The Official Guide for GMAT Review, 11th edition
3.1 Diagnostic Test Quantitative Sample Questions: Data Sufficiency
Supplementary Explanations
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If you are viewing this document on a computer screen, you can jump to any problem by clicking the problem number in the table of contents. Also, you can call up the table of contents at any time by clicking the “Bookmarks” tab on the left side of the Adobe Acrobat Reader display—again, you can click on any item in this table of contents to jump to that point in the document. Click the “Bookmarks” tab a second time to remove the Table of Contents.

Please read this first

I make no guarantees as to your results from using the methods in these explanations.

Most of the methods described in these explanations will work well for most test-takers in most situations, but few methods are perfectly applicable to all situations or all test-takers. Use your judgment.

On the other hand, if you find a method in these explanations challenging to apply, don't give up on it too quickly—give it a fair shot. It is natural for *any* new technique to initially feel unfamiliar and time-consuming. If you keep using the same techniques on the GMAT that you're *already* comfortable with, you can expect to keep getting the same score! If you would like to *change* your GMAT score, the only way to do so is to *change* your GMAT techniques—even though at first any new technique is likely to feel uncomfortable and to slow you down.

This document provides supplementary explanations to the Data Sufficiency Diagnostic questions in *The Official Guide for GMAT Review*, 11th ed, pages 24-25. The *Official Guide* can be purchased from amazon.com at this link:
http://www.amazon.com/gp/product/0976570904/sr=8-1/qid=1154718341/ref=pd_bbs_1/002-6026248-8506436?ie=UTF8
The *Official Guide* provides explanations to all the problems in the *Guide*. However, many of those explanations are insufficient (no pun intended) to give you insight into how to attack GMAT math and data sufficiency **systematically**. The explanations in this document should be used as a supplement to rather than a substitute for the explanations in the *Official Guide*. For problems or statements for which the explanation provided by the *Official Guide* is adequate, no further explanation is provided here.

These explanations illustrate the techniques described in the "GMAT Data Sufficiency Manual", and therefore these explanations should be used in conjunction with that manual. The manual is available at http://www.freelance-teacher.com. Terms in this font are described in more detail in the manual—use the index.

Abbreviations used in these explanations:

? = Question	(S1) = statement 1 alone	CN = choosing numbers
G = given	(S2) = statement 2 alone	✓ = consistent with the givens
S = sufficient	(tog) = both statements together	✂ = inconsistent with the givens
I = insufficient		FA = free answer

25.

?: units digit of n G: n is an integer G: (units digit of n) > 2	Identify the Question—write it down, and label it with a question mark. Identify the initial givens (the word "if" indicates givens)—write them down, and label them with G's.

(S1) G: (units digit of n) = (units digit of n^2)	The relevant givens are the initial givens and statement one. Write down the statement and label it with "(S1) G"
CN $n=0$	Choose numbers. 1. Choose a number. Write down your number. Label your number with a variable.
CN ~~n=0~~	2. Is your number consistent or inconsistent with the relevant givens? Our number is inconsistent with the initial given. Cross it out. Throw the number out and choose a new number—back to step one.
CN $n=1$	1. Choose a number.
CN ~~n=1~~	2. Consistent or inconsistent with the relevant givens? Inconsistent with the initial givens. Back to step one.
CN ~~n=2~~	1. Choose a number. 2. Consistent or inconsistent with the relevant givens? Inconsistent with the initial givens. Back to step one.
CN $n=3$ $n^2=3^2=9$	1. Choose a number. 2. Consistent or inconsistent with the relevant givens? Label your work. Don't write "$3^2=9$". Instead, write "$n^2=3^2=9$".
CN ~~n=3~~ $n^2=3^2=9$	Inconsistent with statement one. Cross the number out. Back to step one.
CN ~~n=4~~ $n^2=4^2=16$	1. Choose a number. 2. Consistent or inconsistent with the relevant givens? Inconsistent with statement one. Back to step one.
CN $n=5$ $n^2=4^2=25$	1. Choose a number. 2. Consistent or inconsistent with the relevant givens?
CN $n=5$ ✓ $n^2=4^2=25$	Consistent. Make a checkmark. Go on to step three.
CN $n=5$ ✓ ?: 5 $n^2=4^2=25$	3. Use your number to answer the Question. Write down the answer. Label it with a "?", and circle it.
CN $n=6$ ✓ ?: 6 $n^2=6^2=36$	4. Choose a new number; try to get a different answer than "5". 2. Consistent or inconsistent with the relevant givens? Consistent. Make a checkmark. Go on to step three. 3. Use your number to answer the Question.
I (S2)	We have gotten two different answers to the Question ("5" and "6") that are consistent with all the relevant givens, so statement one is insufficient. Write down an "I".

?: units digit of n
G: n is an integer
G: (units digit of n) > 2

(S2) G: (units digit of n) = (units digit of n^3)	Statement one was insufficient, so there is no **free answer**. The **relevant givens** are the initial givens and statement two; ignore statement one. **Write down** statement two and **label** it with "**(S2) G**".
CN n=5	1. Choose a number. **Reuse** your numbers from statement one.
CN $n=\underline{5}$ ✓ $n^3 = 5^3 = 5\times5\times5 = \underline{5}\times2\underline{5} = \underline{\quad 5}$	2. <u>Consistent</u> or <u>inconsistent</u> with the relevant givens? **Label** your work (with an "n^3"). We only need to calculate the units digit of n^3. <u>Consistent</u>. Make a checkmark. Go on to step three.
CN $n=\underline{5}$ ✓ ?: 5 $n^3 = 5^3 = 5\times5\times5 = \underline{5}\times2\underline{5} = \underline{\quad 5}$	3. Use your number to answer the Question. From our work on statement one, we already know that when we choose n=5, the answer to the Question is "5".
CN $n=\underline{6}$ ✓ ?: 6 $n^3 = 6^3 = 6\times6\times6 = \underline{6}\times3\underline{6} = \underline{\quad 6}$	4. Choose a new number; try to get a different answer than "5". Continue **reusing** numbers from statement one. 2. <u>Consistent</u> or <u>inconsistent</u> with the relevant givens? We only need to calculate the units digit of n^3. <u>Consistent</u>. Make a checkmark. Go on to step three. 3. Answer the Question. From our work on statement one, we already know that when we choose n=6, the answer to the Question is "6".
I (S2)	We have gotten two different answers to the Question ("5" and "6") that are consistent with all the relevant givens, so statement two is **insufficient**. Write down an *I*.

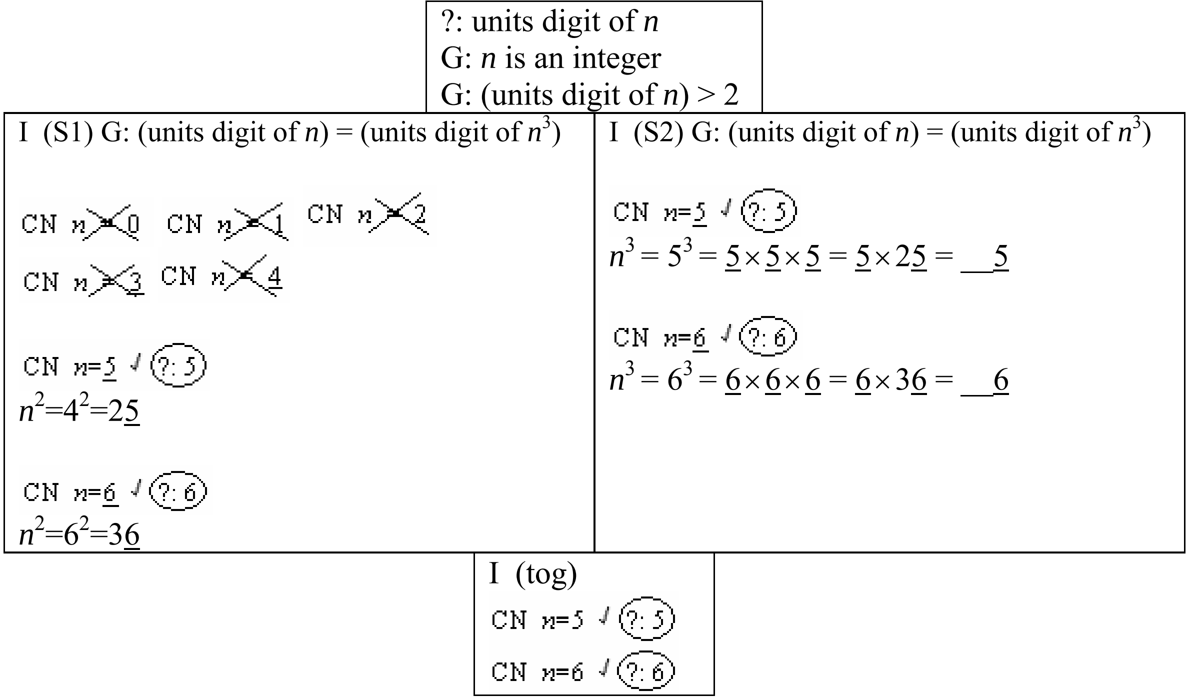
?: units digit of n
G: n is an integer
G: (units digit of n) > 2

(tog)	Since both statements were insufficient alone, we must evaluate the statements together. Write down "(tog)". The relevant givens are now the initial givens, statement one, and statement two.
CN $n=5$	Let's continue using the choosing numbers strategy. 1. Choose a number. Reuse your numbers from statements one and two.
CN $n=5$ √	2. Consistent or inconsistent with the relevant givens? From our work on statements one and two, we already know that $n=5$ is consistent with all the relevant givens. Consistent. Make a checkmark. Go on to step three.
CN $n=5$ √ ?:5	3. Use your number to answer the Question. From our work on statements one and two, we already know that when we choose $n=5$, the answer to the Question is "5".
CN $n=6$ √ ?:6	4. Choose a new number; try to get a different answer than "5". Continue reusing numbers from statements one and two. 2. Consistent or inconsistent with the relevant givens? From our work on statements one and two, we already know that $n=5$ is consistent with all the relevant givens. Consistent. Make a checkmark. Go on to step three. 3. Answer the Question. From our work on statements one and two, we already know that when we choose $n=6$, the answer to the Question is "6".
I (tog) CN $n=5$ √ ?:5 CN $n=6$ √ ?:6	We have gotten two different answers to the Question ("5" and "6") that are consistent with all the relevant givens, so the statements together are insufficient. Write down an *I*.

12TEN: Nothing is sufficient—choice E is correct.

