An Adaptive Multiuser Power Control Algorithm for VDSL

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Abstract—**This paper investigates optimal power control in a frequency selective multiuser interference network. The power control problem is modeled as a non-cooperative game. The existence and uniqueness of a Nash equilibrium in the game is established, and an iterative water-filling algorithm is proposed to efficiently reach the Nash equilibrium. It is shown that the Nash equilibrium point corresponds to a competitively optimal power allocation in the interference network. Based on this result, an adaptive power control algorithm for upstream VDSL power back-off is developed. The power control algorithm takes into account the loop transfer functions and cross-couplings, and allows the loops to negotiate the best use of power and frequency. This new algorithm is found to have a substantial performance improvement when compared to current methods.**

I. Introduction

Optimal power control is a central problem in the design of interference-limited multiuser communication systems. In this paper, the digital subscriber line (DSL) system is considered as a multiuser environment. The aim is to design an optimal power allocation scheme that maximizes the aggregate data rates of the mutually interfering DSL modems.

The DSL technology delivers high speed data services via ordinary telephone copper pairs [1]. DSL is a multiuser environment because DSL lines induce crosstalk into each other and such interference is often the dominant noise source. Although early DSL systems (e.g. ADSL and HDSL) were designed as single-user systems, the next DSL generation must deal with the issues of spectral compatibility and power control explicitly. In the following, the emerging VDSL standard is used as an example. A power control scheme based on the idea of competitive optimality is proposed for VDSL, and it is shown that a multiuser system design with an optimal power allocation scheme can result in a large system performance improvement compared to a single-user design.

The power control problem in DSL systems differs from its more widely-studied counterpart in wireless systems: fading and mobility issues are non-existent in DSL, and consequently, the assumption of perfect channel knowledge is realistic. On the other hand, unlike the usual flat-fading assumption in wireless, DSL loops are severely frequency selective. Thus, a power allocation scheme needs to consider not only the total power allocated for each user, but also the allocation of power in each frequency. Nevertheless, power control schemes designed for wireless systems ([2], [3]) still provide us with considerable insight. For example, the near-far problem in CDMA systems occurs also in DSL systems.

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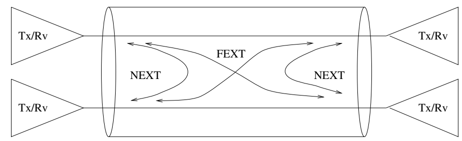

Fig. 1. The DSL crosstalk environment.

The rest of the paper is organized as follows: Section II reviews the DSL environment, and models a typical DSL loop as an interference network. Section III defines and characterizes the competitive equilibrium in such a network, and devises an iterative method to achieve the equilibrium. An adaptive power allocation method for VDSL is proposed in section IV based on the concept of competitive equilibrium. System performance for DSL is characterized in section V and conclusions are drawn in section VI.

II. The VDSL Environment

A DSL binder may consist of up to 100 subscriber lines bundled together. The bundled lines are electromagnetically coupled with each other, and this causes crosstalk noise (see figure 1). Near-end crosstalk (NEXT) refers to the crosstalk created by transmitters located on the same side as the receiver. Far-end crosstalk (FEXT) refers to the crosstalk created by transmitters located on the opposite side. In order to suppress NEXT, the VDSL standard uses frequency division duplex.

In this paper, it is assumed that the transmitters or the receivers in the same bundle are not coordinated. In this case, the DSL environment can be modeled as an interference channel. However, even for the simplest two-user case, the interference channel is little understood, and only partial achievable regions and outer bounds based on multiuser detection are available [4]. In this light, the transmission techniques described below do not use multiuser detection, and focus solely on the problem of optimal power allocation for each user.

