

9.4 PERMUTATIONS

How many different ways are there to arrange your first three classes if they are math, science, and language arts?

- Step 1:** Write math, science, and language arts on index cards.
- Step 2:** Choose one of the subjects as the first class of the day. Choose one of the remaining two subjects for the second class. The third class is the card that remains.
- Step 3:** Record this arrangement of classes.
- Step 4:** Change the order of the classes. Record this arrangement. Continue rearranging the cards until you have found all of the possible arrangements.

- ✓ When you first started to make your list, how many choices did you have for your first class?
 - ✓ Once your first class was selected, how many choices did you have for the second class? Then the third class?
 - ✓ Explain how you can use the Fundamental Counting Principle to find the number of arrangements.
-

A **permutation** is an arrangement, or listing, of objects in which order is important. You can use the Fundamental Counting Principle to find the number of possible arrangements.

$$\begin{array}{c} \frac{3}{\text{There are}} \\ \frac{3 \text{ choices}}{\text{for the 1}^{\text{st}}} \\ \frac{\text{class.}}{\text{}} \end{array} \times \begin{array}{c} \frac{2}{\text{There are}} \\ \frac{2 \text{ choices}}{\text{for the}} \\ \frac{\text{2}^{\text{nd}} \text{ class.}}{\text{}} \end{array} \times \begin{array}{c} \frac{1}{\text{There is 1}} \\ \frac{\text{choice for}}{\text{the 3}^{\text{rd}}} \\ \frac{\text{class.}}{\text{}} \end{array} = \begin{array}{c} \frac{6}{\text{The number}} \\ \frac{\text{of}}{\text{arrangements}} \\ \frac{\text{of 3 classes}}{\text{}} \end{array}$$

The expression $3 \times 2 \times 1$ can be written as $3!$, which is read *three factorial*.

The expression n factorial ($n!$) is the product of all counting numbers beginning with n and counting backward to 1.

Examples: Evaluate Factorials

- ✓ Find the value of each expression.

7!

23!

9!

Examples: Find a Permutation

- ✓ In how many ways can the starting six players of a volleyball team stand in a row for a picture?

This is a permutation that can be written as $6!$.

$$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$$

So, there are 720 ways the six starting players can stand in a row.

- ✓ The finals of the Middle School Appalachian League feature 8 swimmers. In how many ways can the swimmers finish in 1st or 2nd place?

There are 8 choices for 1st place and 7 choices that remain for 2nd place. So, there are $8 \cdot 7$ or 56 choices for 1st and 2nd place.

****Your Turn**

- ✓ In how many ways can a softball manager arrange the first four batters in a lineup of nine players?

- ✓ How many different 5-digit zip codes are there if no digit is repeated?

- ✓ The chromatic scale has 12 notes. In how many ways can a song start with 4 different notes from that scale?

- ✓ There are 7 students waiting at the bus stop. In how many ways can the students board the bus when it arrives?