Students often get confused when they have to evaluate the statements together. Hopefully, this example will demonstrate to you that choosing numbers can simplify the process of evaluating statements together. Choosing numbers for the statements together is the exact same process as choosing numbers for the individual statements—the only difference is that, when you are evaluating the statements together, there are more relevant givens that your numbers have to be consistent with.

Here is the complete recommended scratchwork for #25:



?: units digit of n
G: n is an integer
G: (units digit of n) > 2

I (S1) G: (units digit of n) = (units digit of n^3)

$n^2=4^2=2\underline{5}$

$n^2=6^2=3\underline{6}$

I (S2) G: (units digit of n) = (units digit of n^3)

$n^3 = 5^3 = \underline{5}\times\underline{5}\times\underline{5} = \underline{5}\times 2\underline{5} = \_\_\underline{5}$

$n^3 = 6^3 = \underline{6}\times\underline{6}\times\underline{6} = \underline{6}\times 3\underline{6} = \_\_\underline{6}$

I (tog)

12TEN: Nothing is sufficient—choice E is correct.

If you had trouble finding *any* numbers that were consistent with the givens, remember that there is always at least one answer that is consistent with all the relevant givens. Therefore, there is always at least one set of numbers that is consistent with all the relevant givens.

Some people might not pick to think a number, like "6", with a units digit of 6. Those people will get only one answer to the Question ("5"), so it will seem like each statement is sufficient. The key to making sure you don't miss the two different answers for D25 is to use systematic trial-and-error: noticed how we started with n=0 and *systematically* worked our way up through n=1, 2, 3, 4, 5, and 6. Since D25 focused on units digits, it made sense to systematically test numbers that ended with each of the possible units digits. If we had tested numbers with all ten units digits—0, 1, 2, 3, 4, 5, 6, 7, 8, and 9—and had still gotten only *one* answer, then we could have concluded that there was only *one* answer to the Question and that the statement was sufficient.

Notice how reusing our numbers saved time and simplified our work.

Use good notation. For each number that you choose your notation should either look like √ (?) or like >–<. If your notation ever differs from one of these two patterns, you know that you have made a mistake.■

26.

?: p G: p is an integer	D26 has a hidden given.

(S1) G: 2, 3, and 5 are all factors of p	The relevant givens are the initial given and statement one.
CN $p = 2\times3\times5$	Choose numbers. 1. Choose a number We are choosing p=30, but, for a "factors" problem, it is helpful to express the number we choose in terms of its prime factorization, rather than actually multiplying the factors out.
CN $p = 2\times3\times5$ ✓	2. Is our number <u>consistent</u> or <u>inconsistent</u> with the relevant givens? Clearly, $2\times3\times5$ is an integer. And we can quickly see that 2, 3, and 5 are all factors of $2\times3\times5$—that's the advantage of expressing p in terms of its prime factorization, rather than multiplying it out to get 30. <u>Consistent</u>. Make a check mark. On to step three.
CN: $p = 2\times3\times5$ ✓ ?: $2\times3\times5$	3. Use your number to answer the Question. Avoid needless calculations—there's no need to ever actually calculate $2\times3\times5$.
CN $p = 2\times3\times5\times7$	4. Choose a new number, trying to get a different answer than "$2\times3\times5$". Now we are choosing p=210, but, again, it's best to just express our number in terms of its prime factorization—there's no need for us to ever actually multiply the factors out. (Other numbers we could choose that would also be consistent with statement one are p=$2\times3\times5\times9$ or p=$2\times2\times3\times5$. On the other hand, p=2×3 would be **in**consistent with statement one, since 5 is **not** a factor of 2×3.)
CN $p = 2\times3\times5\times7$ ✓	2. <u>Consistent</u> or <u>inconsistent</u> with the relevant givens? Clearly, $2\times3\times5\times7$ is an integer. And we can quickly see that 2, 3, and 5 are all factors of $2\times3\times5\times7$—that's the advantage of expressing p in terms of its prime factorization, rather than multiplying it out to get 210. <u>Consistent</u>. Make a check mark. On to step three.
CN $p = 2\times3\times5\times7$ ✓ ?: $2\times3\times5\times7$	3. Use your number to answer the Question. Avoid needless calculations—there's no need to ever actually calculate $2\times3\times5\times7$.
I (S1) G: 2, 3, and 5 are all factors of p CN: $p = 2\times3\times5$ ✓ ?: $2\times3\times5$ CN $p = 2\times3\times5\times7$ ✓ ?: $2\times3\times5\times7$	We have gotten two different answers to the Question ("$2\times3\times5$" and "$2\times3\times5\times7$") that are consistent with all the relevant givens. So statement one is insufficient.

?: p
G: p is an integer

(S2) G: 2, 5, and 7 are all factors of p	Statement one was insufficient, so there is no free answer. The relevant givens are the initial given and statement two.
CN $p = 2\times3\times5$	Choose numbers. 1. Choose a number Try to reuse your numbers from statement one.
CN ~~$p = 2\times3\times5$~~	2. Is our number consistent or inconsistent with the relevant givens? We can quickly see that 7 is **not** a factor of $2\times3\times5$—that's the advantage of expressing p in terms of its prime factorization, rather than multiplying it out to get 30. Inconsistent. Cross out the numbers. Back to step one.
CN $p = 2\times3\times5\times7$ ✓	1. Choose a number. Continue reusing your numbers from statement one.
	2. Is our number consistent or inconsistent with the relevant givens? We can quickly see that 7 **is** a factor of $2\times3\times5\times7$—that's the advantage of expressing p in terms of its prime factorization, rather than multiplying it out to get 210. Consistent. Make a checkmark. Go on to step three.
CN $p = 2\times3\times5\times7$ ✓ ?: $2\times3\times5\times7$	3. Use your number to answer the Question.
CN $p=2\times3\times5\times7\times9$	4. Choose a new number, trying to get a different answer than "$2\times3\times5\times7$". (Other numbers we could choose that would also be consistent with statement two are $p=2\times5\times7$ or $p=2\times2\times3\times5\times7$. On the other hand, $p=2\times5$ is **in**consistent with statement two, since 7 is **not** a factor of 2×5.)
CN $p=2\times3\times5\times7\times9$ ✓	2. Is our number consistent or inconsistent with the relevant givens? We can quickly see that 7 **is** a factor of $2\times3\times5\times7\times9$—that's the advantage of expressing p in terms of its prime factorization, rather than multiplying it out. Consistent. Make a checkmark. Go on to step three.
CN $p=2\times3\times5\times7\times9$ ✓ ?: $2\times3\times5\times7\times9$	3. Use your number to answer the Question. Avoid needless calculations—there's no need to ever actually calculate $2\times3\times5\times7\times9$.
I (S2) G: 2, 5, and 7 are all factors of p CN ~~$p = 2\times3\times5$~~ CN $p = 2\times3\times5\times7$ ✓ ?: $2\times3\times5\times7$ CN $p=2\times3\times5\times7\times9$ ✓ ?: $2\times3\times5\times7\times9$	We have gotten two different answers to the Question ("$2\times3\times5\times7$" and "$2\times3\times5\times7\times9$") which are consistent with all the relevant givens. So statement one is insufficient.

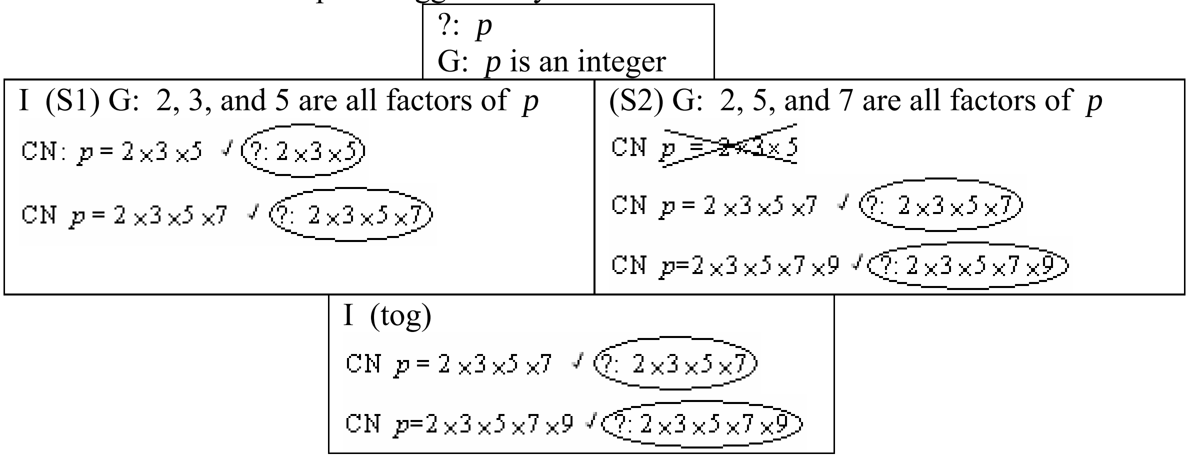
?: *p*
G: *p* is an integer

(tog)	Both statements were insufficient alone, so we have to evaluate the statements together. The relevant givens are now the initial given, statement one, and statement two.
CN $p = 2\times 3\times 5\times 7$	Choose numbers. 1.Choose a number. Reuse your numbers from statement two.
CN $p = 2\times 3\times 5\times 7$ ✓	2. <u>Consistent</u> or <u>inconsistent</u> with the relevant givens? From our work on statements one and two, we already know that this number is <u>consistent</u> with all the relevant givens. Make a checkmark. On to step three.
CN $p = 2\times 3\times 5\times 7$ ✓ $\boxed{?: 2\times 3\times 5\times 7}$	3. Answer the Question. Write down the answer. Label it with a question mark, and circle it.
CN $p=2\times 3\times 5\times 7\times 9$	4. Choose another number, trying to get a different answer than "$2\times 3\times 5\times 7$". Continue reusing your numbers from statement two.
CN $p=2\times 3\times 5\times 7\times 9$ ✓ $\boxed{?: 2\times 3\times 5\times 7\times 9}$	2. <u>Consistent</u> or <u>inconsistent</u> with the relevant givens? From our work on statement two, we already know that this number is <u>consistent</u> with the initial given and statement two. We can quickly see that 3 is a factor of $2\times 3\times 5\times 7\times 9$, so our number is <u>consistent</u> with statement one as well. 3. Answer the Question.
I (tog) CN $p = 2\times 3\times 5\times 7$ ✓ $\boxed{?: 2\times 3\times 5\times 7}$ CN $p=2\times 3\times 5\times 7\times 9$ ✓ $\boxed{?: 2\times 3\times 5\times 7\times 9}$	We have gotten two different answers to the Question ("$2\times 3\times 5\times 7$" and "$2\times 3\times 5\times 7\times 9$") that are consistent with all the relevant givens. So statement one is insufficient.

