf{x}_L + \mathbf{n}'_i \qquad (2)$$

where now $\mathbf{x}_k$ is a vector of N input symbols of user k, and $H_{i,j}$ are circulant matrices. Finally, combining all users together, (2) becomes:

$$\mathbf{y} = H\mathbf{x} + \mathbf{n}' \qquad (3)$$

where $\mathbf{y} = [\mathbf{y}_1\,\mathbf{y}_2\,\ldots\,\mathbf{y}_L]^T$, $\mathbf{x} = [\mathbf{x}_1\,\mathbf{x}_2\,\ldots\,\mathbf{x}_L]^T$, $\mathbf{n}' = [\mathbf{n}'_1\,\mathbf{n}'_2\,\ldots\,\mathbf{n}'_L]^T$, and H is a matrix whose (i,j) block is $H_{i,j}$. Matrix H is always considered to be non-singular, which is a reasonable assumption for DSL systems, as explained later.

III. Zero-Forcing GDFE

The GDFE equalization structure (proposed in [1]) is shown in Fig. 3. It provides a general framework that can be applied to a wide variety of communication scenarios (including multi-user and multi-sensor). Although it shares similarities with the multi-user decision-feedback detectors described in [10], it is much more general in several respects. Starting with the model of (3), and taking into account transmit filtering (realized by the non-singular matrix A) and receiver matched filtering, the *forward canonical model* is obtained:

$$\mathbf{z} = R_f\mathbf{u} + \mathbf{n} \tag{4}$$

where $R_f = A^*H^*HA$, $\mathbf{n} = A^*H^*\mathbf{n}'$ and the noise covariance is $R_{\mathbf{nn}} = R_f$. Next, the Cholesky factorization is computed:

$$R_f = G^*S_oG \tag{5}$$

where S_o is diagonal with positive elements, and G is upper diagonal and monic (having ones along the diagonal). Processing the received vector $\mathbf{z}$ by $S_o^{-1}G^{-*}$ yields:

$$S_o^{-1}G^{-*}\mathbf{z} = G\mathbf{u} + S_o^{-1}G^{-*}\mathbf{n} = G\mathbf{u} + \mathbf{e} \tag{6}$$

where $R_{\mathbf{ee}} = S_o^{-1}$ is diagonal. Since G is upper triangular and the error $\mathbf{e}$ has uncorrelated components, the input $\mathbf{u}$ can be recovered by back-substitution combined with symbol-by-symbol detection. Thus, a decision feedback structure is derived with the feedforward filter being $S_o^{-1}G^{-*}$ and the feedback filter being $I - G$ as in Fig. 3.

IV. Vectored-DMT

A. Coordinating Receivers

Vectored-DMT defines the block diagonal matrix:

$$Q_{mIDFT} = diag\left(\overbrace{Q_{IDFT}, Q_{IDFT}, \ldots, Q_{IDFT}}^{L}\right) \tag{7}$$

where Q_{IDFT} is an $N \times N$ matrix, which, when right-multiplied with a vector, produces its N-point Inverse Discrete Fourier Transform (IDFT). Also, $Q_{DFT} = Q^*_{IDFT}$ is the corresponding Discrete Fourier Transform matrix, and $Q_{mDFT} = Q^*_{mIDFT}$.

A permutation matrix P having NL rows and NL columns is composed of the $N \times N$ blocks $P_{i,j}$ where $i, j = 1, \ldots, L$. The block P_{ij} contains all zeros, except for a one at position (j, i). Some inspection reveals that, when matrix P is right-multiplied with a vector of size NL, it essentially reorders its elements from N groups of L components into L groups of N components. Also, $P^{-1} = P^* = P$.

The transmit filter of the GDFE structure is defined as:

$$A = Q_{mIDFT}P \tag{8}$$

Let us interpret the vector $\mathbf{u}$ as consisting of N groups of L components, where each group holds data symbols of the L users. The matrix P performs a reordering of those symbols, so that each of the L resulting groups contains the N data symbols corresponding to one of the L users. Then, Q_{mIDFT} computes the IDFT on the symbols of each user, exactly as in DMT modulation ([2]). Clearly, the operation of the transmit filter does not result in any energy increase of the transmitted signal. Also, the data symbols of each user are processed independently of those of other users, which is consistent with the assumption that the transmitters may be geographically separated.

