A Multi-user Precoding Scheme achieving Crosstalk Cancellation with Application to DSL Systems *†

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Abstract

This paper describes a precoding scheme capable of achieving crosstalk cancellation in scenarios involving communication from a "base station" to a number of geographically dispersed users. We illustrate that joint processing at the "base station" of the transmitted signals of all users can succeed in removing the otherwise resulting crosstalk. The method employed borrows from the concept of the Tomlinson-Harashima precoder which is often used for equalization against intersymbol interference (ISI). One possible application of the proposed technique is in Digital-Subscriber-Line (DSL) systems, where Far-End-Crosstalk (FEXT) poses a severe transmission constraint. Simulation results show that significant improvements can be realized in such scenarios.

1. Introduction

The Tomlinson-Harashima Precoder [10, 6] is an ISI mitigation structure consisting of a feedback filter in the transmitter and a feedforward filter in the receiver. This device is used to circumvent the problem of error propagation inherent in Decision-Feedback-Equalization (DFE). In the case of Zero-Forcing (ZF) Equalization the feedforward filter is an all-pass filter, which can be moved to the transmitter as discussed in [9].

This paper demonstrates that the precoding concept can also prove valuable for Multiple-Input-Multiple-Output (MIMO) ZF equalization. MIMO equalization with DFE was first proposed in [3]. For V-BLAST ([4]), it can be shown that the interference nulling and interference cancellation steps are equivalent to a ZF-DFE operating in the "space" domain. However, in point-to-multipoint communication scenarios, joint processing at the receiver side is not feasible. Examples include downlink transmission from a base station with multiple antennas to multiple users, and downstream transmission from a central office to a number of DSL users. In such cases, equalization has to be performed at the transmitter side, while taking measures to prevent energy increase.

The proposed equalization structure is described in Section 2. Section 3 outlines a method to generalize this structure for wide-band communication, and introduces a transmission optimization algorithm. In Section 4, simulation results are provided for a DSL system, where the potential benefits of the scheme are illustrated. Section 5 summarizes the conclusions of this paper.

2. Precoding for Crosstalk Cancellation

In the following * denotes the transpose-conjugate operation.

We assume a discrete-time model for the system, and narrow-band transmission with L users. Assuming that $\mathbf{x}$ is a vector whose elements are the MIMO channel input signals, $\mathbf{y}$ is a vector consisting of the signals received from each user, and $\mathbf{n}$ is a vector of the noise samples, we obtain:

$$\mathbf{y} = H\mathbf{x} + \mathbf{n} \tag{1}$$

where H is an $L \times L$ non-singular matrix describing the MIMO channel, and $\mathbf{x}$, $\mathbf{y}$, $\mathbf{n}$ are $L \times 1$ vectors. Also, the noise samples are white and (without loss of generality) they all have variance $N_o/2$.

Next, we show how appropriate signal processing at the transmitter can achieve interference-free reception. It is important to notice that naively attempting to invert the channel is not a good approach. It implies that we give more

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energy to the "weak" users, a step which is certainly against intuition gained from information theory.

The first step is to obtain the QR decomposition of H^*:

$$H^* = QR \tag{2}$$

where Q is an $L \times L$ unitary matrix, and R is an $L \times L$ upper triangular matrix. After defining $\mathbf{x} = Q\mathbf{x}'$, we obtain:

$$\mathbf{y} = R^*\mathbf{x}' + \mathbf{n} \tag{3}$$

Observe that multiplying by Q simply serves to rotate the signal space, thereby causing no energy increase. It is now clear that crosstalk-free reception implies:

$$S\tilde{\mathbf{x}} = R^*\mathbf{x}' \tag{4}$$

where S is an $L \times L$ diagonal matrix, whose elements equal the diagonal elements of R^*. Also, $\tilde{\mathbf{x}}$ is an $L \times 1$ vector consisting of the symbols destined for the L users, and $\mathbf{x}'$ is an $L \times 1$ vector, which when multiplied by Q provides the channel input vector. Expanding the previous equation we get:

$$\begin{bmatrix} r_{11}\tilde{x}_1 \\ r_{22}\tilde{x}_2 \\ \vdots \\ r_{LL}\tilde{x}_L \end{bmatrix} = \begin{bmatrix} r_{11} & 0 & \cdots & 0 \\ r_{21} & r_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ r_{L1} & r_{L2} & \cdots & r_{LL} \end{bmatrix} \cdot \begin{bmatrix} x'_1 \\ x'_2 \\ \vdots \\ x'_L \end{bmatrix} \tag{5}$$

Thanks to the triangular structure, inversion is trivial and it amounts to the following *precoding* operation:

$$\begin{aligned} x'_1 &= \tilde{x}_1 \\ x'_2 &= \tilde{x}_2 - \tfrac{r_{21}}{r_{22}}x'_1 \\ &\vdots \\ x'_L &= \tilde{x}_L - \tfrac{r_{L,L-1}}{r_{LL}}x'_{L-1} - \cdots - \tfrac{r_{L1}}{r_{LL}}x'_1 \end{aligned} \tag{6}$$

However, this computation may lead to significant energy increase of $\mathbf{x}'$ (and subsequently of $\mathbf{x}$). This problem is analogous to the one encountered when the feedback section of a (single-user) DFE is moved to the transmitter, in order to prevent error propagation. In that case, the solution is to employ modulo arithmetic to bound the value of the transmitted signal. The original idea was proposed (almost simultaneously) by Tomlinson in [10] and Harashima in [6].

Extending the concept of modulo arithmetic to the "space" dimension, we modify the precoding operation as follows:

$$\begin{aligned} x'_1 &= \tilde{x}_1 \\ x'_2 &= \Gamma_{M_2}\left[\tilde{x}_2 - \tfrac{r_{21}}{r_{22}}x'_1\right] \\ &\vdots \\ x'_L &= \Gamma_{M_L}\left[\tilde{x}_L - \tfrac{r_{L,L-1}}{r_{LL}}x'_{L-1} - \cdots - \tfrac{r_{L1}}{r_{LL}}x'_1\right] \end{aligned} \tag{7}$$

We define:

$$\Gamma_{M_i}[x] = x - M_i d \left\lfloor \frac{x + \frac{M_i d}{2}}{M_i d} \right\rfloor \tag{8}$$

where M_i is the PAM constellation size for user i, d is the constellation point spacing, and x is real. (If square-QAM is employed, then x is complex and we define $\Gamma_{M_i}[x] = \Gamma_{\sqrt{M_i}}[\Re(x)] + \jmath\Gamma_{\sqrt{M_i}}[\Im(x)]$.) The constellation point distance d is the same for all users, since it is desirable that all users have the same probability of error.

Note that $\Gamma_{M_i}[x]$ does not need to be integer-valued. The operation can be described as "wrapping" the real line around a circle (as illustrated in Figure 1).

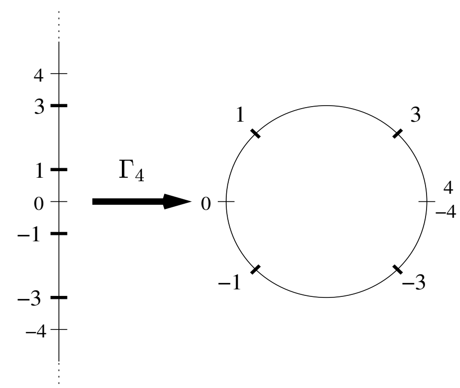

Figure 1. Illustration of the $\Gamma_M[x]$ operator with $M = 4$ and $d = 2$.