12TE<u>N</u>: <u>N</u>othing is sufficient—choice E is correct.

Students often get confused when they have to evaluate the statements together. Hopefully, this example will demonstrate to you that choosing numbers can simplify the process of evaluating statements together. And hopefully, this example also demonstrates that choosing numbers for the statements together is the exact same process as choosing numbers for the individual statements—the only difference is that, when you are evaluating the statements together, there are more relevant givens that your numbers have to be consistent with.

Here is the complete suggested systematic scratchwork for D26:



?: *p*
G: *p* is an integer

I (S1) G: 2, 3, and 5 are all factors of *p*
CN: $p = 2\times3\times5$ ✓ (?: $2\times3\times5$)
CN $p = 2\times3\times5\times7$ ✓ (?: $2\times3\times5\times7$)

(S2) G: 2, 5, and 7 are all factors of *p*
CN ~~$p = 2\times3\times5$~~
CN $p = 2\times3\times5\times7$ ✓ (?: $2\times3\times5\times7$)
CN $p=2\times3\times5\times7\times9$ ✓ (?: $2\times3\times5\times7\times9$)

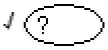
I (tog)
CN $p = 2\times3\times5\times7$ ✓ (?: $2\times3\times5\times7$)
CN $p=2\times3\times5\times7\times9$ ✓ (?: $2\times3\times5\times7\times9$)

12TEN: Nothing is sufficient—choice E is correct.

MISTAKE: A student might choose $p = 2\times3\times5$ for statement one and then say to themself, "This number answers the Question, so statement one must be sufficient". This is the mistake of choosing only one set of numbers. Remember that your number will *always* answer the Question. The issue is not *whether* you can answer the Question, but whether you can find *more than one* answer to the Question. Don't skip step four.

Notice how reusing our numbers saved us time and simplified our work.

Notice how expressing our numbers in terms of their prime factorizations, rather than multiplying them out, made it easier to find numbers that would be consistent with the givens. MORAL: When a problem deals with factors, it is often best to work with prime factorizations.

Use good notation. For each number that you choose your notation should either look like  or like . If your notation ever differs from one of these two patterns, you know that you have made a mistake.

The Question is, "What is the value of *p*?" Therefore, when we choose $p = 2\times3\times5$, the answer to the Question is "$2\times3\times5$". When we choose $p = 2\times3\times5\times7$, the answer to the Question is "$2\times3\times5\times7$". Executing step three is so **simple** in this situation that it makes some students uncomfortable—but it shouldn't! Don't feel uncomfortable about choosing a number for a variable when the Question is asking for the value of that same variable.

"*x* is a factor of *y*" means "$\frac{y}{x}$ is an integer". For example,

$$\frac{2\times3\times5\times7\times9}{3} = \frac{2\times\cancel{3}\times5\times7\times9}{\cancel{3}} = 2\times5\times7\times9$$

is an integer, so 3 is a factor of $2\times3\times5\times7\times9$. On the other hand, $\frac{2\times3\times5}{7}$ is **not** an integer (the 7 doesn't cancel from the denominator), so 7 is **not** a factor of $2\times3\times5$. Notice how writing numbers in terms of their prime factorizations makes it simpler to find their factors.■

27.

?: total number of minutes charged for = M G: M is a positive integer	Write down the Question; label it with a "?". Write down the initial given (the word "if" indicates givens); label it with a "G".

Statement one is shorter and simpler so it's probably easier—start with statement one. The relevant givens are the initial given and statement one.

I (S1) G: total charge = C = \$6.50

Based on statement one, the charge per minute could be almost anything, so we cannot tell how many minutes Wanda had to use to rack up a \$6.50 fee. So the number of minutes Wanda was charged for could be almost anything. So there are many different possible answers to the Question that are consistent with all the relevant givens, so statement one is insufficient.

Next, we evaluate statement two. Statement one was insufficient, so there is no free answer. The relevant givens are now the initial given and statement two—ignore statement one.

I (S2) G: charge for first minute = c + \$.50
G: charge per minute after first minute = c
G: c is positive

We have no idea what the charge per minute or the total charge are, so we have no idea how many minutes Wanda was charged for. So there are many different answers to the Question that are consistent with all the relevant givens. So statement two is insufficient.

Since both statements are insufficient alone, we have to evaluate the statements together. The relevant givens are the initial given, statement one, and statement two.

Since we still don't know what the charge per minute (c) is, we still don't know how many minutes (M) Wanda is able to talk before racking up a total charge of \$6.50. It is possible for Wanda to talk a long time (large M) as long as the charge per minute c is small. On the other hand, it is possible for Wanda to talk only a few minutes (small M) as long as the charge per minute c is big (so that her charge still comes to \$6.50). So there are many different answers (i.e., different M's) that would be consistent with all the relevant givens. So the statements together are insufficient.

I (tog)

12TEN: Nothing is sufficient—choice E is correct.

?: total number of minutes charged for = M
G: M is a positive integer

I (S1) G: total charge = $C = \$6.50$	I (S2) G: charge for first minute = $c + \$.50$ G: charge per minute after first minute = c G: c is positive

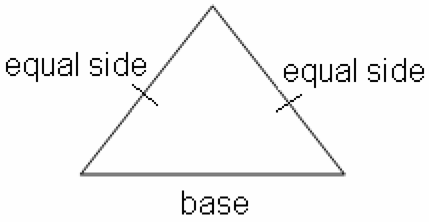
If you were nervous about the intuitive argument presented on the previous page, you could confirm that the statements together are insufficient by choosing specific numbers:

(tog) CN $M=2$	1. Choose numbers We will *choose* a number for M. The variable c has to be positive, so we will use our number for M to *determine* whether c is positive. (Alternatively, we could choose a number for c, and then use our number for c to determine whether M is a positive integer. That alternative approach, however, would be slightly more complicated.)
CN $M=2$ ✓ ?: 2 charge for first minute = $c + \$.50$ charge for second minute = c total charge = [a] $(c + \$.50) + c = \$6.50$	2. Consistent or inconsistent with the relevant givens? We can see that, **if** we solved [a] for c, we would find that c is positive. So our M and c will be consistent with all the relevant givens (there is no need to **actually** solve [a] or to determine a precise value for c). 3. Answer the Question.
CN $M=3$	4. Choose another number, trying to get a different answer than "2".
I (tog) CN $M=2$ ✓ ?: 2 charge for first minute = $c + \$.50$ charge for second minute = c total charge = [a] $(c + \$.50) + c = \$6.50$ CN $M=3$ ✓ ?: 3 charge for first minute = $c + \$.50$ charge for last two minutes = $2c$ total charge = [b] $(c + \$.50) + 2c = \$6.50$	2. Consistent or inconsistent with the relevant givens? We can see that, **if** we solved [b] for c, we would find that c is positive. So our M and c will be consistent with all the relevant givens (there is no need to **actually** solve [b] or to determine a precise value for c). 3. Answer the Question. We have obtained two different answers to the Question ("2" and "3") that are consistent with all the relevant givens, so the statements together are insufficient.

12TEN: Nothing is sufficient—choice E is correct.

Usually you need to obtain specific numbers for **all** the variables to be sure that your numbers are consistent with the givens. For D27, however, we were able to determine that c would be consistent with the givens without spending the time to actually determine a precise value for c. Avoid needless calculations.■

28.

?: perimeter of triangle MNP G: triangle MNP is isosceles 	The initial information has a hidden given. "Isosceles" means "at least two equal sides". "Perimeter" means "the sum of all three sides—i.e., the total distance around the triangle". When a geometry problem fails to provide you with a diagram, draw your own. Indicate the two equal sides with "hash marks". Since we don't know whether the base is segment MN, segment MP, or segment NP, we won't label the corners of the triangle as M, N, or P yet.

I (S1) G: MN = 16

The relevant givens are the initial given and statement one.

Statement one does not give enough information to determine all three sides of the triangle. So there are many different perimeters that would be consistent with statement one. So there are many different answers that are consistent with the relevant givens. So statement one is insufficient.

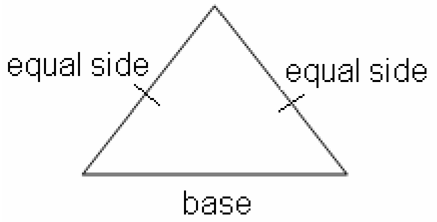
I (S2) G: NP = 20

Statement one was insufficient, so there is no free answer.

The relevant givens are the initial given and statement two—ignore statement one.

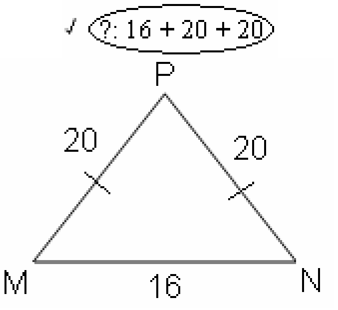
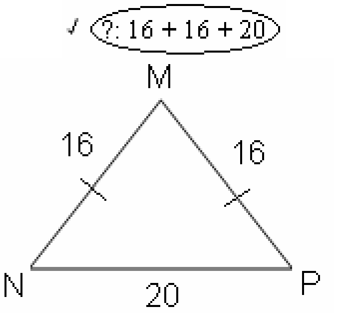
Statement two does not give enough information to determine all three sides of the triangle. So there are many different perimeters that would be consistent with statement two. So there are many different answers that are consistent with the relevant givens. So statement two is insufficient.

?: perimeter of triangle MNP
G: triangle MNP is isosceles



I (S1) G: MN = 16	I (S2) G: NP = 20

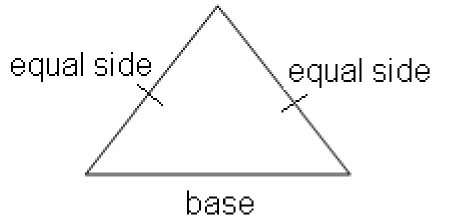
Since both statements were insufficient alone, we have to evaluate the statements together. The relevant givens are the initial given, statement one, and statement two.