Matrix R_f now factors as:

$$R_f = P^*Q_{mDFT}H^*HQ_{mIDFT}P \tag{9}$$

Recalling that the $H_{i,j}$ blocks of matrix H are circulant, $H_{i,j} = Q_{IDFT}\Lambda_{i,j}Q_{DFT}$, where $\Lambda_{i,j}$ is diagonal. Thus:

$$R_f = P^*\Lambda^*\Lambda P \tag{10}$$

where Λ consists of the $\Lambda_{i,j}$ blocks. The reordering imposed by the P matrix makes R_f a block diagonal matrix:

$$R_f = diag\left(R_{f,1}, R_{f,2}, \ldots, R_{f,N}\right) \tag{11}$$

where $R_{f,i}$, $i = 1, \ldots, N$ are $L \times L$ matrices. The interpretation of this result is that each block describes transmission within a "tone" as defined by IDFT modulation. The Cholesky factorization of R_f is:

$$R_f = diag\left(G_1^*S_{o,1}G_1, G_2^*S_{o,2}G_2, \ldots, G_N^*S_{o,N}G_N\right) \tag{12}$$

It offers great aid to define the following decomposition:

$$P^*\Lambda P = diag\left(Q_1R_1, Q_2R_2, \ldots, Q_NR_N\right) \tag{13}$$

where Q_i, $i = 1, \ldots, N$ are $L \times L$ unitary matrices, and R_i, $i = 1, \ldots, N$ are $L \times L$ upper triangular matrices. Simple substitution proves that $R_i = S_{o,i}^{\frac{1}{2}}G_i$.

The GDFE receiver components are now examined. The special form of the feedback filter $G = diag\left(G_1, G_2, \ldots, G_N\right)$ allows its breakup into N independent feedback filters, where each filter operates on the signals of a single tone. The combination of the matched filter and the feedforward filter is:

$$\begin{aligned} S_o^{-1}G^{-*}A^*H^* &= S_o^{-1}G^{-*}P^*\Lambda^*Q_{mDFT} \\ &= S_o^{-1}G^{-*}P^*\Lambda^*PP^*Q_{mDFT} \end{aligned} \tag{14}$$

Using (13):

$$\begin{aligned} &S_o^{-1}G^{-*}A^*H^* = \\ &S_o^{-1}G^{-*}diag\left(R_1^*Q_1^*, R_2^*Q_2^*, \ldots, R_N^*Q_N^*\right)P^*Q_{mDFT} = \\ &S_o^{-1}G^{-*}diag\left(G_1^*S_{o,1}^{\frac{1}{2}}, G_2^*S_{o,2}^{\frac{1}{2}}, \ldots, G_N^*S_{o,N}^{\frac{1}{2}}\right)\cdot \\ &\qquad diag\left(Q_1^*, Q_2^*, \ldots Q_N^*\right)P^*Q_{mDFT} = \\ &diag\left(S_{o,1}^{-\frac{1}{2}}, S_{o,2}^{-\frac{1}{2}}, \ldots, S_{o,N}^{-\frac{1}{2}}\right)diag\left(Q_1^*, Q_2^*, \ldots Q_N^*\right)PQ_{mDFT} \end{aligned} \tag{15}$$

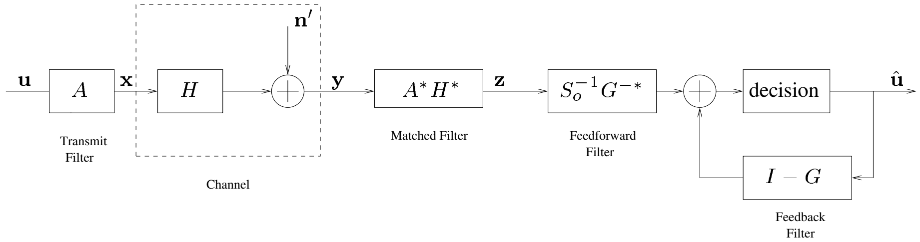

Fig. 3. GDFE equalization structure.

At this point, the function of the "matched-feedforward" filter can be interpreted: The matrix Q_{mDFT} performs the DFT for each user, and matrix P reorders the samples into groups of tones. The Q_i matrices "rotate" the samples corresponding to the i^{th} tone, in order to optimize the effect of decision feedback occuring later, and the $S_{o,i}^{-1/2}$ matrices perform a simple scaling. The operations taking place after reordering can be broken into N independent sections corresponding to tones, just like for the feedback filter.

