9.5 Combinations

Coach Chavez wants to select co-captains for her basketball team. She will select 2 girls from the 4 oldest members of the team: Anita, Bailey, Charlene, and Daniele.

Step 1: Use the first letter of each name to list all of the permutations of co-captains. How many are there?

AB	BA	CA	DA
AC	BC	CB	DB
AD	BD	CD	DC

Step 2: Cross out any arrangement that contains the same letters as another one in the list. How many are there now?

Step 3: Explain the difference between the two lists.

An arrangement, or listing, or objects in which order is *not* important is called a **combination**. For example, in the activity above, choosing Anita and Bailey is the same as choosing Bailey and Anita.

Permutations and combinations are related. You can find the number of combinations of objects by dividing the number of permutations of the entire set by the number of ways each smaller set can be arranged.

A permutation of 4 players taken 2 at a time.

$$\frac{4 \cdot 3}{2!} = \frac{4 \cdot 3}{2 \cdot 1} = \frac{12}{2} = 6$$

There are $2!$ ways to arrange 2 players.

Example: Find the Number of Combinations

✓ Paul's Pizza Parlor is offering a large two-topping pizza for $14.99. There are five toppings from which to choose. How many different two-topping pizzas are possible?

Method 1: Make a List

The five toppings are labeled pepperoni (p), sausage (s), onions (o), mushrooms (m), and green peppers (g).

p,s	p,m	p,o	p,g	s,m
s,o	s,g	m,o	m,g	o,g

Method 2: Use a Permutation

There are $5 \cdot 4$ permutations of two toppings chosen from five. There are $2!$ ways to arrange the two toppings.

$$\frac{5 \cdot 4}{2!} = \frac{20}{2} = 10$$

So, there are 10 different two-topping pizzas.

Example: Use a Combination to Solve a Problem

- ✓ A checkers tournament features each of the top 8 regional players playing every opponent one time. The 2 players with the best records will then play in a final round to determine the champion. How many matches will be played if there are no ties?

Find the number of ways 2 players can be chosen from a group of 8.

There are $8 \cdot 7$ ways to choose 2 people.
There are $2!$ ways to choose 2 people.

$$\frac{8 \cdot 7}{2!} = \frac{56}{2} = 28$$

There are 28 matches plus 1 final match to determine the champion. So, there will be 29 matches played.

****Your Turn**

How many matches will be played if the top 16 players were invited to play?

The difference between permutations and combinations is that the order is important in permutations, while order is *not* important in combinations.

Examples: Identify Permutations and Combinations

Tell whether each situation represents a permutation or combination. Then solve the problem.

- ✓ The six students listed at the right are members of Student Council. How many ways can you choose a president, vice president, and treasurer from this group?

Ballot
Marissa
Samuel
Paige
Travis
Sally
Kenny

This is a permutation because the order of president, vice president, and treasurer is important. So, the number of ways you can choose the 3 officers is $6 \cdot 5 \cdot 4$, or 120 ways.

- ✓ In how many ways can you choose a committee of three students from the six members in student council shown above?

This is a *combination* because the order of the students in the committee is not important.

There are $6 \cdot 5 \cdot 4$ ways to choose 3 people.
There are $3!$ ways to choose 3 people.

$$\frac{6 \cdot 5 \cdot 4}{3!} = \frac{120}{6} = 20$$

So, there are 20 ways to choose the committee.

****Your Turn**

Tell whether each situation represents a permutation or combination. Then solve the problem.

- ✓ How many ways can 10 students finish 1st, 2nd, and 3rd at the science fair?

- ✓ How many ways can you pick 2 puppies from a littler of 7 puppies?

- ✓ There are 21 soccer players trying out for 15 spots on the soccer team. How many ways does the coach have to create the team?

- ✓ How many ways can you select four essay questions out of a total of 10 on the exam?

- ✓ Six children remain in a game of musical chairs. If two chairs are removed, how many different groups of four students remain?

- ✓ How many ways can three flute players be seated in the first, second, or third seats in the orchestra?

- ✓ In how many ways can four paintings be chosen for display from a collection of 15?