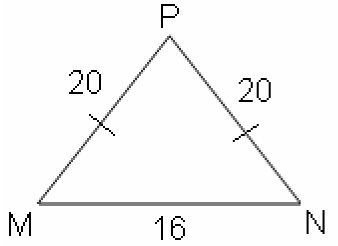
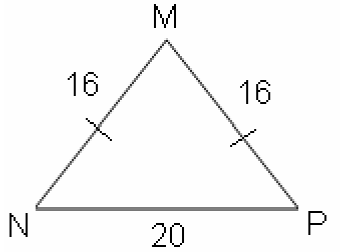
I (tog) CN base = MN = 16, equal sides=NP=MP= 20  CN base=NP=20, equal sides=MN=PN=16  	1. Choose numbers Build the numbers into a sketch. (We won't worry about drawing the sketch to scale.) 2. Consistent or inconsistent with the relevant givens? 16 < 20 + 20 and 20 < 16 + 20, so these numbers satisfy the "triangle inequality" (see below). Consistent: on to step three. 3. Answer the Question. Avoid needless calculations—we will not need to actually calculate the answer to the Question to determine whether we can get more than one answer. 4. Choose new numbers, trying to get a different answer to the Question than "16+20+20". 2. Consistent or inconsistent with the relevant givens? 16 < 16 + 20 and 20 < 16 + 16, so these numbers satisfy the triangle inequality. Consistent: on to step three. 3. Answer the Question. We don't need to calculate the answer to the Question to confirm that we have gotten a different answer. We have gotten two different answers to the Question ("16+20+20" and "16+16+20") that are consistent with all the relevant givens, so the statements together are insufficient.

12TEN: Nothing is sufficient—the correct choice is E.

The "triangle inequality" says that the sum of any two sides of a triangle must be greater than the third side. It is based on the fact that the shortest path between two points is a straight line. E.g., one path from point M to point N is segment MN—the straight-line path. An alternative path from M to N is to first go from point M to point P, and then go from point P to point N; this alternative path must be longer than the straight-line path, so segment MP plus segment PN must be greater than segment MN.

?: perimeter of triangle MNP
G: triangle MNP is isosceles



I (S1) G: MN = 16	I (S2) G: NP = 20
I (tog) CN base = MN = 16, equal sides=NP=MP= 20 ✓ ?: 16 + 20 + 20  	CN base=NP=20, equal sides=MN=PN=16 ✓ ?: 16 + 16 + 20 

12TEN: Nothing is sufficient—the correct choice is E.

Draw diagrams for geometry problems!

MISTAKE: A student might choose base=16 and equal sides=20. Then they might say to themself, “These numbers answer the Question. Sufficient.” This is the mistake of choosing only one set of numbers. Remember, your numbers will *always* answer the Question. The issue is not *whether* you can answer the Question, but whether you can find *more than one* answer to the Question.■

29.

There are two reasonable approaches to D29—the "weighted average" approach, and the "overlapping sets" approach. First, we will describe the "weighted average" approach.

?: % of *all* retailers w/ computers

The statements split the retailers into two groups—"storeowners" and "non-storeowners". Therefore, it is helpful to rephrase the Question as a weighted average of the two groups:

?: (% storeowners)(% of storeowners w/ comp) + (% non-storeowners)(% of nonstoreowners w/ comp)

Statement two is shorter and simpler, so it's probably easier—start with statement two. The relevant given is statement two.

I (S2) G: % storeowners = 40%
⇒ % non-storeowners = 60%
⇒ ?: (.4)(% of storeowners w/ comp) + (.6)(% of non-storeowners w/ comp)

We still have two unknowns in our weighted average. So there are many possible answers to the Question that are consistent with the relevant given. So statement two is insufficient.

Now evaluate statement one. Since statement two was insufficient, there is no free answer. The relevant given is statement one—ignore statement two.

I (S1) % of storeowners w/ comp = 85%
⇒ ?: (% storeowners)(.85) + (% non-storeowners)(% of non-storeowners w/ comp)

We still have three unknowns in our weighted average. So there are many possible answers to the Question that are consistent with the relevant given. So statement one is insufficient.

Since both statements are insufficient alone, we have to evaluate the statements together. The relevant givens are statement one and statement two.

I (tog) ?: (.4)(.85) + (.6)(% of non-storeowners w/ comp)

From statement one we know the percentage of all retailers who are storeowners (40%) and the percentage of all retailers who are non-storeowners (60%). From statement two we know the percentage of all storeowners who purchased computers (85%). That still leaves one unknown ("% of non-storeowners w/ comp") in our weighted average.

The percentage of non-storeowners who purchased computers could be anything. So there are many possible answers to the Question that are consistent with the relevant given. So the statements together are insufficient.

12TEN: Nothing is sufficient—the correct choice is E.

Notice that no calculations were necessary. Avoid needless calculations.

This "weighted average" approach is the best way to attack D29. For completeness, though, the next page describes an "overlapping sets" approach.

Here is an “overlapping sets” approach to D29.

	storeowner	**not storeowner**	**total**
computer			?
no computer			
total			1

Each cell is a part:whole ratio, where the “whole” is *all* the retailers in the survey. The ratios will be expressed as decimals. The “total-total” cell is 100%, which is “1” as a decimal. (We have built the Question into the table.)

(S2)

	storeowner	**not storeowner**	**total**
computer			?
no computer			
total	.4		1

Statement two is a part:whole ratio, where the “whole” is all the retailers in the survey—the same “whole” as the cells in our table.

We have built the given into the table.

I (S2)

	storeowner	**not storeowner**	**total**
computer			?
no computer			
total	.4	.6	1

Statement two is sufficient to determine exactly one number for the percentage of all retailers who are non-storeowners, but it is insufficient to determine exactly one number for the percentage of all retailers who purchased computers.

(S1)

	storeowner	**not storeowner**	**total**
computer	$.85S$		?
no computer			
total	S		1

Statement one is a part:whole ratio, where the “whole” is all the retailers in the survey *who owned their own store*. This is a different “whole” than the cells in our table refer to, so we have to be careful when entering statement one into the table.

(S1)

	storeowner	**not storeowner**	**total**
computer	$.85S$		?
no computer	$.15S$		
total	S		1

Statement one is insufficient to determine exactly one number for any of the percentages in the table.

I (tog)

	storeowner	**not storeowner**	**total**
computer	.85(.4)		?
no computer	.15(.4)		
total	.4	.6	1

The statements together are sufficient to determine exactly one number for the percentage of *storeowners* who purchased computers, but they are insufficient to determine exactly one number for the percentage of *all* retailers who purchased computers.

To bolster your confidence that the statements together are insufficient, you could choose numbers.

(tog)
CN

	storeowner	not storeowner	total
computer	.85(.4)	0	?
no computer	.15(.4)		
total	.4	.6	1

1. Choose a number.
We have chosen “0” for the percentage of all retailers who are non-storeowners-who-purchased-computers. (It’s good to choose 0.)

CN √

	storeowner	not storeowner	total
computer	.85(.4)	0	?=.85(.4)
no computer	.15(.4)	.6	.15(.4)+.6
total	.4	.6	1

Use your number to determine the other cells in the table.
2. <u>Consistent or inconsistent</u> with the relevant givens?
<u>Consistent</u>: on to step 3.
3. Answer the Question
It is not necessary to actually *calculate* the answer.

CN

	storeowner	not storeowner	total
computer	.85(.4)	.6	?
no computer	.15(.4)		
total	.4	.6	1

4. Choose another number, trying to get a different answer than “.85(.4)”.
We have chosen “.6” for the percentage of retailers who are non-storeowners-who-purchased-computers. (It’s good to choose boundary values.)

I (tog)
CN √

	storeowner	not storeowner	total
computer	.85(.4)	0	?=.85(.4)
no computer	.15(.4)	.6	.15(.4)+.6
total	.4	.6	1

CN √

	storeowner	not storeowner	total
computer	.85(.4)	.6	?=.85(.4)+.6
no computer	.15(.4)	0	.15(.4)
total	.4	.6	1

Use your number to determine the other cells in the table.
2. <u>Consistent or inconsistent</u> with the relevant givens?
<u>Consistent</u>: on to step 3.
3. Answer the Question
Avoid needless calculations—it is not necessary to actually calculate the answer.

We have obtained two different answers to the Question [“.85(.4)” and “.85(.4)+.6”] that are consistent with all the relevant givens. So the statements together are insufficient.

12TEN: Nothing is sufficient—the correct choice is E.■

30.

?: number of $10 certificates sold G: some $10 certificates, some $100 certificates G: total of 20 certificates sold	Write down the Question; label it with a "?". Write down the initial givens (the word "if" indicates givens); label them with "G's".

Statement two is shorter and simpler, so it's probably easier—start with statement two. The relevant givens are the initial givens and statement two.

(S2) G: more than fifteen $100 certificates were sold

CN number of $100 certificates sold = 16	Choose numbers. 1. Choose a number. It's good to choose a number close to the boundary.
CN number of $100 certificates sold = 16 ⇒ number of $10 certificates sold = 4 √ ?: 4	We *chose* a number for how many $100 certificates were sold. Now we use that number and the initial givens to *determine* a number for how many $10 certificates were sold. 2. Consistent or inconsistent with the relevant givens? Consistent. Make a "√" .On to step three. 3. Answer the Question.
CN number of $100 certificates sold = 17	4. Choose another number, trying to get a different answer than "4".
I (S2) G: more than fifteen $100 certificates were sold CN number of $100 certificates sold = 16 ⇒ number of $10 certificates sold = 4 √ ?: 4 CN number of $100 certificates sold = 17 ⇒ number of $10 certificates sold = 3 √ ?: 3	Use the number we *chose* for the $100 certificates, plus the initial given, to *determine* a number for the $10 certificates. 2. Consistent or inconsistent with the relevant givens? Consistent. Make a "√" .On to step three. 3. Answer the Question. We have gotten two different answers to the Question ("4" and "3") that are consistent with all the relevant givens. So statement two is insufficient.

?: number of $10 certificates sold
G: some $10 certificates, some $100 certificates
G: total of 20 certificates sold

Statement two was insufficient, so there is no free answer. The relevant givens are now statement one and the initial given.

(S1) G: total value of the certificates sold was between $1650 and $1800	
CN number of $100 certificates sold = 16 ⇒ number of $10 certificates sold = 4	1. Choose a number. Try to reuse your numbers from statement one.
CN ~~number of $100 certificates sold = 16~~ ~~⇒ number of $10 certificates sold = 4~~ ⇒ total value of the certificates sold = 16($100) + 4($10) = $1600+$40=$1640	2. <u>Consistent or inconsistent</u> with the relevant givens? Label your calculations, as shown at left. <u>Inconsistent</u> ($1640<$1650): cross these numbers out and go back to step one.
CN number of $100 certificates sold = 17 ⇒ number of $10 certificates sold = 3	1. Choose a number. Try to reuse your numbers from statement one.
CN number of $100 certificates sold = 17 ⇒ number of $10 certificates sold = 3 total value of the certificates sold = 17($100) + 3($10) = $1700+$30=$1730 √ ?: 3	2. <u>Consistent or inconsistent</u> with the relevant givens? Label your calculations, as shown at left. <u>Consistent</u> ($1650<$1730<$1800): make a "√"; go on to step three. 3. Answer the Question.
CN number of $100 certificates sold = 18	4. Choose another number, trying to get a different answer than "3".
CN number of $100 certificates sold = 18 ⇒ number of $10 certificates sold = 2	We *chose* a number for the $100 certificates; now we use that number, together with the initial given, to *determine* a number for the $10 certificates.
CN ~~number of $100 certificates sold = 18~~ ~~⇒ number of $10 certificates sold = 2~~ total value of the certificates sold = 18($100) + 2($10) > 1800	2. <u>Consistent or inconsistent</u> with the relevant givens? Avoid needless calculations. <u>Inconsistent</u>: cross the numbers out.