The precoding operation is shown in Figure 2. Making the approximation that the i^{th} precoder output has a uniform distribution over the interval $[-M_i d/2, M_i d/2]$, it can be shown ([1]) that the energy increases by a factor of $\frac{M_i^2}{M_i^2-1}$ for PAM, and by $\frac{M_i}{M_i-1}$ for square-QAM transmission. Therefore, the increase is not excessive, and becomes negligible for large constellations.

At each of the receivers the following operation is performed prior to detection:

$$\hat{y}_i = \Gamma_{M_i}\left[\frac{y_i}{r_{ii}}\right] \tag{9}$$

Direct substitution using Equations 3 and 7 yields:

$$\hat{y}_i = \Gamma_{M_i}\left\{\Gamma_{M_i}\left[\tilde{x}_i - \sum_{j=1}^{i-1}\frac{r_{ij}}{r_{ii}}x'_j\right] + \sum_{j=1}^{i-1}\frac{r_{ij}}{r_{ii}}x'_j + \frac{n_i}{r_{ii}}\right\} \tag{10}$$

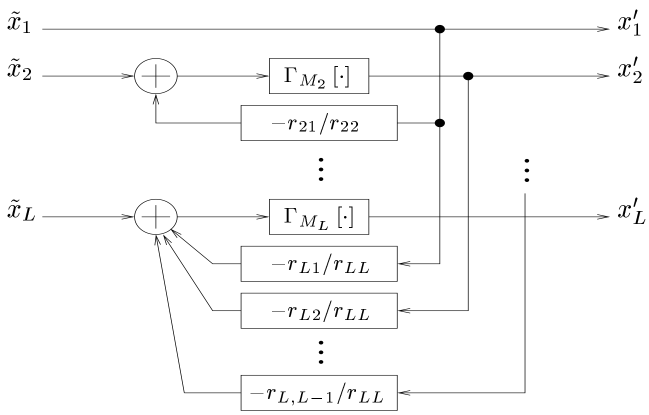

Figure 2. Tomlinson-Harashima Precoder in the "space" domain.

Observing that $\Gamma_M[x \pm y] = \Gamma_M\{\Gamma_M[x] \pm \Gamma_M[y]\}$, it is then straightforward to show that:

$$\hat{y}_i = \Gamma_{M_i}\left[\tilde{x}_i + \frac{n_i}{r_{ii}}\right] \tag{11}$$

In a well-designed system, $Pr(|n_i/r_{ii}| > d/2)$ is very small, so with fairly large M_i the following will hold with a very high probability:

$$\hat{y}_i = \tilde{x}_i + \frac{n_i}{r_{ii}} \tag{12}$$

Evidently, the proposed precoder achieves ZF equalization of the MIMO channel. The complete equalization structure is presented in Figure 3.

3. Combining Precoding with Multi-Carrier Modulation

A fundamental assumption of the previous section was the memoryless property of the MIMO channel. An elegant way to remove this constraint is by combining Discrete-Multitone (DMT) transmission ([7]) with the proposed precoding structure. The idea is to employ DMT, in order to partition the available bandwidth into a large number of orthogonal bands, where precoding can be applied individually.

A key requirement is that the cyclic prefix length of DMT must be at least as long as the maximum memory (expressed in number of symbols) of the transfer and coupling functions. Additionally, the IFFT windows of all users have to be synchronized. Under these assumptions, it can be shown ([5]) that transmission in the k^{th} DMT frequency bin (tone) can be modelled as:

$$\mathbf{y}_k = H_k\mathbf{x}_k + \mathbf{n}_k \tag{13}$$

where $\mathbf{x}_k$ is a vector consisting of the L channel inputs for tone k, $\mathbf{y}_k$ is a vector of the samples received by the L users for tone k, H_k is the channel matrix for tone k, and $\mathbf{n}_k$ is the noise vector for tone k.