Consider the interference channel model with N transmitters and N receivers depicted in figure 2. The channel from user i to user j is modeled as an ISI channel, whose transfer function is denoted as $H_{ij}(f)$, where $0 \le f \le F_s$, $F_s = \frac{1}{2T_s}$, and T_s is the sampling rate. Each receiver also experiences background noise with power-spectral-density (PSD) $\sigma_i(f)$. The power allocation for each transmitter is denoted as $P_i(f)$, which has to satisfy a power constraint:

$$\int_0^{F_s} P_i(f)df \le \mathbf{P_i}. \tag{1}$$

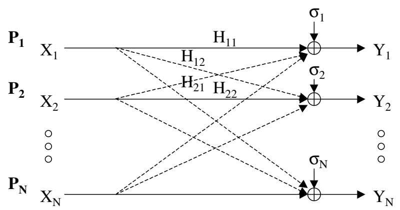

Fig. 2. A Gaussian interference network.

Treating all interference as noise, the achievable data rate for user i is:

$$R_i = \int_0^{F_s} \log_2 \left[1 + \frac{P_i(f)|H_{ii}(f)|^2}{\Gamma \left(\sigma_i(f) + \sum_{j \neq i} P_j(f)|H_{ji}(f)|^2 \right)} \right] df \tag{2}$$

where Γ denotes the SNR-gap. The objective of the system design is to maximize the set of rates $(R_1, \cdots R_N)$ subject to the power constraints (1). A convenient way to characterize the trade-offs among the user data rates is through the concept of a rate region, defined as:

$$\begin{aligned} \mathcal{R} \;\; = \;\; & \{(R_1, \ldots, R_N) : \exists (P_1(f), \ldots, P_N(f))\,, \\ & \text{satisfying (1) and (2)}\}\,. \end{aligned} \tag{3}$$

Although in theory the rate region can be found by an exhaustive search through all possible power allocations, or by a series of optimization involving weighted sums of data rates, the computational complexity of these approaches is prohibitively high, due to the non-convexity of the achievable rate formula. This difficulty is later circumvented by adopting the concept of competitive optimality.

Current DSL systems are designed as single-user systems, where PSD constraints are used to limit the worst-case crosstalk emissions. This approach is problematic in certain situations. For example, figure 3 illustrates a scenario where two loops of different lengths emanate from the central office (CO) to the customer premises (CP). When both transmitters at the CP-side transmit with the same PSD, the FEXT caused by the short line severely degrades the upstream performance of the long line. This is known as the near-far problem and a typical solution requires the short lines to reduce its upstream PSD through some mechanism known as upstream power back-off (UPBO) [5], [6], [7]. Note that the downstream direction does not suffer from a similar problem.

Several upstream power back-off algorithms have been proposed for VDSL, all of which attempt to reduce the interference caused by the shorter loops by forcing them to “emulate” the behavior of a longer loop. In the *constant power back-off* method, the PSD is reduced by a constant factor across all frequencies in the upstream transmission bands, so that at a particular reference frequency the received PSD level of shorter loops is the same as the received PSD level of a longer reference loop. A generalization of this method is the *reference length* method, where a frequency-dependent amount of back-off is applied, so

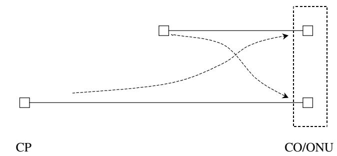

Fig. 3. A situation requiring upstream power back-off.

that the received PSD of a shorter loop is the same as that of a longer reference loop. Extensions of these methods include the *multiple reference length* method, the *equalized-FEXT* method and the *reference noise* method. A detailed review of these methods can be found in [7]. However, none of these methods is truly optimal, as finding the true optimum requires the solution of a non-convex optimization problem, which is computationally prohibitive. The first attempt in finding the true global optimum is due to Cherubini [8], where simulated annealing is used to solve the non-convex problem. The approach adopted here differs, since, instead of searching for a global optimum, we search for so-called competitively optimal points. The competitively optimal power allocation has the intuitive appeal of being the locally optimal solution that all users have the incentive to move toward.

