`EEE523 Home Work #1

Fuding Ge

1. **Question:** The transition (cutoff) frequency, f_T, is defined as follows:

$$f_T = \frac{g_m}{2\pi(C_{gd} + C_{gs})}$$

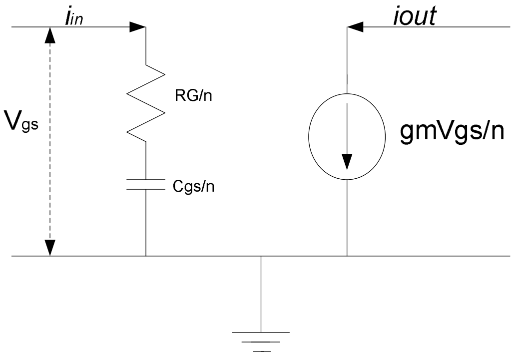
(1) Suppose the gate resistance is significant (R_G) and the device is modeled as a distributed set of n transistors each with a gate resistance equal to R_G/n. Find f_T.

(2) For a given bias current, the minimum allowable drain-source voltage for operation in saturation can be reduced by increasing the width and hence the capacitance of the transistor. Prove the following relationship

$$f_T = \frac{3}{2}\frac{\mu_n}{2\pi}\frac{V_{gs} - V_t}{L^2}$$

Answer:

(1) We built a simplified model as shown in the following figure:



$$Vgs = i_{in} \bullet (\frac{RG}{n} + \frac{1}{\omega \frac{Cgs}{n}}) = i_{in} \bullet (\frac{RG}{n} + \frac{n}{\omega Cgs})$$

$$i_{out} = \frac{g_m}{n} \bullet i_{in} \bullet (\frac{RG}{n} + \frac{n}{\omega Cgs})$$

Let

$$\frac{i_{out}}{i_{in}} = \frac{g_m}{n} \bullet (\frac{RG}{n} + \frac{n}{\omega Cgs}) = 1$$

We get

$$\omega_T = \frac{\frac{1}{g_m} - RG}{C_{gs}}$$

We can rewrite it as:

$$f_T = \frac{\frac{1}{g_m} - RG}{2\pi C_{gs}}$$

We can see that the transition frequency becomes smaller when we consider the gate resistance.

(2) When the transistor is in saturation region, Cgd is quite small and can be neglected. We also have:

$$Cgs = \frac{2}{3} WLCox$$

and

$$g_m = \mu_n Cox \frac{W}{L}(V_{gs} - V_t)$$

Then

$$f_T = \frac{3}{2}\frac{g_m}{2\pi WLCox} = \frac{3}{2}\frac{\mu_n Cox \frac{W}{L}(V_{gs} - V_t)}{2\pi WLCox} = \frac{3}{2}\frac{\mu_n (V_{gs} - V_t)}{2\pi L^2}$$

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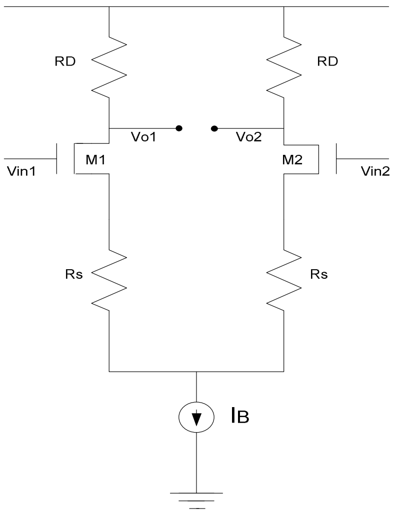


Calculate the small signal voltage gain:

$$A_v = \frac{V_{o2} - V_{o1}}{V_{in2} - V_{in1}}$$

Answer: This is a fully differential amplifier with source degeneration. We can use the "half-circuit" technique to calculate the gain.

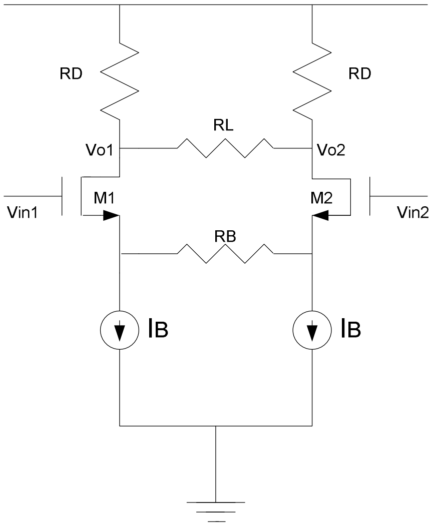
We know that the equivalent transconductance of the circuit is:

$$G_m = \frac{\partial I_D}{\partial V_{in}} = \frac{g_m}{1 + g_m Rs}$$

Then

$$A_v = \frac{V_{o2} - V_{o1}}{V_{in2} - V_{in1}} = -G_m R_D = \frac{-g_m R_D}{1 + g_m Rs}$$

3 Calculate the differential-mode and common-mode gain.



Answer:
Differential-mode gain:
The circuit can treated as amplifier with source degeneration with a resistor of RB/2. The load will be RD//RL/2.