Choosing sixteen $100 certificates produced a total value that was inconsistent with the givens because it was too low. If we choose fewer than sixteen $100 certificates, the total value will be even lower and hence even more inconsistent.

Choosing eighteen $100 certificates produced a total value that was inconsistent with the givens because it was too high. If we choose more than eighteen $100 certificates, the total value will be even higher and hence even more inconsistent.

Therefore, the **only** number of $100 certificates that is consistent with all the relevant givens is seventeen. So the there is only one answer to the Question ("3") that is consistent with all the relevant givens. So statement one is sufficient.

S (S1)

12TEN: Only statement 1 is sufficient—choice A is correct.

Remember that we started by evaluating statement two.

?: number of $10 certificates sold
G: some $10 certificates, some $100 certificates
G: total of 20 certificates sold

S (S1) G: total value of the certificates sold was between $1650 and $1800

~~CN number of $100 certificates sold = 16~~
~~⇒ number of $10 certificates sold = 4~~
⇒ total value of the certificates sold = 16($100) + 4($10) = $1600+$40=$1640

CN number of $100 certificates sold = 17
⇒ number of $10 certificates sold = 3
total value of the certificates sold = 17($100) + 3($10) = $1700+$30=$1730
✓ ?: 3

~~CN number of $100 certificates sold = 18~~
~~⇒ number of $10 certificates sold = 2~~
total value of the certificates sold = 18($100) + 2($10) > 1800

I (S2) G: more than fifteen $100 certificates were sold

CN number of $100 certificates sold = 16
⇒ number of $10 certificates sold = 4
✓ ?: 4

CN number of $100 certificates sold = 17
⇒ number of $10 certificates sold = 3
✓ ?: 3

12TEN: Only statement 1 is sufficient—choice A is correct.

Although we did have to do some calculations to check whether our numbers were consistent or inconsistent with statement one, the calculations were very minimal.

Although statement two only mentioned the $100 certificates, we needed to obtain a number for the $10 certificates as well—you usually need numbers for all the variables. But notice that, instead of *choosing* a number for the $10 certificates, we instead used the number for chose for the $100 certificates, together with the initial given, to *determine* a number for the $10 certificates. This is the strategy of *choosing* numbers for some variables and *determining* numbers for the other variables.

Because statement one gives us a range of values, rather than an exact value, it is tempting to assume that it must be insufficient. Notice how choosing numbers greatly clarifies the analysis for statement one and gives us a systematic, straightforward, non-algebraic way to recognize that, perhaps surprisingly, statement one is sufficient.

MORAL: Choosing numbers can save you from traps!

Here is an algebraic approach to D30:

?: C G: [a] $C + E = 20$ G: E is an integer, [b] C is an integer G: $E>0$, [c] $C>0$	C = the number of “cheap” \$10 certificates E = the number of “expensive” \$100 certificates. Use easy-to-remember variables. “C” and “E” are better than “x” and “y”.
G: [d] total value of the certificates = $10C + 100E$	
⇒ [e] $E = 20 - C$	Solve [a] for E.

(S2) G: [2a] $E > 15$	
⇒ $20 - C > 15$	Substitute [e] into [2a]. Now we try to solve for C.
⇒ $-C > -5$	Isolate C by subtracting 20 from both sides.
⇒ [2b] $C < 5$	Multiply both sides by -1. Multiplying by a negative number “flips” the inequality from a “>” to a “<”.
⇒ [2c] $0 < C < 5$	Combine [c] and [2b]
⇒ $C = 1, 2, 3,$ or 4	From [b] and [2c]
I (S2) ?: 1 ?: 2 ?: 3 ?: 4	There is more than one answer to the Question that is consistent with all the relevant givens, so statement two is insufficient.

(S1) G: $1650 < 10C + 100E < 1800$	Combine [d] with statement one.
⇒ $1650 < 10\,(C + 10E) < 1800$	Factor a 10 out of the middle expression.
⇒ [1a] $165 < C + 10E < 180$	Divide the left, middle, and right expressions by 10.
⇒ $165 < C + 10(20-C) < 180$	Reduce the number of variables by substituting [e] into [1a]. Now, try to solve for C.
⇒ $165 < C + 200 - 10C < 180$	Use the distributive law on the middle expression.
⇒ $165 < -9C + 200 < 180$	Simplify the middle expression. $C - 10C = -9C$
⇒ $-35 < -9C < -20$	Start isolating C by subtracting 200 from the left, middle, and right expressions.
⇒ $35 > 9C > 20$	Multiply the left, middle, and right expressions by -1. Multiplying by a negative number “flips” the inequalities from “>” to “<”.
⇒ $20 < 9C < 35$	
⇒ [1b] $2\frac{2}{9} < C < 3\frac{6}{9}$	Isolate C by dividing the left, middle, and right expressions by 9.
⇒ $C = 3$	From [1b] and [b].
S (S1) ?: 3	There is exactly one answer to the Question (“3”) that is consistent with all the relevant givens. So statement one is sufficient.

12TEN: Only statement 1 is sufficient—choice A is correct.

You can see that, unless your algebra skills are quite strong, you were probably better off choosing numbers for D30. Remember that the good news about D30 is that, by choosing numbers, you can correctly and confidently evaluate even the tricky statement one—and you can do so reasonably quickly, with minimal calculations and no algebra.■

31.

Consider n data points: $x_1, x_2, x_3, \ldots, x_n$

$$\text{mean} = \overline{X} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

$$\text{variance} = \frac{(x_1 - \overline{X})^2 + (x_2 - \overline{X})^2 + (x_3 - \overline{X})^2 + \ldots + (x_n - \overline{X})^2}{n}$$

$$[a]\ \text{standard deviation} = \sqrt{\text{variance}}$$

$$\Rightarrow [b]\ \text{standard deviation} = \sqrt{\frac{(x_1 - \overline{X})^2 + (x_2 - \overline{X})^2 + (x_3 - \overline{X})^2 + \ldots + (x_n - \overline{X})^2}{n}}$$

The "mean" is the average of the data points. It may be helpful to think of the "variance" as the average squared deviation from the mean.

G: n=20
?: Is the standard deviation < 3?

Statement one is shorter, simpler, and easier—start with statement one. The relevant givens are the initial given and statement one.

(S1) G: [1a] variance = 4	
$\Rightarrow$ standard deviation $= \sqrt{4}$	Substitute [1a] into [a].
$\Rightarrow$ standard deviation = 2	
S (S1) ?: yes	There is exactly one answer to the Question ("yes") that is consistent with all the relevant givens. So statement one is sufficient.

(S2) FA ?: yes	Since statement one was sufficient, it gives us a free answer ("yes") that we automatically know must be consistent with all the relevant givens for statement two. ("FA" = "free answer") The relevant givens are now the initial given and statement two.
G : [2a] $x_1 - \overline{X} = 2, x_2 - \overline{X} = 2, x_3 - \overline{X} = 2, \ldots, x_{20} - \overline{X} = 2$	From statement two.
$\Rightarrow [2b]$ standard deviation $= \sqrt{\frac{(2)^2 + (2)^2 + (2)^2 + \ldots + (2)^2}{20}}$	Substitute [2a] into [b].

It is clear that, if we perform the calculations in [2b], we will obtain exactly one value for the standard deviation. Therefore, will obtain exactly one answer to the Question that is consistent with all the relevant givens. So statement two is sufficient.

S (S2)

12TEN: Each of the statements is sufficient alone—choice D is correct.

Consider n data points: $x_1, x_2, x_3, \ldots, x_n$

$$\text{mean} = \overline{X} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

$$\text{variance} = \frac{\left(x_1 - \overline{X}\right)^2 + \left(x_2 - \overline{X}\right)^2 + \left(x_3 - \overline{X}\right)^2 + \ldots + \left(x_n - \overline{X}\right)^2}{n}$$

$$[\text{a}]\ \text{standard deviation} = \sqrt{\text{variance}}$$

$$\Rightarrow [\text{b}]\ \text{standard deviation} = \sqrt{\frac{\left(x_1 - \overline{X}\right)^2 + \left(x_2 - \overline{X}\right)^2 + \left(x_3 - \overline{X}\right)^2 + \ldots + \left(x_n - \overline{X}\right)^2}{n}}$$

G: n=20
?: Is the standard deviation < 3?

S (S1) G: variance = 4 $\Rightarrow$ standard deviation $= \sqrt{4}$ $\Rightarrow$ standard deviation = 2 $\Rightarrow$?: yes	S (S2) FA ?: yes $\text{G}: x_1 - \overline{X} = 2, x_2 - \overline{X} = 2, x_3 - \overline{X} = 2, \ldots, x_{20} - \overline{X} = 2$ $\Rightarrow [\text{2b}]\ \text{standard deviation} = \sqrt{\frac{(2)^2 + (2)^2 + (2)^2 + \ldots + (2)^2}{20}}$

12TEN: Each of the statements is sufficient alone—choice D is correct.

(For the sake of clarity, the above demonstration of how to solve D31 includes a lot more than you would likely want to write down to solve the problem during the test.)

Avoid needless calculations. It is not necessary to actually perform the calculations in [2b], for two reasons: (1) Once we know that there is **exactly one** answer to the Question, we know that statement two is sufficient, regardless of **what** that one answer is. (2) When we have a free answer for a statement, and when that statement is sufficient, the one answer to the Question must be the same as our free answer. Therefore, we know that the calculations in [2b] must produce an answer of "yes".

For completeness, here is how to perform the calculations in [2b]:

$\Rightarrow [\text{2c}]\ \text{standard deviation} = \sqrt{\frac{4+4+4+\ldots+4}{20}}$	From [2b]
$\Rightarrow\ \text{standard deviation} = \sqrt{\frac{20(4)}{20}}$	There are twenty data points in the set, so there are twenty terms in the numerator of [2c].
$\Rightarrow\ \text{standard deviation} = \sqrt{4}$	Cancel the "20's" from the numerator and denominator.
$\Rightarrow$ standard deviation = 2	

As predicted, the result of our calculations is consistent with our free answer.