III. Competitive Optimality

The interference channel can be modeled as a non-cooperative game, where each user adjusts its power allocation to maximize its own data rate, while regarding all other interference as noise. If such power adjustment is done continuously for all users, it is natural to ask whether an equilibrium can eventually be reached. Such an equilibrium is a desirable system operating point, since then, all users have reached their own local maxima, and nobody has an incentive to “move away” from the current power allocation. From a game theory perspective, this point is called a Nash equilibrium, and it is defined as a strategy profile in which each player’s strategy is an optimal response to the other player’s strategy [9].

Without loss of generality, the following model can be assumed for a two-user interference channel:

$$\mathbf{y}_1 = \mathbf{x}_1 + \mathcal{A}_2 \mathbf{x}_2 + \mathbf{n}_1 \tag{4}$$

$$\mathbf{y}_2 = \mathbf{x}_2 + \mathcal{A}_1 \mathbf{x}_1 + \mathbf{n}_2. \tag{5}$$

The squared magnitude of $\mathcal{A}_1$ and $\mathcal{A}_2$ are denoted as $\alpha_1(f)$ and $\alpha_2(f)$, and $N_1(f)$ and $N_2(f)$ are the noise PSD’s. The transmitters are considered as two players in a game, with the structure of the game, i.e., the interference coupling functions and noise power, being common knowledge. The strategy for each player is its transmit power spectrum, $P_1(f)$ and $P_2(f)$, subject to the power constraints $\int_0^{F_s} P_1(f)df \leq \mathbf{P}_1$, and $\int_0^{F_s} P_2(f)df \leq \mathbf{P}_2$. (Only deterministic, or pure strategy is considered here.) The payoff for each user is its respective data rate:

$$R_1 = \int_0^{F_s} \log\left(1 + \frac{P_1(f)}{N_1(f) + \alpha_2(f) P_2(f)}\right) df \qquad (6)$$

$$R_2 = \int_0^{F_s} \log\left(1 + \frac{P_2(f)}{N_2(f) + \alpha_1(f) P_1(f)}\right) df. \qquad (7)$$

Note that by choosing $N_1(f) = \frac{\Gamma\sigma_1(f)}{|H_{11}(f)|^2}$, $\alpha_2(f) = \frac{|H_{21}(f)|^2}{\Gamma|H_{11}(f)|^2}$, and similarly for $N_2(f)$ and $\alpha_1(f)$, one arrives at (2).

Since for each user the optimal power allocation is the one resulting from water-filling, a Nash equilibrium is reached if water-filling is simultaneously achieved for all users. A complete characterization of the simultaneous water-filling point is hard to obtain. Here, sufficient conditions for the existence and uniqueness of the Nash equilibrium are presented for the two-user case.

Theorem 1: Suppose that $\alpha_1(f)\alpha_2(f) < 1$, $\forall f$, then at least one pure strategy Nash equilibrium in the Gaussian interference game exists. Further, let $\epsilon_1 = \sup\{\alpha_1(f)\alpha_2(f)\}$, $\epsilon_2 = \sup\{\alpha_1(f)\}\frac{1}{F_s}\int_0^{F_s}\alpha_2(f)df$, and $\epsilon_3 = \sup\{\alpha_2(f)\}\frac{1}{F_s}\int_0^{F_s}\alpha_1(f)df$. If either $\epsilon_1 < 1$, or $\epsilon_1 + \epsilon_2 < \frac{1}{2}$, or $\epsilon_1 + \epsilon_3 < \frac{1}{2}$, then the Nash equilibrium is unique, and is stable.

Proof: This result is an improvement of an earlier result [10], which contains the existence proof. The condition for uniqueness is strengthened, and its proof is presented below.