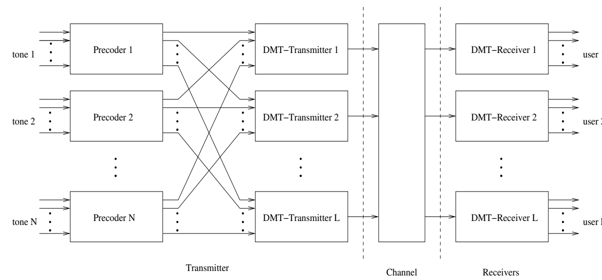

Figure 4. Combining Precoding with DMT.

Then, the use of the technique of Section 2 on each tone succeeds in removing the crosstalk, and thus establishes parallel channels as follows:

$$\hat{\mathbf{y}}_k = \tilde{\mathbf{x}}_k + S_k^{-1}\mathbf{n}_k \tag{14}$$

where the interpretation of $\tilde{\mathbf{x}}_k$, $\hat{\mathbf{y}}_k$, and S_k is the same as that of $\tilde{\mathbf{x}}$, $\hat{\mathbf{y}}$, and S in the specific case of tone k. The combined DMT-Precoding structure is shown in Figure 4.

Since assigning equal energy to all channels is clearly sub-optimal, a remaining issue is the allocation of energy to each of the parallel channels of the L users. Assuming that the energy constraint applies to the whole transmitter, we have:

$$\sum_{k=1}^{N}\sum_{l=1}^{L} \varepsilon_{kl} \leq E \tag{15}$$

where ε_{kl} is the energy per symbol at channel input l and tone k, L is the number of users, N is the number of tones and E is the maximum energy. Using a rate maximization criterion, we formulate the following optimization problem:

$$\max\left[\sum_{k=1}^{N}\sum_{l=1}^{L} a_l \log_2\left(1 + \frac{\left|S_k^l\right|^2 \tilde{\varepsilon}_{kl}}{\Gamma\frac{N_0}{2}}\right)\right] \tag{16}$$

subject to Equation 15, where Γ denotes the SNR Gap, $N_0/2$ is the noise variance, $\tilde{\varepsilon}_{kl}$ is the energy per symbol assigned to user l at tone k, S_k^l is the complex sub-channel gain for user k at tone l, and a_l is the weighting factor for user l such that $\sum_{l=1}^{L} a_l = 1$.

Assuming that the constellation sizes are fairly large, the energy increase due to precoding can be ignored. Thus, $\varepsilon'_{kl} \approx \tilde{\varepsilon}_{kl}$, where ε'_{kl} is the l^{th} output of the feedback section of the precoder operating at tone k. We note that:

$$\sum_{l=1}^{L} \varepsilon_{kl} = tr\ E\left(\mathbf{x}_k\mathbf{x}_k^*\right) = trE\left(\mathbf{x}'_k\mathbf{x}'^*_k\right) = \sum_{l=1}^{L} \varepsilon'_{kl} \approx \sum_{l=1}^{L} \tilde{\varepsilon}_{kl} \tag{17}$$

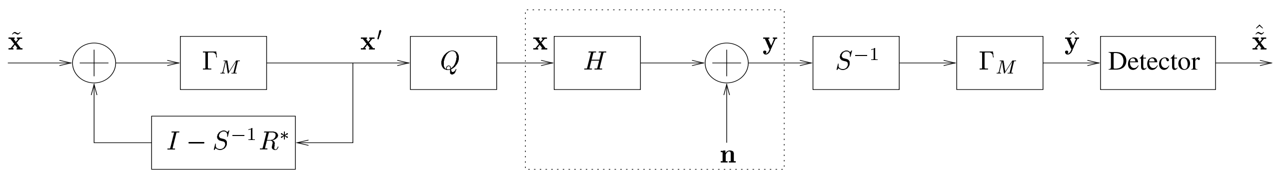

Figure 3. Multi-user Precoder structure.