Again, these calculations are provided only for the sake of completeness—they were not necessary for D31. In fact, while it is useful to know the formula for standard deviation for the GMAT, the GMAT problems will be designed so that you will probably **never** have to actually **calculate** a standard deviation.■

32.

$\text{G: integers} = \begin{cases} 3, 4, 5, 6 \\ x, y \end{cases}$?: Is range > 9?	D32 has hidden givens. Write down the known integers in order from low to high.

Although it is slightly longer, statement two is simpler than statement one, so it might be easier. Let's start with statement two. The relevant givens are the initial given and statement two.

(S2) G: $3 < x < y$	It's more natural to write the inequality with the smaller numbers to the left.
CN x=4, y=5 √	Choose numbers. 1. Choose numbers Start by choosing numbers close to the boundary. 2. <u>Consistent or inconsistent</u> with the relevant givens? The initial given never said that x and y had to be *distinct* from the other integers, so our numbers are <u>consistent</u> with all the relevant givens: make a "√"; on to step three.
CN x=**4**, y=**5** √ ?: no integers = 3, 4, **4**, 5, **5**, and 6 range = 6-3 = 3	3. Use your numbers to answer the Question. The "range" is defined as the largest number in the group minus the smallest number in the group. D32 is a yes/no Question, not a number Question, so the answer is "no", not "3".
I (S2) G: $3 < x < y$ CN x=**4**, y=**5** √ ?: no integers = 3, 4, **4**, 5, **5**, and 6 range = 6-3 = 3 CN x=**4**, y = **1 million**, √ ?: yes integers = 3, 4, **4**, 5, 6, and **1 million** range = 1 million – 3 > 9	4. Choose new numbers, trying to get an answer of "yes"—i.e., trying to get a range greater than 9. To get big range, choose a big number for y. (After choosing a number close to a boundary, it's good to choose a number far from the boundary.) 2. <u>Consistent or inconsistent</u> with the relevant givens? <u>Consistent</u>: make a "√"; on to step three. 3. Use your numbers to answer the Question. We don't need to calculate a precise value for the range. We have found two different answers to the Question ("no" and "yes") that are consistent with all the relevant givens. So statement two is insufficient.

$$\text{G : integers} = \begin{cases} 3, 4, 5, 6 \\ x, \text{y} \end{cases}$$

?: Is range > 9?

Since statement two was insufficient, there is no free answer. The relevant givens are now statement one and the initial given.

(S1) G: $3x < y$	It's more natural to write the inequality with the smaller numbers to the left.
CN x=4, y=5 √	Choose numbers. 1. Choose numbers Try to reuse your numbers from statement two.
CN ~~x = 4, y = 5~~ 3x=3(4)=12	2. Consistent or inconsistent with the relevant givens? Label your calculations. Inconsistent with statement one: cross the numbers out and go back to step one.
CN x=4, y = 1 million √ ?: yes 3x=12	1. Choose numbers Continue trying to reuse your numbers from statement two. 2. Consistent or inconsistent with the relevant givens? Consistent: make a √; on to step three. 3. Answer the Question. From our work on statement two, we already know that when we choose these numbers, the answer to the Question is "yes".
I (S1) G: $3x < y$ CN ~~x = 4, y = 5~~ 3x=3(4)=12 CN x=4, y = 1 million √ ?: yes 3x=12 CN x=**0**, y=**1** √ ?: no 3x=3(0)=0 integers = **0**, **1**, 3, 4, 5, and 6 range = 6 - 0 = 6	4. Choose new numbers, trying to get an answer of "no"—i.e., trying to get a range less than or equal to 9. To get big range, choose a small number for *y*. It's good to choose 0. 2. Consistent or inconsistent with the relevant givens? Zero is an integer. Consistent: make a "√"; on to step three. 3. Use your numbers to answer the Question. We don't need to calculate a precise value for the range. We have found two different answers to the Question ("yes" and "no") that are consistent with all the relevant givens. So statement one is insufficient.

$$\text{G: integers} = \begin{cases} 3, 4, 5, 6 \\ x, y \end{cases}$$

?: Is range > 9?

(S1) G: $3x < y$	(S2) G: $3 < x < y$

Since both statements were insufficient alone, we must evaluate the statements together. The relevant givens are now the initial given, statement one, and statement two.

(tog) CN $x=4, y = 1$ million √ ?: yes	Choose numbers. 1. Choose numbers Reuse your numbers from statement one. 2. Consistent or inconsistent with the relevant givens? From our previous work, we already know that these numbers are consistent with all the relevant givens: make a √; on to step three. 3. Answer the Question. From our previous work, we already know that when we choose these numbers, the answer to the Question is "yes".
CN ~~$x = 0, y = 1$~~	4. Choose new numbers, trying to get an answer of "no"—i.e., trying to get a range less than or equal to 9. Continue reusing your numbers. 2. Consistent or inconsistent with the relevant givens? Inconsistent with statement two: cross the numbers out; back to step one.
CN $x=$**4**$, y=$**13** √ ?: yes $3x=3(4)=12$ integers = 3, 4, **4**, 5, 6, and **13** range = 13-3 = 10	1. Choose numbers, trying to get an answer of "no"—i.e., trying to get a range less than or equal to 9. To get a small range, we want to choose numbers that are close to 3. 2. Consistent or inconsistent with the relevant givens? Consistent: make a √; on to step three. 3. Answer the Question.
CN $x = 3.1, \quad y = 10$	4. Choose new numbers, trying to get an answer of "no"—i.e., trying to get a range less than or equal to 9.
CN ~~$x = 3.1, y = 10$~~	2. Consistent or inconsistent with the relevant givens? x=3.1 is inconsistent with the initial given, which says that x and y are integers: cross the numbers out.

If you keep choosing new numbers, you won't be able to find any numbers that are consistent with all the relevant givens that give an answer to the Question of "no". Therefore, eventually you should conclude that there is only one answer to the Question ("yes") that is consistent with all the relevant givens. So the statements together are sufficient.

S (tog)

12TEN: The statements are only sufficient together—choice C is correct.

Remember that we started by evaluating statement two.

$$\text{G : integers} = \begin{cases} 3, 4, 5, 6 \\ x, y \end{cases}$$

?: Is range > 9?

I (S1) G: $3x < y$

CN ~~$x = 4, y = 5$~~

$3x$=3(4)=12

CN x=4, y = 1 million ✓ (?: yes)

$3x$=12

CN x=**0**, y=**1** ✓ (?: no)

$3x$=3(0)=0

integers = **0**, **1**, 3, 4, 5, and 6

range = 6 - 0 = 6

I (S2) G: $3 < x < y$

CN x=**4**, y=**5** ✓ (?: no)

integers = 3, 4, **4**, 5, **5**, and 6

range = 6-3 = 3

CN x=**4**, y = **1 million**, ✓ (?: yes)

integers = 3, 4, **4**, 5, 6, and **1 million**

range = 1 million – 3 > 9

S (tog)

CN x=4, y = 1 million ✓ (?: yes)

CN ~~$x = 0, y = 1$~~

CN ~~$x = 3.4, y = 10$~~

CN x=**4**, y=**13** ✓ (?: yes)

$3x$=3(4)=12

integers = 3, 4, **4**, 5, 6, and **13**

range = 13-3 = 10

12TEN: The statements are only sufficient together—choice C is correct.

Here is a MISTAKE to avoid when choosing numbers for the statements together: During step two, a student might ask themself, "Is x=0, y=1 *consistent with the givens*?" Then they might say, "**No**, these numbers are not consistent with statement two." This might make them think that they have obtained an *answer to the Question* of "**no**". This will make it seem like they have obtained two answers to the Question ("yes" and "no"), which will make it seem like the statements together are insufficient.

This is the MISTAKE of treating the given like a Question. x=0, y=1 are *inconsistent with the givens*, but that does not mean that we have obtained an *answer to the Question* of "no". When our numbers are inconsistent with the givens, we do **not** go on to step three, so we do **not** get an answer to the Question.

TO AVOID THIS MISTAKE: (1) Do *not* ask yourself "Is x=0, y=1 consistent with the relevant givens?" This is bad wording because it prompts you to respond "yes" or "no". *Instead*, ask yourself "Is x=0, y=1 consistent or inconsistent with the relevant givens?" This is better wording because it prompts you to respond "consistent" or "inconsistent", rather than "yes" or "no". (2) **Never** write down the word *no* to indicate that a number is inconsistent with the givens; instead, just cross the number out. Notice that, if you made the mistake of treating the given like a Question, your notation would look like this:

x=0, y=1 no

Hopefully, you would see immediately that this notation is **wrong** because it contains neither a ✓ nor a ✗, and this would alert you that you must have made a mistake.■

33.

D33 can be solved algebraically or by choosing numbers. Here is the choosing numbers approach:

$$?:\ \text{Is } \frac{5^{x+2}}{25} < 1?$$

Statement two is simpler so it's probably easier—start with statement two. The relevant given is statement two.

(S2) G: $x<0$	
CN $x = -1$ ✓ ?: yes $\frac{5^{x+2}}{25}=\frac{5^{-1+2}}{25}=\frac{5^{1}}{25}=\frac{5}{25}=\frac{1}{5}$	Choose numbers. 1. Choose a number 2. Consistent or inconsistent with the relevant givens? Consistent: make a "✓"; on to step three. 3. Answer the Question. Label your calculations. D33 has a yes/no Question, not a number Question, so the answer is "yes", not "$\frac{1}{5}$".
CN $x = -2$ ✓ ?: yes $\frac{5^{x+2}}{25}=\frac{5^{-2+2}}{25}=\frac{5^{0}}{25}=\frac{1}{25}$	4. Choose another number, trying to get a different answer than "yes"—i.e., trying to get $\frac{5^{x+2}}{25}\geq 1$. 2. Consistent or inconsistent with the relevant givens? Consistent: make a "✓"; on to step three. 3. Answer the Question. $p^0=1$ for any number p.
CN $x = -3$ ✓ ?: yes $\frac{5^{x+2}}{25}=\frac{5^{-3+2}}{25}=\frac{5^{-1}}{25}=\frac{\left(\frac{1}{5^{1}}\right)}{25}=\frac{\left(\frac{1}{5}\right)}{25}=\ldots$ $\ldots=\frac{\left(\frac{1}{5}\right)}{\left(\frac{25}{1}\right)}=\frac{1}{5}\div\frac{25}{1}=\frac{1}{5}\times\frac{1}{25}<1$	4. Choose another number, trying to get $\frac{5^{x+2}}{25}\geq 1$. 2. Consistent or inconsistent with the relevant givens? Consistent: make a "✓"; on to step three. 3. Answer the Question. $p^{-q}=\frac{1}{p^{q}}$ for any numbers p and q.