The idea is to start with an arbitrary power distribution for user 1, and water-fill for the two users alternatively regarding the other user as noise. Denote $P_1^{(0)}(f)$ as the initial power allocation for user 1. Water-fill for user 2 regarding $P_1^{(0)}(f)$ as noise, call the resulting power allocation $P_2^{(0)}(f)$. Then water-fill for user 1 regarding $P_2^{(0)}(f)$ as noise to get $P_1^{(1)}(f)$, then $P_2^{(1)}(f), P_1^{(2)}(f)$, etc. Assume the existence of a Nash equilibrium $(P_1^N(f), P_2^N(f))$. We will show that the iterative water-filling process converges to the Nash equilibrium in $\mathbf{L}_1$-norm, $||P_1^k(f) - P_1^N(f)||_1 = \frac{1}{F_s}\int_0^{F_s}|P_1^{(k)}(f) - P_1^N(f)|df$. Denote the positive part of a function as $(\cdot)_+$, and the negative part as $(\cdot)_-$. Let $Q_i^{(k)}(f) = P_i^{(k)}(f) - P_i^N(f)$. Then:

$$\begin{aligned}
&\max\left\{\int_0^{F_s}\left(Q_1^{(k+1)}(f)\right)_+ df, \int_0^{F_s}\left(Q_1^{(k+1)}(f)\right)_- df\right\} \\
&\le \sup\alpha_2(f)\max\left\{\int_0^{F_s}\left(Q_2^{(k)}(f)\right)_+ df, \int_0^{F_s}\left(Q_2^{(k)}(f)\right)_- df\right\} \\
&\le \sup\alpha_2(f)\sup\alpha_1(f) \\
&\quad \max\left\{\int_0^{F_s}\left(Q_1^{(k)}(f)\right)_+ df, \int_0^{F_s}\left(Q_1^{(k)}(f)\right)_- df\right\}
\end{aligned}$$

which is a contraction if $\sup\alpha_1(f)\sup\alpha_2(f) = \epsilon_1 < 1$. So, $P_1^{(k)} \to P_1^N$ in $\mathbf{L}_1$-norm as $k \to \infty$.

The above condition may be too restrictive in certain cases. To derive the second and third sufficient conditions, let $\Delta_1^{(k)}(f) = P_1^{(k)}(f) - P_1^N(f)$ be the difference in power allocation from a Nash equilibrium at the kth iteration. The difference in interference is then $\alpha_1(f)\Delta_1^{(k)}(f)$. This difference in interference would cause user 2's power allocation to differ by at most $\alpha_1(f)\Delta_1^{(k)}(f) - \frac{1}{F_s}\int_0^{F_s}\alpha_1(f)\Delta_1^{(k)}(f)df$. (The mean is subtracted because the water-filling process is sensitive only to the relative interference level change, and not to the absolute interference level change.) This difference in user 2's power allocation in turn causes an interference level difference in user 1: $\alpha_2(f)\alpha_1(f)\Delta_1^{(k)}(f) - \alpha_2(f)\frac{1}{F_s}\int_0^{F_s}\alpha_1(f)\Delta_1^{(k)}(f)df$. Finally, this difference in interference would cause user 1's power allocation to differ by at most:

$$\begin{aligned}
\Delta_1^{(k+1)}(f) \le\ & \alpha_2(f)\alpha_1(f)\Delta_1^{(k)}(f) - \\
& \alpha_2(f)\tfrac{1}{F_s}\textstyle\int_0^{F_s}\alpha_1(f)\Delta_1^{(k)}(f)df - \\
& \tfrac{1}{F_s}\textstyle\int_0^{F_s}\alpha_2(f)\alpha_1(f)\Delta_1^{(k)}(f)df - \\
& \tfrac{1}{F_s}\textstyle\int_0^{F_s}\alpha_2(f)df\tfrac{1}{F_s}\int_0^{F_s}\alpha_1(f)\Delta_1^{(k)}(f)df
\end{aligned}$$