Therefore, a VDMT system with coordinating receivers has the structure shown in Fig. 4. Each of the "canceler" blocks eliminates crosstalk within a tone. A canceler block is shown in more detail in Fig. 5.

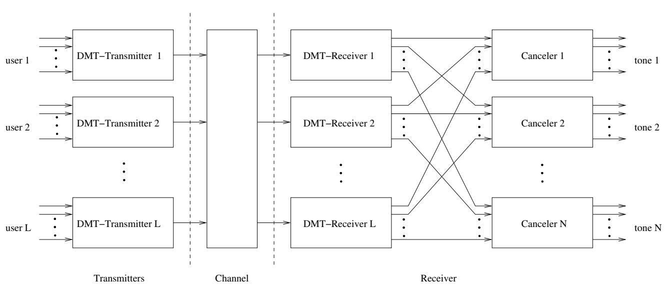

Fig. 4. Vectored-DMT system with coordinating receivers.

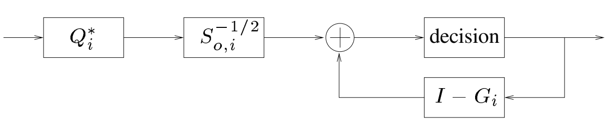

Fig. 5. Canceler block at the VDMT receiver.

B. Coordinating Transmitters

In this case, the receivers are assumed to be geographically separated, which implies that the receiver section must be separable into N independent components corresponding to the N users. However, there is now freedom in choosing the transmit filter as:

$$A = Q_{mIDFT} P B \tag{16}$$

where the matrix B will be derived shortly. Then:

$$R_f = B^* P^* \Lambda^* \Lambda P B \tag{17}$$

Observe that:

$$P^* \Lambda^* \Lambda P = diag\left(G_1^* S_{o,1} G_1, G_2^* S_{o,2} G_2, \ldots, G_N^* S_{o,N} G_N\right) \tag{18}$$

where G_i, $S_{o,i}$ are the same as those in (12). The aim is to find an appropriate B, which can make R_f diagonal so that the receiver feedback section disappears. To this end, the following decomposition is defined:

$$P^* \Lambda^* P = diag\left(Q_1' R_1', Q_2' R_2', \ldots, Q_N' R_N'\right) \tag{19}$$

where Q_i', $i = 1, \ldots, N$ are $L \times L$ unitary matrices, and R_i', $i = 1, \ldots, N$ are $L \times L$ upper triangular matrices. Choosing:

$$B = diag\left(Q_1' R_1^{'-*} S_{o,1}^{\frac{1}{2}}, Q_2' R_2^{'-*} S_{o,2}^{\frac{1}{2}}, \ldots, Q_N' R_N^{'-*} S_{o,N}^{\frac{1}{2}}\right) \tag{20}$$

it is easily determined that $R_f = diag\left(S_{o,1}, S_{o,2}, \ldots, S_{o,N}\right)$, which results in a trivial Cholesky factorization $R_f = G^{'*} S_o G'$ with $G' = I$. Therefore, there is no feedback section and the matched-feedforward filter becomes:

$$\begin{aligned}
&S_o^{-1} G^{'-*} A^* H^* = \\
&S_o^{-1} B^* P^* \Lambda^* Q_{mDFT} = \\
&S_o^{-1} diag\left(S_{o,1}^{\frac{1}{2}} R_1^{'-1} Q_1^{'*}, S_{o,2}^{\frac{1}{2}} R_2^{'-1} Q_2^{'*}, \ldots, S_{o,N}^{\frac{1}{2}} R_N^{'-1} Q_N^{'*}\right) \cdot \\
&\quad diag\left(Q_1' R_1', Q_2' R_2', \ldots, Q_N' R_N'\right) P^* Q_{mDFT} = \\
&S_o^{-\frac{1}{2}} P Q_{mDFT}
\end{aligned} \tag{21}$$

Interpreting the receiver function, it is seen that the DFT is computed on the samples of each user and the output is scaled.