evaluation of statement two continued on next page …

$$\boxed{?:\ \text{Is } \frac{5^{x+2}}{25}<1?}$$

… evaluation of statement two continued from previous page

(S2) G: $x<0$

CN $x=-1$ ✓ ?: yes

$$\frac{5^{x+2}}{25}=\frac{5^{-1+2}}{25}=\frac{5^{1}}{25}=\dots$$

$$\dots=\frac{5}{25}=\frac{1}{5}$$

CN $x=-2$ ✓ ?: yes

$$\frac{5^{x+2}}{25}=\frac{5^{-2+2}}{25}=\frac{5^{0}}{25}=\frac{1}{25}$$

CN $x=-3$ ✓ ?: yes

$$\frac{5^{x+2}}{25}=\frac{5^{-3+2}}{25}=\frac{5^{-1}}{25}=\frac{\left(\frac{1}{5^{1}}\right)}{25}=\frac{\left(\frac{1}{5}\right)}{25}=\dots$$

$$\dots=\frac{\left(\frac{1}{5}\right)}{\left(\frac{25}{1}\right)}=\frac{1}{5}\div\frac{25}{1}=\frac{1}{5}\times\frac{1}{25}<1$$

CN $x=-4$ ✓ ?: yes

$$\frac{5^{x+2}}{25}=\frac{5^{-4+2}}{25}=\frac{5^{-2}}{25}=\frac{\left(\frac{1}{5^{2}}\right)}{25}=\frac{\left(\frac{1}{25}\right)}{25}=\dots$$

$$\dots=\frac{\left(\frac{1}{25}\right)}{\left(\frac{25}{1}\right)}=\frac{1}{25}\div\frac{25}{1}=\frac{1}{25}\times\frac{1}{25}<1$$

CN $x=-20$ ✓ ?: yes

$$\frac{5^{x+2}}{25}=\frac{5^{-20+2}}{25}=\frac{5^{-18}}{25}=\frac{\left(\frac{1}{5^{18}}\right)}{25}=\dots$$

$$\dots=\frac{\left(\frac{1}{5^{18}}\right)}{\left(\frac{25}{1}\right)}=\frac{1}{5^{18}}\div\frac{25}{1}=\frac{1}{5^{18}}\times\frac{1}{25}<1$$

CN $x=-\frac{1}{2}$ ✓ ?: yes

$$\frac{5^{x+2}}{25}=\frac{5^{-\frac{1}{2}+2}}{25}=\frac{5^{\frac{3}{2}}}{25}=\frac{\sqrt[2]{5^{3}}}{25}=\frac{\sqrt{125}}{25}$$

$$<\frac{25}{25}=1$$

$p^{\frac{q}{r}}=\sqrt[r]{p^{q}}$ for any numbers *p*, *q*, and *r*.

$125<25^2$, so $\sqrt{125}<25$, so $\frac{\sqrt{125}}{25}<\frac{25}{25}=1$.

If you continue choosing numbers, you will not be able to find any numbers that are consistent with statement two that give an answer to the Question of "no". So, eventually, you should stop choosing new numbers and conclude that there is only answer to the Question ("yes") that is consistent with all the relevant givens. So statement two is sufficient.

$\boxed{\text{S (S2)}}$

$\boxed{?: \text{ Is } \frac{5^{x+2}}{25} < 1?}$

The relevant given is now statement one—ignore statement two.

(S1) FA ?: yes G: $5^x < 1$	Since statement two was sufficient, it gives us a free answer (“yes”).
CN x=0	1. Choose a number, trying to get an answer of “no”—i.e., trying to get $\frac{5^{x+2}}{25} \geq 1$. There is no point reusing our numbers from statement one, since they would simply give us another answer of “yes”. It’s good to choose 0.
CN $\xcancel{x=0}$ $5^x = 5^0 = 1$	2. Consistent or inconsistent with the relevant given? Label your work. p^0=1 for any number p. 1 is **not** less than 1, so x=0 is inconsistent with statement one: cross the number out; back to step one.
CN $\xcancel{x=1}$ $5^x = 5^1 = 5$	1. Choose a number, trying to get $\frac{5^{x+2}}{25} \geq 1$. 2. Consistent or inconsistent with the relevant given? Inconsistent with statement one: cross the number out; back to step one.
CN $\xcancel{x = \frac{1}{2}}$ $5^x = 5^{\frac{1}{2}} = \sqrt[2]{5^1} = \sqrt{5} > 1$	1. Choose a number, trying to get $\frac{5^{x+2}}{25} \geq 1$. 2. Consistent or inconsistent with the relevant given? $p^{\frac{q}{r}} = \sqrt[r]{p^q}$ for any numbers p, q, and r. Inconsistent with statement one: cross the number out; back to step one.

If you continue choosing numbers, you will not be able to find any numbers that are consistent with statement one that give an answer to the Question of “no”. So eventually you should stop choosing numbers and conclude that there is only one answer to the Question that is consistent with statement one (“yes”, our free answer). So statement one is sufficient.

$\boxed{\text{S (S1)}}$

12TEN: Each of the statements is sufficient alone—choice D is correct.

By the way, we have seen that, when x=0 (which is inconsistent with statement one), 5^x=1. Choosing numbers demonstrates that increasing x always increases 5^x, and decreasing x always decreases 5^x. So the only way for 5^x to be less than 1 is for x to be less than zero—i.e., for x to be negative. So statement one actually means the same thing as statement two.

Remember that we started with statement two.

?: Is $\frac{5^{x+2}}{25} < 1$?

(S1) FA ?: yes

G: $5^x < 1$

CN $\cancel{x = 0}$

$5^x = 5^0 = 1$

CN $\cancel{x = 1}$

$5^x = 5^1 = 5$

CN $\cancel{x = \frac{1}{2}}$

$5^x = 5^{\frac{1}{2}} = \sqrt[2]{5^1} = \sqrt{5} > 1$

(S2) G: $x<0$

CN $x = -1$ ✓ ?: yes

$$\frac{5^{x+2}}{25} = \frac{5^{-1+2}}{25} = \frac{5^1}{25} = \frac{5}{25} = \frac{1}{5}$$

CN $x = -2$ ✓ ?: yes

$$\frac{5^{x+2}}{25} = \frac{5^{-2+2}}{25} = \frac{5^0}{25} = \frac{1}{25}$$

CN $x = -3$ ✓ ?: yes

$$\frac{5^{x+2}}{25} = \frac{5^{-3+2}}{25} = \frac{5^{-1}}{25} = \frac{\left(\frac{1}{5^1}\right)}{25} = \frac{\left(\frac{1}{5}\right)}{25} = \frac{\left(\frac{1}{5}\right)}{\left(\frac{25}{1}\right)} = \frac{1}{5} \div \frac{25}{1} = \frac{1}{5} \times \frac{1}{25} < 1$$

CN $x = -4$ ✓ ?: yes

$$\frac{5^{x+2}}{25} = \frac{5^{-4+2}}{25} = \frac{5^{-2}}{25} = \frac{\left(\frac{1}{5^2}\right)}{25} = \frac{\left(\frac{1}{25}\right)}{25} = \frac{\left(\frac{1}{25}\right)}{\left(\frac{25}{1}\right)} = \frac{1}{25} \div \frac{25}{1} = \frac{1}{25} \times \frac{1}{25}$$

CN $x = -20$ ✓ ?: yes

$$\frac{5^{x+2}}{25} = \frac{5^{-20+2}}{25} = \frac{5^{-18}}{25} = \frac{\left(\frac{1}{5^{18}}\right)}{25} = \frac{\left(\frac{1}{5^{18}}\right)}{\left(\frac{25}{1}\right)} = \frac{1}{5^{18}} \div \frac{25}{1} = \frac{1}{5^{18}} \times \frac{1}{25} < 1$$

CN $x = -\frac{1}{2}$ ✓ ?: yes

$$\frac{5^{x+2}}{25} = \frac{5^{-\frac{1}{2}+2}}{25} = \frac{5^{\frac{3}{2}}}{25} = \frac{\sqrt[2]{5^3}}{25} = \frac{\sqrt{125}}{25} < \frac{25}{25} = 1$$

12TEN: Each of the statements is sufficient alone—choice D is correct.

It is helpful to notice that, when x=0 (which would be inconsistent with both statements), $\frac{5^{x+2}}{25}=\frac{5^{0+2}}{25}=\frac{5^2}{25}=\frac{25}{25}=1$. From the numbers we have chosen, we can see that, the more negative x becomes, the smaller $\frac{5^{x+2}}{25}$ becomes. So, for **any** negative x, $\frac{5^{x+2}}{25}$ will be less than 1. So, for **any** negative x, the answer to the Question will be “yes”.

Here is the algebraic approach to D33:

$?:\ \text{Is}\ \frac{5^{x+2}}{25}<1?$	Let’s try to rephrase this Question.
$?:\ \text{Is}\ \frac{5^x5^2}{25}<1?$	$p^a p^b = p^{a+b}$, so $p^{a+b} = p^a p^b$, so $5^{x+2}=5^x5^2$.
$?:\ \text{Is}\ \frac{5^x(25)}{25}<1?$	
$?:\ [a]\ \text{Is}\ 5^x<1?$	Cancel the 25’s from the numerator and denominator.

Rephrasing the Question as [a] should make it obvious that there is only one answer to the Question (“yes”) that is consistent with statement one, so statement one is sufficient.

Rephrasing the Question as [a] should simplify the process of choosing numbers to evaluate statement two. Or, you can do more algebra:

$?:\ \text{Is}\ 5^x<5^0\ ?$	From [a]. p^0=1, so 1=5^0. When you have a variable exponent, it is helpful to express everything as a power of the same base. In this case, we have expressed both sides of the inequality as a power of the base “5”.
$?:\ [b]\ \text{Is}\ x<0?$	Increasing x always increases 5^x, and decreasing x always decreases 5^x. Therefore, the only way for 5^x to be less than 5^0 is for x to be less than 0. You can see how expressing everything as a power of the same base made it easier to compare the exponents.

Rephrasing the Question as [b] should make it obvious that there is only one answer to the Question (“yes”) that is consistent with statement two, so statement two is sufficient.

Here is a summary of the exponent rules that it might be useful to know for D33:

p^0=1, so 1=p^0.
$p^{-q}=\frac{1}{p^q}$
$p^{\frac{q}{r}}=\sqrt[r]{p^q}$
$p^a p^b = p^{a+b}$, so $p^{a+b} = p^a p^b$.

■

34.