Therefore:

$$\sum_{k=1}^{N}\sum_{l=1}^{L}\tilde{\varepsilon}_{kl} = \sum_{k=1}^{N}\sum_{l=1}^{L}\varepsilon_{kl} \leq E \tag{18}$$

Now, the optimization problem acquires a familiar form, whose solution can be easily obtained using Lagrange multipliers:

$$\begin{aligned} \tilde{\varepsilon}_{kl} &= a_l K - \frac{\Gamma\frac{N_0}{2}}{|S_k^l|^2} \\ \tilde{\varepsilon}_{kl} &\geq 0 \end{aligned} \tag{19}$$

The constant K is chosen so that the energy constraint is satisfied with equality:

$$K = \frac{1}{\sum_{l=1}^{L} a_l |T_l|}\left[E + \Gamma\frac{N_o}{2}\sum_{l=1}^{L}\sum_{k\ in\ T_l}\frac{1}{|S_k^l|^2}\right] \tag{20}$$

where T_l is the set of tones of user l with non-zero energy, and $|T_l|$ denotes its cardinality. It is rather obvious that this solution amounts to a two-dimensional case of the well-known waterfilling technique ([2]).

4. Application to DSL and Simulation Results

A typical DSL system topology is shown in Figure 5. The customers are connected to the central office through twisted-pair copper wires. Electromagnetic coupling between the wires results in crosstalk, which is characterized as Far-End-Crosstalk (FEXT) when the crosstalking signal originates from the same side as the affected signal, and Near-End-Crosstalk (NEXT) when the crosstalking signal originates from the opposite side of the affected signal.

Very-High-Speed DSL (VDSL) promises to deliver rates yet higher than the ones provided through the existing DSL systems by utilizing more bandwidth. Although the choice of Frequency-Division-Duplexing (FDD) removes the concern for NEXT, FEXT is a major noise component causing significant performance degradation. Therefore, employing multi-user precoding to cancel the FEXT in VDSL downstream transmission is expected to offer considerable benefits.

Next, we provide simulation results verifying this claim. The parameters used were chosen to comply with a recent

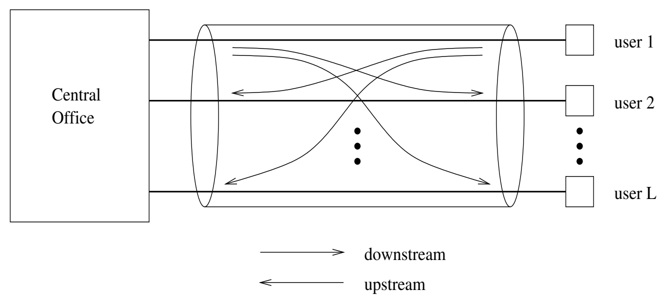

Figure 5. DSL system topology.

version of the VDSL standard currently being drafted by the T1E1.4 committee ([11, 8]). These are summarized in Table 1. The frequency bands used for downstream are 0.138 - 2.5 MHz, 3.75 - 8.5 MHz, and 12 - 17.7 MHz. An important difference between the standard and the simulations is that instead of applying an energy constraint on each user individually, a limit is imposed on the total energy transmitted by all users.

Number of DMT Tones	4096
Tone Width	4.3125 KHz
Symbol Rate	4 KHz
Coding Gain	3.8 dB
Noise Margin	6 dB
Required BER	$< 10^{-7}$
Power per user	11.5 dBmW
Cable Type	26-Gauge (0.4 mm)
Alien Crosstalk	10 ADSL, 4 HDSL, 10 ISDN

Table 1. Simulation parameters.

In the figures, we show the achievable rates for the 2 user case with and without precoding. The plots are traced by changing the weights of the users. Figure 6 refers to a scenario where the loop length for both users is 800 meters. We see that the improvement that can be realized in terms of data rate is almost 40%. For Figure 7 the loop lengths are 1000 meters (user 1) and 700 meters (user 2). In this asymmetric topology, the resultant improvement for user 2 is much larger than that for user 1. This is attributed to the fact that FEXT in short loops is more severe, and therefore cancelling it by employing precoding has a bigger positive effect on the data rate.