The $\mathbf{L}_1$ norm of $\Delta_1^{(k+1)}(f)$ above can be bounded using the triangular inequality as shown below:

$$\begin{aligned}
&\tfrac{1}{F_s}\textstyle\int_0^{F_s}|\Delta_1^{(k+1)}(f)|df \le \\
&\quad \sup\{\alpha_2(f)\alpha_1(f)\}\tfrac{1}{F_s}\textstyle\int_0^{F_s}|\Delta_1^{(k)}(f)|df + \\
&\quad \sup\{\alpha_1(f)\}\tfrac{1}{F_s}\textstyle\int_0^{F_s}\alpha_2(f)df\tfrac{1}{F_s}\int_0^{F_s}|\Delta_1^{(k)}(f)|df + \\
&\quad \sup\{\alpha_2(f)\alpha_1(f)\}\tfrac{1}{F_s}\textstyle\int_0^{F_s}|\Delta_1^{(k)}(f)|df + \\
&\quad \sup\{\alpha_1(f)\}\tfrac{1}{F_s}\textstyle\int_0^{F_s}\alpha_2(f)df\tfrac{1}{F_s}\int_0^{F_s}|\Delta_1^{(k)}(f)|df
\end{aligned}$$

Thus, if $\epsilon_1 + \epsilon_3 < \frac{1}{2}$, the iterative water-filling algorithm becomes a contraction, and $P_1^{(k)}(f) \to P_1^N(f)$ in $\mathbf{L}_1$-norm as $k \to \infty$. The same analysis can be applied to $P_2(f)$ yielding the third condition.

The convergence of the iterative water-filling process implies that the Nash equilibrium is unique. This is because the starting point is arbitrary, so in particular, the starting point could be a different Nash equilibrium if the Nash equilibrium were not unique. But each Nash equilibrium is its own fixed point, so this cannot happen. The stability of the Nash equilibrium also follows from the convergence of the iterative procedure. □

Corollary 1: If any of the conditions for existence and uniqueness of the Nash Equilibrium is satisfied, then an iterative water-filling algorithm, where in every step each modem updates its PSD regarding all interference as noise, converges to the unique Nash equilibrium from any starting point.

Proof: This is a direct consequence of the iterative water-filling procedure in the proof of the above theorem. □

IV. Adaptive Power Control

The DSL channel is severely frequency-selective. So, a power control algorithm for DSL must allocate power optimally both across the frequency and among the users. However, if one considers only the competitively optimal power allocations, and assumes that the existence and uniqueness conditions for the Nash equilibrium are satisfied, then total power alone is sufficient to represent all such power allocations.

We now propose an adaptive power control algorithm based on competitive optimality. The proposed algorithm runs in two stages, aiming to achieve certain target rates for each user. The inner stage takes specific power constraints for each user as input, and derives the competitively optimal power allocations and data rates as output with iterative water-filling. In other words, each user updates its power allocation regarding all other users' crosstalk as noise. The water-filling is successively applied to the first user, the second user, and so on, then again to the first user, second user, etc, until the power allocations of all users have converged.

The outer stage finds the optimal total power constraint for each user by adjusting each user's total power based on the outcome of the inner iterative water-filling. If a user's data rate is below its target rate, its power is increased, unless it is already at its maximum power limit. If a user's data rate is much above its target rate, its power is decreased. If the data rate is just above the target rate, its power remains unchanged. The outer procedure converges when the set of target rates is achieved. The algorithm can be expressed as follows:

Algorithm 1: Let K be the number of users, $\mathbf{P}$ be the modem power limit and T_i be the target rate of the ith modem.