D34 is an Overlapping Sets problem—use an Overlapping Sets Table:

	computer	**no computer**	**total**
writing	20%		
no writing		?	
total			100%

Each cell is a part:whole ratio (expressed as a percentage), where the “whole” is *all* the companies in the survey.

The Question is asking for a part:whole ratio, where the whole is all the companies in the survey, so we can build the Question directly into the table.

The initial given gives a part:whole ratio (20%), where the whole is all the companies in the survey, so we can build the initial given’s ratio directly into the table.

Statement two is simpler and slightly shorter—start with statement two.

(S2)

	computer	**no computer**	**total**
writing	20%	45%	
no writing		?	
total			100%

The relevant givens are the initial given and statement two.

Make a new table for statement two. Statement two gives a part:whole ratio (45%), where the “whole” is all companies in the survey, so we can build statement two’s ratio directly into the table.

I (S2)

	computer	**no computer**	**total**
writing	20%	45%	65%
no writing		?	35%
total			100%

The answer to the Question could be anything from 0% to 35%. So there is more than one answer to the Question that is consistent with all the relevant givens. So statement two is insufficient.

	computer	**no computer**	**total**
writing	20%		
no writing		?	
total			100%

Statement two is insufficient, so there is no free answer. The relevant givens are now statement one and the initial given—ignore statement two. Draw a new table.

Statement one gives a part:whole ratio (½), but the "whole" is all the companies *that required computer skills*. We are using a different "whole" in our table (*all* the companies), so we cannot enter the ½ ratio *directly* into the table.

(S1)

	computer	**no computer**	**total**
writing	20%		
no writing	20%	?	
total	40%		100%

Statement two says that of all companies *that require computer skills*, ½ require writing skills, which implies that ½ do *not* require writing skills. Therefore, the table entry for companies that require computer *and* writing skills should be *equal* to the table entry for companies that require computer skills but *don't* require writing skills.

I (S1)

	computer	**no computer**	**total**
writing	20%		
no writing	20%	?	
total	40%	60%	100%

The answer to the Question could be anything from 0% to 60%. So there is more than one answer to the Question that is consistent with all the relevant givens. So statement one is insufficient.

Since both statements are insufficient alone, we have to evaluate the statements together. The relevant givens are now the initial given, statement one, and statement two.

S (tog)

	computer	**no computer**	**total**
writing	20%	45%	65%
no writing	20%	?	35%
total	40%	60%	100%

Build all the information from statements one and two into the table.

	computer	**no computer**	**total**
writing	20%	45%	65%
no writing	20%	?: 15%	35%
total	40%	60%	100%

There is exactly one answer to the Question ("15%") that is consistent with all the relevant givens. So the statements together are sufficient.

12TEN: The statements are only sufficient together—choice C is correct.

Remember that we started with statement two.

	computer	**no computer**	**total**
writing	20%		
no writing		?	
total			100%

I (S1)

	computer	**no computer**	**total**
writing	20%		
no writing	20%	?	
total	40%	60%	100%

I (S2)

	computer	**no computer**	**total**
writing	20%	45%	65%
no writing		?	35%
total			100%

S (tog)

	computer	**no computer**	**total**
writing	20%	45%	65%
no writing	20%	?: 15%	35%
total	40%	60%	100%

12TEN: The statements are only sufficient together—choice C is correct.

Use Overlapping Sets Tables when you have two overlapping sets, set A and set B, and people can be in one set, the other set, neither, or both. Try to build the Question and givens into your table.

When the cells of your Overlapping Sets Table represent part:whole ratios, the “whole” should be *all* the people involved in the table. If you are given a ratio that uses a smaller “whole” than your table (such as statement one for D34), you will not be able to enter that ratio *directly* into your table. Instead, you will have to translate the ratio involving the smaller whole into a ratio that involves the same whole as your table.

right way to construct an overlapping sets table:

	computer	**no computer**	**total**
writing			
no writing			
total			

wrong way to construct an overlapping sets table:

	computer	**writing**	**total**
no computer	*		
no writing		**	
total			

*This cell doesn’t make sense—how could a company both require computer skills and not require computer skills?

**This cell doesn’t make sense—how could a company both require writing skills and not require writing skills?■

Note

From this point on, to avoid infringing GMAC's copyright, I will avoid copying the Question and givens into this document. Please remember, however, that in your own scratchwork for data sufficiency, I recommend that you should almost always write down the Question, and that you should usually write down the givens as well.

35.

The best way to solve D35 is by using the algebraic approach described in the *Official Guide*'s explanation. However, if you couldn't come up with the algebraic approach on your own, you could also have solved the problem by choosing numbers:

(S1) G: [1a] $3w=3-3q$ $q=0$	The only relevant given is statement one. 1. **Choose** a number for q. It's good to choose "0".
$q=0$ $3w=3-3(0) \Rightarrow 3w=3 \Rightarrow w=1$	Use the number you chose for q, together with statement one, to **determine** a number for w.
$q=0$ ✓ $3w=3-3q \Rightarrow 3w=3-3(0) \Rightarrow 3w=3 \Rightarrow w=1$	2. Are your numbers consistent or inconsistent with the relevant given? Since we used statement one to determine w, we **already** know that our numbers are consistent with statement one—there is no need to plug our numbers back into [1a]. On to step three.
$q=0$ ✓ (?: 1) $3w=3-3(0) \Rightarrow 3w=3 \Rightarrow w=1$ $w+q=1+0=1$	3. Answer the Question. Label your work with "$w+q=$".
$q=1$	4. **Choose** a new number for q, trying to get a different answer than "1". It's good to choose "1".
$q=1$ $3w=3-3(1) \Rightarrow 3w=3-3 \Rightarrow 3w=0 \Rightarrow w=1$	Use the number you chose for q, together with statement one, to **determine** a number for w.
$q=1$ ✓ (?: 1) $3w=3-3(1) \Rightarrow 3w=3-3 \Rightarrow 3w=0 \Rightarrow w=0$ $w+q=0+1=1$	2. Are your numbers consistent or inconsistent with the relevant given? Consistent: On to step three. 3. Answer the Question

If you continue choosing numbers, you will not be able to find any numbers that are consistent with statement one that give a different answer to the Question than "1". So eventually you should stop choosing numbers and conclude that there is only one answer to the Question ("1") that is consistent with statement one. So statement one is sufficient.

S (S1)

Now we evaluate statement two. The only relevant given is statement two; ignore statement one.

(S2) FA (?: 1) [2a] G: $5w+5q=5$	Since statement one was sufficient, it gave us a free answer.
$q=2$	1. **Choose** a number for *q*. There is no point reusing our numbers from statement one, since they would simply give us another answer of "1".
$q=2$ $5w+5(2)=5 \Rightarrow 5w+10=5 \Rightarrow 5w=-5$ $\Rightarrow w=-1$	Use the number you chose for *q*, together with statement two, to **determine** a number for *w*.
$q=2$ ✓ $5w+5(2)=5 \Rightarrow 5w+10=5 \Rightarrow 5w=-5$ $\Rightarrow w=-1$	2. Are your numbers <u>consistent</u> or <u>inconsistent</u> with the relevant given? Since we used statement two to determine *w*, we **already** know that our numbers are <u>consistent</u> with statement two—there is no need to plug our numbers back into [2a]. On to step three.
$q=2$ ✓ (?: 1) $5w+5(2)=5 \Rightarrow 5w+10=5 \Rightarrow 5w=-5$ $\Rightarrow w=-1$ $w+q=-1+2=1$	3. Answer the Question. Label your work with "$w+q=$".

If you continue choosing numbers, you will not be able to find any numbers that are consistent with statement two that give a different answer to the Question than "1". So eventually you should stop choosing numbers and conclude that there is only one answer to the Question ("1") that is consistent with statement two. So statement two is sufficient.

S (S1) $q=0$ ✓ (?: 1) $3w=3-3(0) \Rightarrow 3w=3 \Rightarrow w=1$ $w+q=1+0=1$ $q=1$ ✓ (?: 1) $3w=3-3(1) \Rightarrow 3w=3-3 \Rightarrow 3w=0 \Rightarrow w=0$ $w+q=0+1=1$	S (S2) FA (?: 1) $q=2$ ✓ (?: 1) $5w+5(2)=5 \Rightarrow 5w+10=5 \Rightarrow 5w=-5$ $\Rightarrow w=-1$ $w+q=-1+2=1$

12TEN: Each of the statements is sufficient alone—the correct choice is D.

The *Official Guide* describes the best algebraic approach to D35: rephrase the givens so they look like the Question ("$w+q$"). However, even if you can't figure out how to make the statements look like the Question, you should still be able to use algebra to evaluate the statements by solving the equations.

(S1) [1a] $w = \frac{3-3q}{3}$	Solve statement one for w by dividing both sides by 3.
$\Rightarrow$ [1b] $w = \frac{3(1-q)}{3}$	Factor a 3 out of the numerator. (If you apply the distributive law to the numerator for [1b], you will get the numerator for [1a]. This proves that [1b] is equivalent to [1a].)
$\Rightarrow$ [1b] $w = 1-q$	Cancel a 3 out of the numerator and denominator.
S (S1) $\Rightarrow w + q = (1-q) + q$ $= 1 - q + q$ $= 1$ (?: 1)	Use [1b] to substitute for w. We have used algebra to prove that there is exactly one answer to the Question ("1") that is consistent with statement one. So statement one is sufficient.

(S2) FA (?: 1) $5w = 5 - 5q$	Since statement one was sufficient, it gives us a free answer ("1"). We will solve statement two for w. We need to get w by itself. Begin by subtracting $5q$ from both sides.
$\Rightarrow w = \frac{5-5q}{5}$	Continue isolating w by dividing both sides by 5.
$\Rightarrow w = \frac{5(1-q)}{5}$	Factor a 5 out of the numerator.
$\Rightarrow$ [2a] $w = 1-q$	Cancel a 5 out of the numerator and denominator.

Since [2a] is the same as [1b], we have proven that statement two is equivalent to statement one. Therefore, since statement one is sufficient, statement two must also be sufficient.

S (S2)

S (S1) $w = \frac{3-3q}{3}$ $\Rightarrow w = \frac{3(1-q)}{3}$ $\Rightarrow w = 1-q$ $\Rightarrow w + q = (1-q) + q$ $= 1 - q + q$ $= 1$ (?: 1)	S (S2) FA (?: 1) $5w = 5 - 5q$ $\Rightarrow w = \frac{5-5q}{5}$ $= \frac{5(1-q)}{5}$ $= 1-q$

12TEN: Each of the statements is sufficient alone—the correct choice is D.

Best approach for D35: Rephrase the givens to look like the Question. If you can't see how to do that, solve the equations. If you can't see how to do that, choose numbers.

36.

… coming soon …