The transmitter function deserves some more attention. Prior to IDFT modulation, the matrices $Q_i' R_i^{'-*} S_{o,i}^{1/2}$ operate independently on the symbols of each tone. The matrices Q_i' simply rotate the symbols, but $R_i^{'-*} S_{o,i}^{1/2}$ may actually cause undesired energy increase.

A solution to this problem (based on the concept of Tomlinson-Harashima precoding [11], [12]) is presented in [13] and briefly described here. It is not difficult to see that $R_i^{'-*} S_{o,i}^{1/2}$ can be replaced by a feedback loop with $I - S_{o,i}^{-1/2} R_i^{'*}$ in the feedback path. Then, in order to prevent energy increase, a modulo operator is placed in the forward path of the feedback loop, and the same operation is also applied at the receivers after scaling.

The VDMT system with coordinating transmitters is depicted in Fig. 6. Each of the "precoder" blocks eliminates crosstalk within a tone. A precoder block is shown in more detail in Fig. 7.

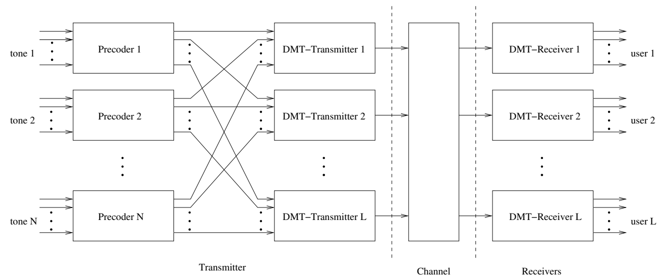

Fig. 6. Vectored-DMT system with coordinating transmitters.

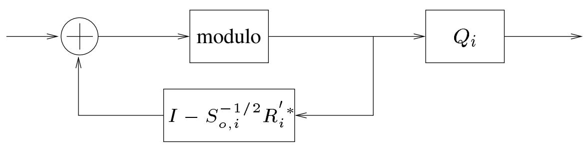

Fig. 7. Precoder block at the VDMT transmitter.

V. Application to DSL and Simulation Results

Vectored-DMT delivers considerable benefits in communication environments such as in DSL, where FEXT is a primary factor degrading performance. Very-High-Speed DSL (VDSL) ([14]) uses bandwidth up to 20 MHz, which results in a severe FEXT problem. Most often (see [16]), the twisted pairs belonging to a single cable meet at either a Central Office (CO) (Fiber-To-The-Exchange (FFTEx) scenario), or at a remotely deployed Optical Network Unit (ONU) (Fiber-To-The-Cabinet (FFTCab) scenario). So, upstream communication has co-located receivers as in Fig. 1, while downstream communication has co-located transmitters as in Fig. 2. Typically, crosstalk is significantly smaller than the useful signal at all frequencies employed for communication. This implies that the $R_{f,i}$ matrices of (11) (which describe transmission within a tone) are close to being diagonal, and they have full rank. Therefore, R_f is non-singular, and from (9), it can be deduced that H is also non-singular. Thus, VDMT can be employed for VDSL upstream and downstream transmission.

Simulation results are next presented quantifying the advantage of employing VDMT in the VDSL setting. The parameters used (shown in Table I) were taken mainly from [14], and the DMT specifications were chosen so that backwards compatibility with ADSL is maintained. The upstream-downstream frequency division is shown in Table II, and the noise models used are specified in [17]. No transmit optimization is performed, since this topic was not examined in this paper. The transmit power of each user is essentially constrained by the PSD mask. Also, infinite granularity of the number of bits at each tone is presumed, and error propagation effects are ignored.

TABLE I

Simulation parameters.

Number of DMT Tones	4096
Tone Width	4.3125 KHz
Symbol Rate	4 KHz
Coding Gain	3.8 dB
Noise Margin	6 dB
Required BER	$< 10^{-7}$
Number of self-FEXT Disturbers	20
Bit Cap on each Tone	11
Cable Type	26-Gauge (0.4 mm)
Source/Load Resistance	100 Ohm
Amateur Radio Bands	as specified in [14]
Power Spectral Density Mask	as specified in [15]

TABLE II

Upstream-downstream frequency assignment.

