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AC machine equation set in d, q axes of ordinates

$p_\tau\Psi_d = u_d + \omega\Psi_q - ri_d$, $i_d = 1/x_\sigma (\Psi_d - \Psi_{ad})$, (1)

$p_\tau\Psi_q = u_q - \omega\Psi_d - ri_q$, $i_q = 1/x_\sigma (\Psi_q - \Psi_{aq})$, (2)

$p_\tau\Psi_{fd} = u_{fd} - r_{fd}i_{fd}$, $i_{fd} = 1/x_{\sigma f} (\Psi_{fd} - \Psi_{ad})$, (3)

$p_\tau\Psi_{jd} = - r_{jd}i_{jd}$, $i_{jd} = 1/x_{\sigma jd} (\Psi_{jd} - \Psi_{ad})$, $j = 1,...,n$, (4)

$p_\tau\Psi_{iq} = - r_{iq}i_{iq}$, $i_{iq} = 1/x_{\sigma iq} (\Psi_{iq} - \Psi_{aq})$, $i = 1,...,m$, (5)

$p_\tau\Psi_{ad} = l_d\, p_\tau i_{\Sigma d} + m_{dq}\, p_\tau i_{\Sigma q}$, (6)

$p_\tau\Psi_{aq} = l_q\, p_\tau i_{\Sigma q} + m_{dq}\, p_\tau i_{\Sigma d}$, (7)

$p_\tau\omega = 1/ H_j\, (\Psi_d\, i_q - \Psi_q\, i_d - m_c)$, (8)

The quantities in equations (1-8) are

Ψ_d, Ψ_q = stator flux linkages related to d- and q –axes,

Ψ_{fd}= excitation flux linkage,

Ψ_{jd}, Ψ_{iq}= damper contours flux linkages,

i_d, i_{dq}, i_{fd}, i_{jd}, i_{iq}= corresponding currents,

r, r_{fd}, r_{jd}, r_{iq} = non-reactive resistances,

x_σ, $x_{\sigma f}$, $x_{\sigma jd}$, $x_{\sigma iq}$, = leakage reactances related to stator, excitation and damper contours, they can be regarded as constant in many transients. Still these reactances vary when the machine is short-circuited.

l_d, l_q = differential inductances, they vary on each integration step and can be calculated in accordance with [1],

ω = angular speed of a machine,

H_j = inertial constant,

m_c is a torque due to a mechanic load.

As to AC salient pole synchronous generators that don’t have solid (or non-laminated) poles, most often it is possible to take into consideration one damper contour related to direct and perpendicular axes (j=1, i=1 in equations 4 and 5). But the rotation speed of induction motors can significantly vary during transients, so the number of these contours can be increased to 2 or 3 [2] which is sufficient for most applications. My program calculates the parameters of these two contours, but it isn’t still suitable for all possible machine slot shapes. So the algorithm is given here to illustrate one of the possible ways to solve this problem.

Node voltage equations

It is preferable to divide the equations of all the elements of a power system, calculating the corresponding characteristics in separate blocks. The node equations are intended to do the following:

- determine the system structure and its possible changes;
- mutually co-ordinate the axes of different power system elements.

In conformity with [3], calculation of the node voltages can be obtained by the matrix equation derived from Kirchhoff’s first low written in differential form:

$$[u] = [B_{S\Sigma}]^{-1}[M][p_\tau i_{u=0}] = [B_{S\Sigma}]^{-1}[J_{node}], \qquad (9)$$

where $[B_{S\Sigma}]$= node conductance matrix, obtained from the reactances of the machines,

$[p_\tau i_{u=0}]= p[i_{d1d}, i_{q1d}, \ldots\, i_{dnd}, i_{qnd}]$= derivatives vector obtained from the currents i_d and i_q of all machines, transmission lines and loads written in common axes,

[pτiu=0] = vector which is calculated assuming that the nodal voltages are equal to zero. In [4] the term **[J_{node}]** is called the nodal vector of truncated derivatives of stator currents; **[M]** is a connection matrix of power system branches. The strings and columns of the matrix **[M]** correspond to the nodes of the power system and its branches respectively. The module of each matrix element is equal to 1, if the appropriate node is connected to the branche, otherwise it equals 0. Non-zero matrix elements are defined as positive if the current enters the node and negative if the current flows out off the node. Initially the currents related to any synchronous **Generator k** are calculated in reference to its own axes of ordinates. Then these currents have to be changed to common (or synchronous) axes of ordinates by the conversion [3]:

$$[p\tau i_{SGk}] = [T_k]\,[p\tau i_{SGk}] + [F_k]\,[i_{SGk}] \qquad (10)$$

where **[$p\tau i_{SGk}$]** is a current derivatives vector of a synchronous generator converted to common axes,
[i_{SGB}] is a current vector of a generator written in its own axes of ordinates,
[T_k] is a matrix needed to convert the axes of ordinates,

[F_k] =pτ [T_k] is its derivative.
The matrixes **[T_k]** and **[F_k]** can be written in conformity with [3]. I haven't used them in my program as it is written in phase axes of ordinates.

The power system structure determines the system node equations via the matrixes **[M]** and **[$B_{S\Sigma}$]**.

I designed the node equation set capable of simulating saturation phenomenon in electric machines. These equations contain matrices with differential inductances l_d, l_q (see eq.6, 7). I offered another form of matrices **[$B_{S\Sigma}$]** and truncated derivatives vectors of AC machines stator currents. It was published in [5]. I'll present this equation set in the next version of the current pdf-file. If you are interested in a more detailed description, send me a message.

As to the power system described on my web page node conductance matrix blocks **[$B_{S\Sigma1}$]... [$B_{S\Sigma3}$]** are calculated by adding up matrices of a synchronous generator, induction motor and transmission lines:

$$[B_{S\Sigma1}] = [B_{SG}] + [B_{L1}],$$
$$[B_{S\Sigma2}] = [B_{IM1}] + [B_{L1}] + [B_{L2}], \qquad (11)$$
$$[B_{S\Sigma3}] = [B_{IM2}] + [B_{L2}],$$

where

$$[B_{L1}] = \mathrm{diag}[1/x_{AL1}\ ,\ 1/x_{BL1}\ ,\ 1/x_{CL1}]\ ,$$

$$[B_{L2}] = \mathrm{diag}[1/x_{AL2},\ 1/x_{BL2},\ 1/x_{CL2}],$$

x_{AL1}, x_{BL1}, x_{CL1}, x_{AL2}, x_{BL2}, x_{CL2} = transmission lines reactances.

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