Then the differential model gain is:

$$A_{vd} = \frac{V_{o2} - V_{o1}}{V_{in2} - V_{in1}} == \frac{-g_m R_D \parallel \frac{RL}{2}}{1 + g_m \frac{Rs}{2}}$$

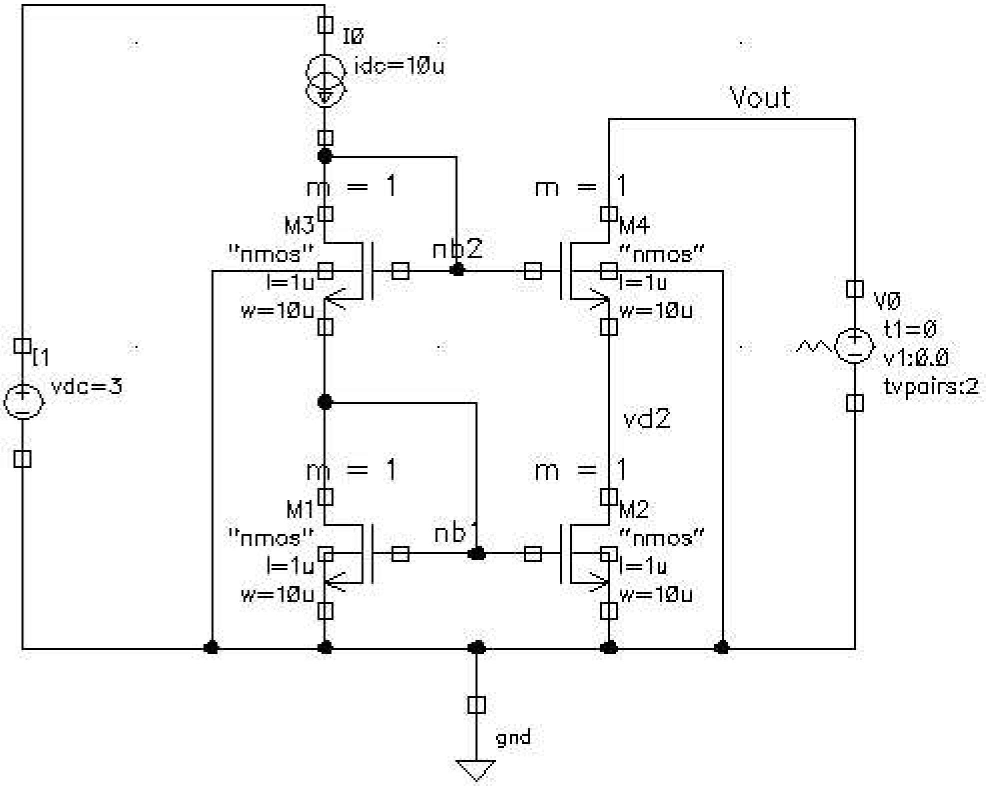
Common mode gain:

When the small signal input vin=0, there is no current flow through RB or RL. If we assume the resistance of the current source IB is RSS, then the equivalent resistance of the 2 IB is RSS/2. Us the equation (4.28) in the textbook, we have:

$$A_{vc} = \frac{RD/2}{1/(2gm) + RSS/2} = \frac{RD}{1/gm + RSS}$$

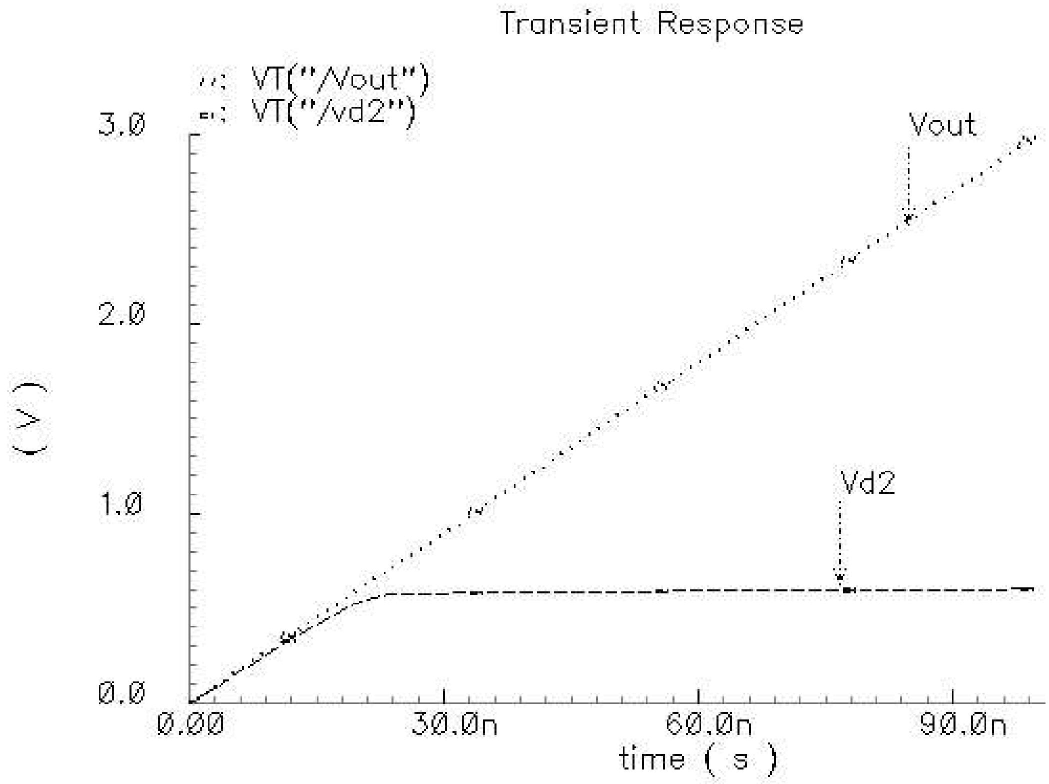
4 Show the shielding effect of cascode current mirror circuit. Use the transistor parameter in the textbook (Table 2.1). You can use any voltage, current, and transistor size -Simulation

The test circuit is shown in the following figure. The voltage Vout is swept from 0 V to 3 V in 100 ns. We checked the voltage Vd2.



Test circuit for shielding effects in cascode current mirror

The simulated result is shown in the following figure. It can be see that once the output voltage Vout in the range that M2 and M4 working in the saturation region, though Vout change a lot, Vd2 almost keep the same value. This is the shielding effects of cascode current mirror.



Simulated result showing the shielding effects

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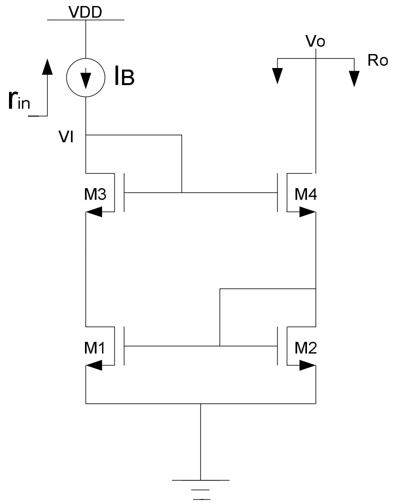

(1)Calculate the output impedance Ro
(2) Calculate the minimum output voltage Vomin
(3) Calculate the minimum input voltage Vimin

Answer: This is the Wilson current mirror.

(1) The output impedance is:

$$Ro = R(1 + A_L)$$

Where R is the output impedance without the feedback due to M1 and M3. Treated M2 as the source degeneration resistance of M4, we have:

$$R = 2r_{o4}$$

We also have:

$$A_L = \frac{gm1(r_{o1} \parallel r_{in})}{2}$$

Assume $r_{in} = r_{o1}$ we have:

$$Ro = 2r_{o4}(1 + \frac{g_{m1}r_{o1}}{4}) = r_{o4}\frac{g_{m1}r_{o1}}{2}$$

(2) Minimum output voltage Vomin:

For the operation of M2 in saturation region, the voltage of M2 drain should be:

$$V_{d2} = V_{GS}$$

$$V_{d4} = V_{d2} + V_{GS} - V_T = 2V_{GS} - V_T$$

Hence we have:

$$V_{o\min} = 2V_{GS} - V_T$$

(3) Minimum input voltage Vimin:

$$V_{in\min} = V_{o\min} = 2V_{GS} - V_T$$