5. Conclusion

A precoding scheme for crosstalk cancelling in point to multi-point communication scenarios was proposed. This scheme bears similarities to the Tomlinson-Harashima precoder used for equalization. For channels with memory, precoding can be combined with DMT, and transmit optimization becomes analogous to waterfilling. Application of this method to DSL results in significant performance gains.

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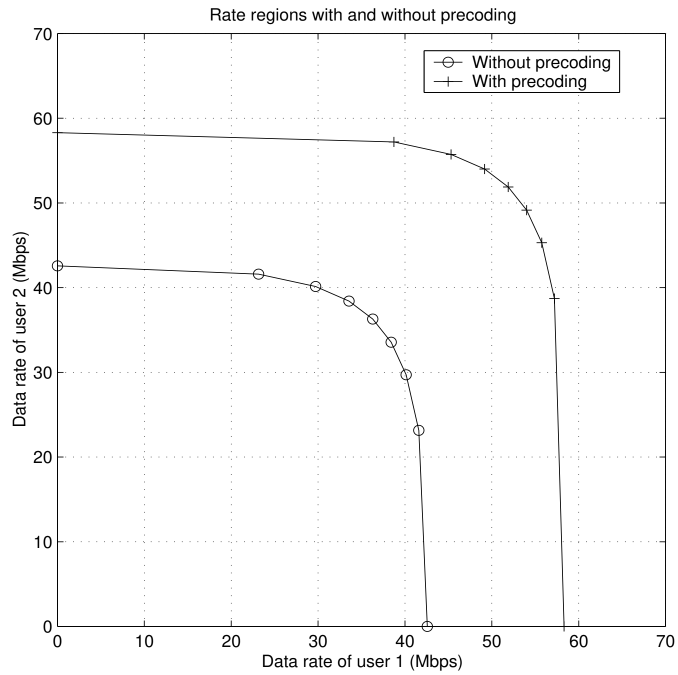

Figure 6. VDSL downstream rate regions for user 1 and user 2 at 800m.

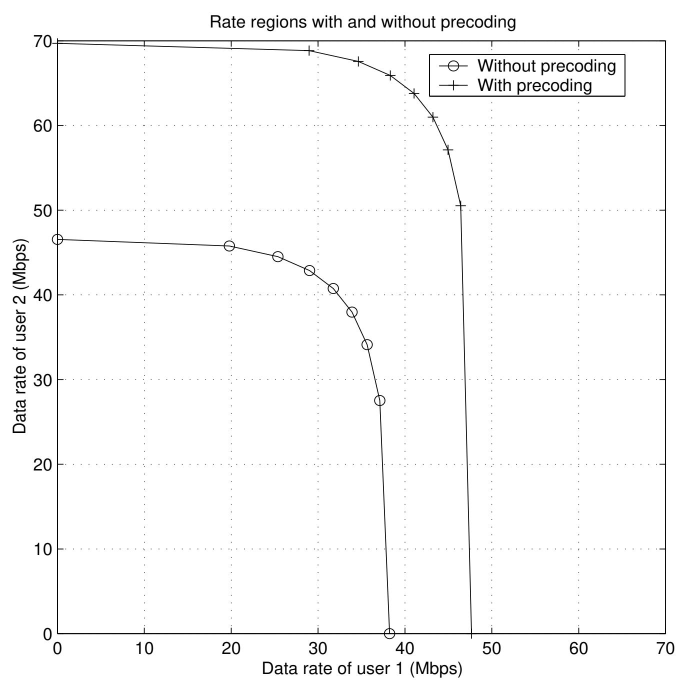

Figure 7. VDSL downstream rate regions for user 1 at 1000m and user 2 at 700m.