Initialize $\mathbf{P}_i = \mathbf{P}$, $i = 1, \dots K$
repeat
  repeat
    for $i = 1$ to K
      $$N(f) = \sum_{j=1, j\neq i}^{K} |H_{ji}(f)|^2 P_j(f) + \sigma_i(f)$$
      $P_i(f)$ = water-filling spectrum with channel $|H_{ii}(f)|^2$, noise $N(f)$, and power constraint $\mathbf{P}_i$
      R_i = data rate on channel $|H_{ii}(f)|^2$ with power allocation $P_i(f)$, and noise $N(f)$
    end
  until the desired accuracy is reached
  for $i = 1$ to K
    If $R_i > T_i + \epsilon$, set $\mathbf{P}_i = \mathbf{P}_i - \delta$
    If $R_i < T_i$, set $\mathbf{P}_i = \mathbf{P}_i + \delta$
    If $\mathbf{P}_i > \mathbf{P}$, set $\mathbf{P}_i = \mathbf{P}$
  end
until $R_i > T_i$ for all i

The above algorithm has been found to work well with $\delta = 3$ dB and ϵ equal to 10% of the target rate. The outer iteration converges only if the set of target rates is achievable, which has to be determined *a priori*. Alternatively, if full knowledge of all channel and crosstalk transfer functions is available, then a central agent may perform the computational steps of the power control algorithm "off-line", and "command" the modems to adopt the specified power allocations.

Compared to conventional power control methods, this new method offers two key advantages. First, the interference levels are implicitly restricted, therefore PSD constraints are not needed, thus allowing a more efficient use of total power. Secondly, the different loops in a binder are effectively given the opportunity to negotiate the best use of frequency, so that each loop has an incentive to "move away" from those frequencies where interference is strong, and "concentrate" on those frequencies that it can most efficiently utilize.

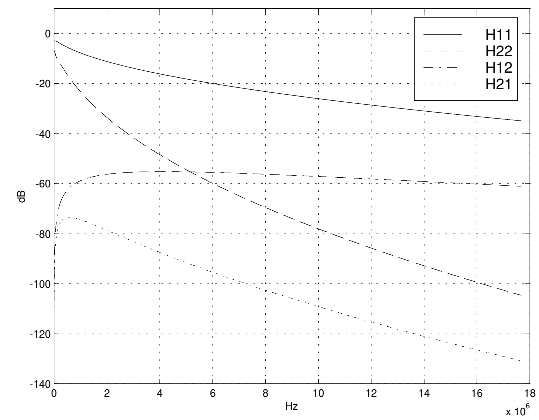

Fig. 4. Channel and crosstalk transfer functions: 3000ft vs 1000ft.

V. Performance

Figure 4 shows the plots of the channel and crosstalk transfer functions for two users located 3000ft and 1000ft away from the CO, where H_{ij} refers to the upstream transfer function from user i to user j. The twisted pairs are assumed to be 26 AWG, and the crosstalk transfer functions are computed using the well-known FEXT models [11]. For all $j \neq i$, $H_{ii}(f) \gg H_{ij}(f)$, $\forall f$, and this difference exceeds 20dB. So, $\alpha_1(f)\alpha_2(f) = \frac{|H_{12}(f)|^2}{\Gamma|H_{22}(f)|^2}\frac{|H_{21}(f)|^2}{\Gamma|H_{11}(f)|^2} < 1$, $\forall f$, thus, a Nash equilibrium exists. Also, the first condition of Theorem 1 turns out to be satisfied, so the Nash equilibrium is unique, and the iterative water-filling procedure converges. The authors have not encountered any realistic DSL scenario, where iterative water-filling does not converge.

Next, the performance of the power control scheme for a binder with 8 VDSL lines is evaluated. Four of the lines are at a distance of 3000 feet away from the CO, while the other 4 are at the a distance of L feet, where L varies between 500 and 2500 feet. The maximum transmission power of each modem is 11.5dBm [11], but no PSD constraint applies, except at frequencies below 1.1MHz for the protection of ADSL and other services. Crosstalk noise model A [12] is assumed, and the 998 frequency plan [13] is used to separate upstream and downstream. Also, frequency bands corresponding to the amateur radio frequencies [11] are notched off.