Upstream Bands (MHz)	Downstream Bands (MHz)
$0.03 - 0.138$	$0.138 - 2.5$
$2.5 - 3.75$	$3.75 - 7.5$
$7.5 - 14.5$	$14.5 - 17.7$

Figures 8 and 9 compare the achievable data rates of DMT and VDMT versus the loop length with either noise A or noise F, for upstream and downstream transmission respectively. Evidently, the use of VDMT improves performance significantly, and this can be viewed in two ways: For a given loop length, VDMT allows the achievement of much higher data rates. The rate increases are considerable for lengths shorter than 4000 feet in noise A, or 3000 feet in noise F. The gains become spectacular in short loops, where obviously transmission is FEXT-limited. Also, it is seen that VDMT extends the maximum loop length given a data rate requirement. As an example, in Fig. 8, a downstream rate requirement of 40 Mbps limits DMT systems with noise of type A to loops shorter than 1500 feet, while employing VDMT extends the reach to 2700 feet.

Simulation results estimating the performance degradation due to channel identification errors are also presented. Assuming that the crosstalk coefficients are imperfectly estimated, Figures 10 and 11 give the achievable data rates versus the loop length with noise A for downstream and upstream transmission respectively. The curves are parameterized by the relative estimation error. It is seen that as the channel identification errors increase, the performance of VDMT gracefully diminishes to that of DMT.

VI. Conclusion

In this paper, we proposed a new modulation scheme (VDMT) capable of achieving FEXT cancellation in communication scenarios where coordination between users is possible either at the transmitter or at the receiver. The scheme can be interpreted as a combination of DMT with either successive cancellation or precoding. Simulation results indicate that VDMT can provide significantly higher performance in DSL systems. The superiority of VDMT is more evident in short loops and high data rates, which makes it an attractive choice for VDSL applications.

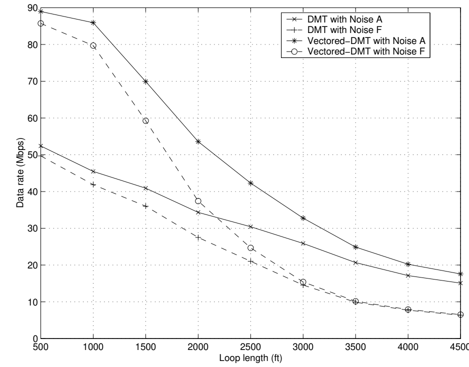

Fig. 8. Downstream VDSL data rates.

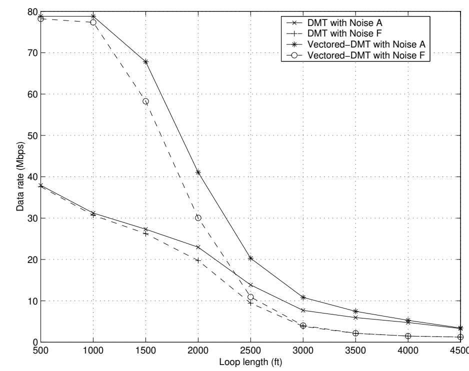

Fig. 9. Upstream VDSL data rates.

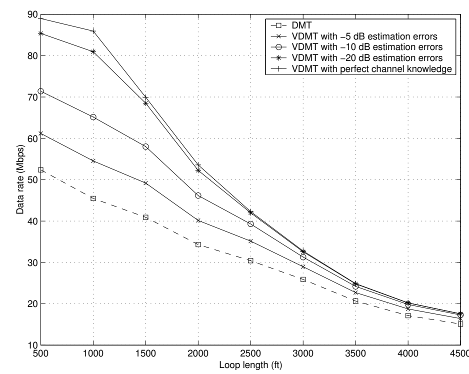

Fig. 10. Downstream VDSL data rates with channel estimation errors.

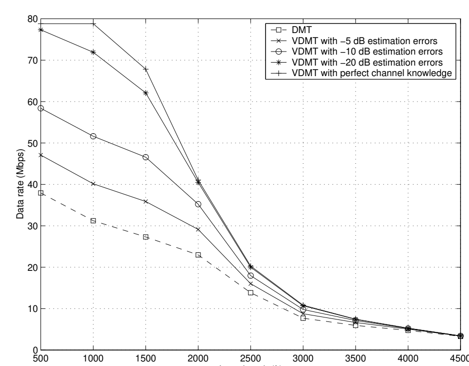

Fig. 11. Upstream VDSL data rates with channel estimation errors.

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