Figure 5 illustrates the convergence of the algorithm, where the two sets of loops have lengths 1000ft and 3000ft. The total power constraint is set at -15.5dBm for the 1000ft loops, and at 11.5dBm for the 3000ft loops. The algorithm successively

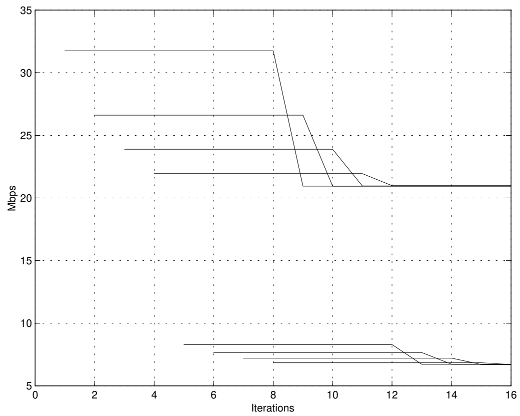

Fig. 5. Convergence of iterative water-filling algorithm.

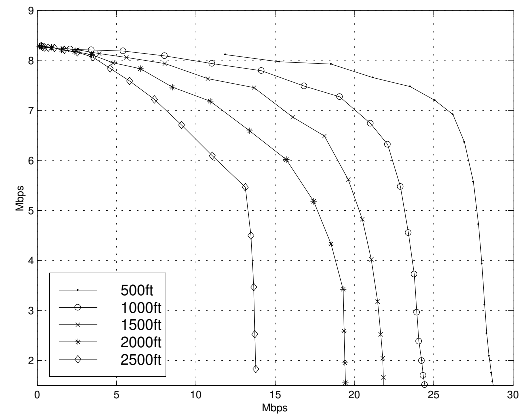

Fig. 6. Competitively optimal rate regions: 3000ft vs various lengths.

TABLE I

Reference-noise power back-off vs iterative water-filling.

loop length (ft)	reference noise (Mbps)	iterative water-filling (Mbps)
500	12.5	26.5
1000	10.1	21.0
1500	8.9	16.5
2000	8.0	12.5
2500	7.3	9.0

performs water-filling for each of the loops, while keeping the power allocation of the other 7 loops unchanged. After the first water-filling, the 1000ft loop achieves a rate of 32Mbps in the absence of any interference, however, subsequent loops achieve smaller data rates due to the crosstalk from previously water-filled loops. Eventually, when the first loop is revisited at the 9th iteration, its data rate is also reduced. The algorithm converges after only two water-fillings per loop.

The data rates for each set of 4 users are the same, so the rate region can be depicted as two-dimensional, as shown in figure 6. Different total power constraints result in alternative rate-tuples. Using the curve corresponding to 500ft as an example, one observes that 7.8Mbps for the 3000ft loops and 18Mbps for the 500ft loops are achievable. With a different total power allocation, 7Mbps for the 3000ft loops and 26Mbps for the 500ft loops are also achievable. The data rate trade-offs are easy to visualize, implying the possibility of supporting different classes of service on the same binder.

Finally, the proposed scheme is compared with the reference noise power back-off method, where the reference noise level equals the FEXT caused by a 3000ft loop. This means that all loops are forced to emit the same amount of interference as a 3000ft loop, regardless of their actual length. It is found that each of the 3000ft loops achieves a rate of 6.7Mbps. The performance of the other 4 loops is tabulated in Table I. Evidently, the competitively optimal power allocation method offers a substantial increase in performance.

VI. Conclusion

This paper considers the problem of optimal power control in a frequency selective multiuser interference network. The interference network is modeled as a non-cooperative game. Under a set of sufficient conditions, the existence and uniqueness of a Nash equilibrium in the game are shown. The Nash equilibrium corresponds to a competitively optimal power allocation, and it can be reached using an iterative water-filling algorithm. This iterative algorithm is used as the core of an adaptive power control scheme. The new scheme allows the loops to negotiate the best use of power and frequency with each other. When applied to the VDSL upstream power back-off problem, it is found to outperform current power back-off schemes substantially.

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